# Professional Learning Communities for Teacher Development: The Collaborative Enquiry Process in the Data-Informed Practice Improvement Project (DIPIP)

School of Education, University of the Witwatersrand

Funded by the Gauteng Department of Education

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## Overview of report

The context of this project conducted by the School of Education at the University of Witwatersrand (WSoE) in partnership with the Gauteng Department of Education (GDE) in South Africa is the emerging use of test data for purposes of accountability. The Delivery Agreement for Improved Basic Education notes that a "number of international testing programmes have demonstrated that South Africa's learner performance in reading, writing and mathematics is well below what it should be" (Department of Basic Education 2011, p.10). As a result of this, the Minister of Basic Education is accountable to the President for the improvement of the quality of teaching and learning, and specifically, improvement in the results of the National Senior Certificate and the Annual National Assessments in Grades 3, 6, and 9. However, research on accountability (Elmore, 2000) confirms that without internal accountability in schools, it is unlikely that measures for external accountability such as testing will make a significant difference. One of the most important elements of internal accountability is systematic and collaborative reflection of teachers and school leaders on the teaching and learning processes. A key strategy for facilitating this is to stimulate collaborative enquiry in "professional learning communities" in which teachers and "critical friends" (those with expertise and experience but not directly involved in classroom teaching) engage with the meaning of systemic test data, and how it can be used to inform practice.

The goal of the Data-Informed Practice Improvement Project (DIPIP) research and development project was to create a context for professional conversations in which mathematics teachers, together with university academics, graduate students, and Education Department-based subject advisors, discussed what information test data provided which could be used to think about reasons for learner errors and how these might be addressed through collaborative lesson planning, teaching and reflection. The project aimed to develop and study teachers' capacity to understand, decode and act upon learners' errors when marking and when teaching. It achieved this empirically by

1. designing a sequence of activities to structure teachers' participation in analysing learners' errors in different teaching contexts;
2. devising tools to analyse teachers' understanding of learners' errors across the different activities.

The project, carried out for three years from 2007 to 2010 on a small, but intensive scale, yielded new insights into how to understand and work with learners' errors in mathematics classrooms. At the same time it tested a model of teacher development in which teachers worked in professional learning communities to research/conduct inquiries on their own practice.

This project report addresses the first project outcome:

> A documented, collaborative enquiry process through which academics, subject facilitators and school teachers together discuss what data suggests about reasons for learner errors and how the insights gained from such discussion might be addressed through joint lesson planning and reflection.

In addition to describing the project aims, context and approach, implementation strategy and activities, the report presents the criteria which were used to analyse the data generated during the project.

The full evaluation report will address the other project outcomes:

Evidence of

- depth of teachers' reasoning about and response to learners' errors;
- teachers' awareness of their own learning and learning needs;
- categories of diagnostic judgement derived from teachers' reasoning about learners' errors.

It will also contain a set of reflections on collaborative enquiry in professional learning communities derived not only from the experience of the project team in this project, but from the engagement of the external evaluators with project participants.

# Project implementation process

## Description of the project and its aims

The Data-Informed Practice Improvement Project (DIPIP) was a three-year research and development programme for 62 mathematics teachers from Grades 3 to 9 from a variety of Johannesburg schools. The project worked with the teachers both to design and to reflect on lessons, tasks and instructional practices, and build professional learning communities. It focused on building teachers' understandings of learners' errors, both more generally and in terms of particular topics. A focus on errors as evidence of reasonable and interesting mathematical thinking on the part of learners can help teachers to understand learner thinking (Borasi, 1994; Nesher, 1987; Smith, DiSessa, & Roschelle, 1993), adjust the ways they engage with learners in the classroom situation, as well as revise their teaching approach. The ultimate goal is improved learner performance.

To provide a basis for systematic analysis of learners' errors, the results of Gauteng learners on an international standardized, multiple-choice test, the ICAS test, were used. International Competitions and Assessments for Schools (ICAS) is conducted by Educational Assessment Australia (EAA), University of New South Wales (UNSW) Global Pty Limited. Students from over 20 countries in Asia, Africa, Europe, the Pacific and the USA participate in ICAS each year. EAA produces ICAS papers that test students in a range of subject areas including Mathematics. Certain schools in Gauteng province used the ICAS tests in 2006, 2007 and 2008, and it was the results of learners from these schools on the 2006 and 2007 tests that provided the starting point for teacher engagement with learner error.

The project set up the following activities:

1. analysis of learner results on the ICAS mathematics tests (with a focus on the multiple choice questions and the possible reasons behind the errors that led to learner choices of the distractors[1]);
2. mapping of ICAS test items in relation to the South African mathematics curriculum;
3. readings and discussions of texts about learners' errors in relation to two central mathematical concepts;
4. drawing on the above three analyses, development of lesson plans for between one and five lessons which engage with learners' errors in relation to the central mathematical concepts;
5. reflections on videotaped lessons of some teachers teaching from the lesson plans;

[1] "Distractors" are the three or four incorrect answers in multiple choice test items. They are designed to be close enough to the correct answer to 'distract' the person answering the question.

6. design of a test on one of the central mathematical concepts, analysis of learners' errors on this test and an interview with one learner to probe his/her mathematical reasoning in relation to errors made on the test.

The results of the project are being analysed in an evaluative research report, externally validated by the South African Institute for Distance Education (***Saide***).

## Project approach

**Responding to the increased concern for accountability**

During the last decade of educational transformation in South Africa, the national Department of Education focused on a number of options to improve education. These included redressing past social and economic inequalities, improving efficiency, increasing resource allocations, building capacity, and developing and phasing in three versions of a new curriculum. As access and redress started improving, there were expectations of improved quality of education and learner attainment.

As a policy lever for benchmarking standards and for monitoring performance, the Education Department has embarked on a number of initiatives to collect data. International and local comparative evaluations have been conducted including: Trends in International Mathematics and Science Study (TIMSS, 1995, 1999, 2003); Systemic Evaluation for grades 3 and 6 in Numeracy and Literacy; Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), the Progress in International Reading Literacy Study (PIRLS) and so on.

Not enough emphasis is placed, however, on the potential value of the data available from these systemic evaluations for informing teaching and learning practices. International research shows that merely having another set of data in the form of benchmarking, targets and progress reports that 'name and shame' schools leads to resentment and compliance but not to improvement of learning and teaching (McNeil, 2000; Earl and Fullan; 2003, and Fuhrman and Elmore, 2004).

In South Africa, Kanjee (2007, p. 493) sums up the challenge:

> For national assessment studies to be effectively and efficiently applied to improve the performance of all learners, the active participation of teachers and schools is essential. … Teachers need relevant and timeous information from national (as well as international) assessment studies, as well as support on how to use this information to improve learning and teaching practice. Thus a critical challenge would be to introduce appropriate polices and systems to disseminate information to teachers. For example, teacher-support materials could be developed using test items administered in national assessments.

International research has begun to engage with the question of how to use test data beyond benchmarking (Earl and Fullan, 2003: Earl and Katz, 2005, Katz et al., 2009). Katz et al (2005) draw an important distinction between two very different kinds of practices in benchmarking. They draw a distinction between "accounting", which is the practice of gathering and organising of data and "accountability", which refers to teacher-led educational conversations about what the data means and how it can inform teaching and learning. This develops Elmore's important argument that teachers can be held to account for their performance **only if** they have a deep sense of the demands made upon them. Although this may seem obvious, the challenge lies in what counts as making standards accountability explicit.

Curriculum statements about assessment standards together with results of various standardised assessments, do not, in themselves, make standards clear. In themselves, they do not and cannot disclose what instructional practice should look like. Hence Katz et al are correct when they say (2009, p.28):

> Data don't "tell" us anything; they are benign... The meaning that comes from data comes from interpretation, and interpretation is a human endeavour that involves a mix of insights from evidence and the tacit knowledge that the group brings to the discussion. ...

For Hargreaves, focussing teachers' learning from data is important for building collegiality. He argues (2001, p.524) that the future of collegiality may best be addressed by (inter alia)

> taking professional discussion and dialogue out of the privacy of the classroom and basing it on visible public evidence and data of teachers' performance and practices such as shared samples of student work or public presentations of student performance data

The key issue is how to transform data collected from testing into structured learning opportunities for teachers. This raises three questions:

- In what ways can teachers be involved in analyzing public evidence?
- What counts as a "defensible focus" for this kind of professional development (Katz et al, 2009)?
- In what ways can analysis of data be integrated into the work that teachers do in school?

**Professional learning communities**

The positive outcomes of research done on the efficacy of "Professional Learning Communities" (PLCs) and the related concept of "Networked Learning Communities", served to inform the approach used in this project. The term professional learning communities generally refers to structured professional groups, usually school-based, providing teachers with opportunities for processing the implications of new learning (Timperley et al, 2007, p.201). Commonly PLCs are created in a school and consist of school staff members or a cross section of staff members from different schools in a specific area of specialisation. "Networked Learning Communities", by contrast (Curriculum Services Canada, 2008, p1):

> provide educators with opportunities to interact with each other within the boundaries of their own schools and boards or far beyond those traditional boundaries.

The groups used in the DIPIP project had some elements of each.

- The small groups consisted of a group leader (a mathematics specialist – Wits School of Education staff member or post graduate student who could contribute knowledge from outside the workplace), a Gauteng Department of Education (GDE) mathematics subject facilitator/advisor and two or three mathematics teachers (from the same grade but from different schools). This meant that the groups were structured to include different authorities and different kinds of knowledge bases. These were called **small grade-level groups** (or small groups). As professional learning communities, the groups worked together for a long period of time (weekly meetings during term time at the Wits Education Campus for up to three years), sharing ideas and learning from each and exposing their practice to each other. In these close knit communities, teachers worked collaboratively both on curriculum mapping/error analysis, lesson and interview planning, test setting and reflection.

- For certain tasks (such as presenting lesson plans, video clips of lessons taught or video clips of learner interviews) the groups were asked to present to **large groups**. A large group consisted of the grade-level groups coming together into larger combined groups, each consisting of four to six small groups (henceforth the large groups). This further expanded the opportunities for learning across traditional boundaries.

**Evidence-based learning**

An "Evidence-based Learning" approach was adopted. This approach was used by the various groups to:

- identify learners' errors that occur during *teaching* and *assessment*;
- identify and develop strategies for addressing these errors in teaching and assessment (application of learning);
- identify categories of diagnostic judgement derived from teachers' reasoning about learners' errors.

The ICAS tests provided the evidence base for error analysis to develop the teachers' diagnostic abilities. The test items as well as the results of Gauteng learners on the tests were available for analysis.

The key features which informed the rationale for the selection of the various activities in the project are:

- **Learners' needs inform teachers' learning needs**: This idea is based on recent literature in the field of teacher professional learning and development. The main argument emerging in this literature is that there is a positive (although not conclusive) relationship between teacher learning and student learning. In view of this, the project intended to provide data for teachers on student learning (in the form of tests results, for example) and structure activities for the teachers around this data with the intention that they draw on the data and activities to refine for themselves what they need to learn.
- **Learners' mathematical errors are viewed as reasoned rather than random, and as such, error analysis can inform teaching and provide a focus for teacher development:** This idea comes from decades of research in mathematics education which suggests that many of learners' errors are underlined by conceptual misunderstandings (or misconceptions). This view of errors insists

that the role of teachers should not only be to correct errors, but to understand the reasoning behind the errors that learners make. Then errors can be used productively when teaching.

- **Reflection on assessment and classroom teaching data provides the evidence for learner errors and learner needs:** Working with teachers on classroom teaching is a very common activity in teacher development projects. Classroom teaching data is then used in variety of ways to analyse and develop teachers' teaching ability. Less common is working with teachers on test data to diagnose learners' learning. In this project these two sets of data were combined sequentially into a series of activities. In this project we intended to help teachers focus their classroom teaching (planning and enacting) on what the error analysis has shown them.

## Programme implementation

### Teacher selection

The initial selection of teachers for participation in the project was guided by the Gauteng Department of Education. In particular, teachers from "better performing schools" that had participated in the ICAS tests were selected. However as various teachers dropped out of the project, they were replaced by other mathematics teachers selected from schools with easy access to the Wits education campus.

Over the three year period, a total of 62 teachers participated. Some teachers left during the project and new ones joined. At the end of the project there were 42 teachers, of whom six had participated since the project's inception in September 2007 and 31 had joined at the beginning of 2008. Four joined in 2009. A core of 37 teachers were thus part of the project for two or more years.

### GDE district mathematics facilitators/subject advisors

In total, 21 facilitators participated in the project at various times during its implementation, the majority (15) participated for between three and ten months and only six participated in an on-going way for a period of about two years. The reason for the shorter period of participation for the majority of the facilitators was that they were only appointed to their offices late in 2009, after which they joined the project.

### Group leaders from the Wits School of Education

A total of 23 group leaders were involved in facilitating the groups at various points during the period of the project implementation. The majority (14) were involved in the project for between a year-and-a-half and two years. The remainder (9) participated for between three and twelve months.

### Wits School of Education project management

The project was coordinated at both management and conceptual levels by a team of four project leaders – two professors, and two part-time academics appointed specifically for this project. The project managers were also responsible for training and supporting the group leaders.

**Table 1 Participants' profile**

| | Teachers | GDE Maths Facilitators | Wits group leaders |
|---|---|---|---|
| **Position/ location** | From 31 Gauteng schools:<br>• 14 ex-model C schools[2],<br>• 14 township schools,<br>• 1 inner city school, and<br>• 2 independent/private schools. | From Gauteng districts:<br>• Ekurhuleni South District(1);<br>• Gauteng North District (3); Gauteng West District(1); Jhb Central District(3);<br>• Jhb South District(1); Sedibeng West District(1);Tshwane West District(2) | Wits teaching staff members and maths masters and doctoral students. |
| **Experience** | Of the 42 teachers that were part of the project when it ended in October 2010, the majority had extensive teaching experience:<br>• 19 had between 15 and 30 years<br>• 10 had between 10 and 14 years<br>• 4 had between 5 and 9 years ,and<br>• 4 had between 1 and 4 years. | Facilitators' teaching experience:<br>• one with less than 10 years<br>• 5 with between 10 and 15 years of experience<br>• 4 with between 17 and 21 years<br>• 2 with 32 years of experience. | Typically most of the group leaders had between 15 – 30 years of school teaching experience. |
| **Role in group** | Participant, with the following responsibilities:<br>• Attend weekly meetings<br>• Voluntarily help with group record taking, writing up lesson plans and reflections.<br>• Take part in group activities such as videoed teaching sessions in their schools, learner testing and interviews. | Participant, with the following responsibilities:<br>• Attend weekly meetings<br>• Voluntarily help with group record taking, writing up lesson plans and reflections. | Facilitator, with responsibility for one group<br>Supported by six hours of training by the Wits project management team before three key points in the project:<br>• curriculum mapping;<br>• error analysis; and<br>• the first session of reflections on lesson presentation videos. |

### Project duration, contact time and arrangements for meetings

The groups met weekly during term time for two hours per week. In total, the project ran for 86 weeks from September 2007 to October 2010. This amounts to 172 hours of contact time for the teachers. The venue for the project was centralised: the Wits Education Campus in Parktown was used for all project meetings.

Other than three readings which the group participants were required to go through independently (estimated to be approximately 6 hours) the participants were not required to do any additional work outside of the group meetings. On a voluntary

[2] Formerly 'whites-only' schools in the suburbs which opened up to other racial groups in 1991 (shortly before the change of government in South Africa)

basis certain group members did do other additional work related to their DIPIP groups at home (such as writing up lesson plans, making notes of error analysis discussions or completing records of any other group discussions that were not completed during the two hour meeting period).

The teachers and GDE Facilitators received an honorarium and travel expenses were paid for each session attended. Lunch was provided at all sessions. The group leaders were also paid for their facilitation of the groups.

**Activities to widen access**

Apart from reaching the 62 teachers who participated weekly in the group activities at the university, the project widened access to the project in the following ways:

1. Six newsletters were produced by the mathematics teacher educators at the university, and distributed to all schools in the province by the Gauteng Department of Education. These newsletters, with permission of the Gauteng Department of Education are available as Open Educational Resources at www.oerafrica.org/teachered . The newsletters were designed to bring some of the DIPIP learning experience to teachers in the province. They focused on misconceptions relating to different topics which surfaced from the 2006 ICAS tests. The focus topics were:
    a. Newsletter 1: Ratio and rate
    b. Newsletter 2: The equal sign
    c. Newsletter 3: Solution of equations
    d. Newsletter 4: Subtraction
    e. Newsletter 5: Spatial reasoning in the primary school
    f. Newsletter 5: Spatial reasoning in the high school
2. Four seminars were organized to publicize the project.
    a. Official launch of the project on 4 November 2008.
    b. Presentation on the project to the Wits School of Education on 20 April 2010.
    c. Report on the DIPIP project to the Gauteng Department of Education Provincial Assessment Team on 28 July 2010.
    d. Final meeting of all participants and interested parties on 25 November 2010 – an open meeting with general reflection on the DIPIP project by participants.

## The activities: a full description

The project focus on using learner tests (both ICAS and own tests) brought learners' errors in those tests into the centre of most of the group discussions. It should be noted that some activities were more directly error focused (error analysis, learner interviews and large group presentations) while others were error related since they built on thinking around learners' errors but were not focused only on the errors in learners work and how teachers dealt with these errors (curriculum mapping, lesson planning and test setting).

The tables that follow describe all of the project's activities, the material selected for each activity (such as readings, templates and guidelines given to groups for completion of an activity), the time allocated and the group format in which the activity and its constituent tasks were enacted. This should help the reader to get a full overview of the process.

**Activity One: Mapping of ICAS 2006 test items against the National Curriculum Statement (NCS)**

14 weeks: February to May 2008

The purpose of this activity was to analyse the ICAS test items to establish what assessment standards in the South African curriculum learners would need to have met before they would be able to answer the item correctly. The mapping activity involves a circular process, starting with a test item, moving to the NCS, back to the test item, and then back to the NCS. The activity structured a context for professional conversations in the small group about what the ICAS test data (the examined curriculum) means, how it aligns with the conceptual demands of the NCS (the intended curriculum), and how it fits with teachers' professional knowledge and experience (the enacted curriculum).This circular process was intended to help the small groups go deeper into analysing and discussing the curriculum, and through these discussions strengthen their understanding of the item and their knowledge of the curriculum. The purpose was not only to ascertain the match between and international test and the South African curriculum, but to provide an opportunity for teachers to use the test items as a spur to understand the conceptual base of their own curriculum more deeply.

**Table 2: Activity One – Round 1 Curriculum mapping**

| Material | Tasks | Group type |
|---|---|---|
| Exemplar templates<br>Completed mapping document | Group leader training (by project management team):<br>• A role-play on working with the template.<br>• All group leaders used the template to map two items collectively.<br>• Group leaders in grade-specific pairs worked on mapping some items from their own grade.<br>• Group leaders asked to think about how they would probe the teachers for the mathematical concepts underlying the test items and the assessment criteria. | All group leaders with project team |
| ICAS 2006 test items<br>A modified NCS (curriculum) document developed by the RADMASTE Centre (Scheiber, 2005)<br>Curriculum mapping template | Map ICAS items to the NCS by completing a template:<br>• Identify mathematical concepts being tested by the ICAS item,<br>• Find the relevant NCS assessment standard(s) relating to these concepts,<br>• Give reasons for choice, and<br>• State when/if teach the content, and whether teach it | 14 grade-specific small groups |

| Material | Tasks | Group type |
|---|---|---|
| Exemplar of completed mapping | directly or indirectly. | |

**Activity Two: Analysis of learners' errors in ICAS 2006 tests**

10 weeks: July to October 2008

The purpose of this activity was to discuss the mathematical reasoning that is required to select the correct option in the ICAS multiple choice test items, as well as to provide explanations for learners' choices of each of the distractors. Groups needed to provide several explanations for the choices learners made in order to deepen their conceptual understanding and develop an appreciation of learners' errors and a more differentiated understanding of reasons underlying the errors.

**Table 3: Activity Two – Round 1 Error analysis**

| Material | Tasks | Group type |
|---|---|---|
| Exemplar templates<br>Guidelines for the completion of the error analysis template | Group leader training:<br>• Discussion of error analysis of selected items to generate completed templates for teacher groups to work with | All group leaders with project team |
| Test items from 2006 ICAS test with statistical analysis of learner performance for correct answer as well as 3 distractors<br>Error analysis template<br>Two exemplars of completed error analysis | Error analysis of learner performance using template:<br>• Identification of what enabled learners to give the correct response<br>• Provision of possible explanations for learners' choice of each of the three distracters.<br>The focus was on multiple choice items because the test designers provided statistical analysis of learner responses for these, although some groups also discussed the open ended items. | 14 grade-specific small groups |

Teachers were allowed to reflect on the process of both curriculum mapping and error analysis through focus group interviews that followed after the completion of the first two activities. The interviews took the form of an open discussion facilitated by the DIPIP project leaders, guided by an interview questionnaire. Participants focused on their participation in the curriculum mapping and error analysis activities completed to date.These interviews also probed how their teaching had been affected by the process. Two large groups were interviewed – one comprising the grades 3 to 6 small groups and the other the grades 7 to 9 small groups.

**Activity Three: Round 1 of lesson planning and teaching on a central mathematical concept**

22 weeks: November 2008 to August 2009

The concept of the Equal Sign was selected as a focus for this series of tasks. It was felt that the concept can be operationalized for any specific mathematical content in the maths curriculum. The selection of this concept was also informed by the relevant maths education literature and from issues identified by small groups when completing their error analysis activity. Whereas the equal sign is used in maths to show relationships between quantities or values (equivalence), learners from primary school interpret the equal sign as a 'find the answer' symbol (1 + 1 = ?). This interferes with their understanding as they proceed with solving of equations in secondary school algebra (solve for 'x', where x could appear on the left or the right of the equal sign).

The purpose of the activity was to deepen teachers' understanding of a concept that is not directly taught but is often misunderstood and can lead to learners' errors. Through collaborative lesson planning and reflection on teaching experience, teachers not only analysed learners' errors, but also reflected critically on practical examples of how to engage learners' errors in classroom situations. The input of maths teacher educators from the university as well as comments from peers in large group presentations assisted with this process.

**Table 4: Activity Three – Round 1 Teaching**

| Material | Tasks | Group style |
|---|---|---|
| Pre reading probing questions guide | Preparing for teaching the Equal Sign<br>Teachers were asked to reflect on:<br>• the different meanings of the equal sign<br>• the different ways the equal sign can be used in different exercises<br>• learners' understanding of the equal sign (including examples of different uses of the equal sign in number sentences)<br>• common errors in using the equal sign in a teacher's class<br>• examples of number sentences written by learners | 14 grade-specific small groups |
| Two readings on the Equal Sign[3]<br><br>Guiding questions for the readings on the equal sign | Pre-reading discussion, with guiding questions:<br>• What do the readings add to what groups feel they already know?<br>• Is the new information useful, informative for teaching, surprising?<br>• Are there any criticisms of the ideas in the readings?<br>• Do the assessment standards in the national curriculum address any of the issues about the equal sign raised in the readings?<br>• What are the explanations provided by the readings for learners' conceptions and misconceptions of the meaning of the equal sign?<br>• Does the group have any additions to make based on teaching/professional experience? | Small groups |

[3] Essien, A. & Setati, M. (2006). Revisiting the Equal Sign: Some Grade 8 and 9 Learners' Interpretations. *African Journal of Research in SMT Education, 10(1), 47-58.*

Molina, M. & Ambrose, R. C. (2006). Fostering Relational Thinking while Negotiating the Meaning of the Equal Sign. *Teaching Children Mathematics, Sept, 111-117*

| Material | Tasks | Group style |
|---|---|---|
| Guiding questions for post reading discussion | Post-reading discussion was structured around the following:<br>• Typical number sentences that teachers deal with in their grade (e.g. Flow diagrams, equations, word problems etc.).<br>• Possible misconceptions that may arise when learners learn about equations, number sentences and related concepts<br>• Understanding of the equal sign that learners need to acquire in a grade below or above the grade-level small group.<br>• Number sentences teachers can use to address learners' misconceptions about the equal sign | Small groups |
| Lesson planning guide | Collaborative lesson planning.<br>• Agreement on lesson plan format (based on existing templates/formats used by group members)<br>• Situation of the lesson in the school plan for the term<br>• Selection of one of the test items associated with the equal sign in their grade level ICAS test and linking of this to their first lesson plan.<br>• Searching for resources on the content area selected for the lesson plan.<br>• Development of lesson plan. | Small groups |
| Lesson planning presentation guideline | Lesson plan presentation by one or two group members to the large group<br>One or two people from the group were selected (by the group) to present a short overview of the lesson and explanation of one specific activity designed to address an assessment standard and the possible learners' misconceptions that might arise in the course of the lesson in relation to the chosen mathematical content. The presentation included:<br>• The concepts focussed on.<br>• How the teacher who will teach the lesson will present the concept.<br>• The assessment standard and the ICAS test item(s) selected for planning the lesson.<br>• The misconception(s) the lesson is aiming to address.<br>• How the lesson is intending to address the misconception(s).<br>• Overview of the activities selected for the lesson, and the pacing<br>• Problems the teachers expect to face when teaching the lesson.<br>Feedback on each presentation was given by the DIPIP project leader, group leaders as well as individual large group members. | Three different large groups:<br>Grades 3 and 4<br>Grades 5 and 6<br>Grades 7, 8 and 9 |
| Interview questionnaire | Focus group interview carried out after Round 1 teaching, guided by the following: | Two large groups<br>Grades 3 to 6 |

| Material | Tasks | Group style |
|---|---|---|
| | • What do the teachers and GDE facilitators think they have learnt from the lesson planning activity.<br>• What do they think the aim of the lesson planning activity was?<br>• In what ways did the previous activities (Curriculum mapping and error analysis) help them in planning the lesson?<br>• How different was the process of lesson planning from what they are used to in their schools and what would they change in view of the new experience?<br>• What did they learn about learners' errors?<br>• What new in mathematics did they learn?<br>• Were the readings useful?<br>• What part of the discussion in the presentation to the large group was specifically useful and why?<br>• Did the presentation help them understand better the lesson planned by other groups? | Grades 7, 8 and 9 |
| Verbal instructions to groups. | Lesson plan revision.<br>Revise lesson plans incorporating the feedback received from feedback notes coordinated by the group leader. | Small groups |
| Verbal instructions to groups. | Teaching lessons in schools.<br>Volunteers from each of the 14 groups taught the lesson to their class in their school.<br>The lessons were videotaped[4]. | Individual teachers |
| Exemplar video reflection templates<br>Exemplar lesson video | Lesson reflection Group leader training.<br>The video reflection templates were explained and the trialled by all group leaders. | All group leaders with project team |
| Lesson Reflection Guide | Lesson reflection in small groups.<br>Although one member of the group taught the lesson the reflection was done collaboratively, and focused on how the teacher dealt with the learners' understanding of what s/he taught, in relation to the equal sign.<br>In preparation for the presentation to the large group, groups were requested to select two episodes (see below) for a presentation.<br>Whilst watching the video the groups were requested to fill in a table. They needed to state:<br>• a time they were impressed by, surprised by, concerned with, unsure about learner(s) thinking and why<br>• something that the teacher did/did not plan and how that was dealt with during the lesson<br>• any idea that came up in the reflection and which the teacher did not think about at the time of the lesson<br>• A new challenge the teacher thinks s/he needs to take up | Small groups |

[4] Consent was obtained from all teachers and their principals, and all of the learners in the classes which were videoed, in accordance with ethical research requirements.

| Material | Tasks | Group style |
|---|---|---|
| | for her/his further development. | |
| Lesson teaching presentation guide | Lesson presentation – two five minute episodes from each group illustrating points at which:<br>The teacher dealt well with learner issues and misconceptions relating to the equal sign.<br>The teacher did not deal well with learner issues and misconceptions relating to the equal sign.<br>The presentation[5] was written up, and the presenter had to:<br>• situate the episode in the lesson,<br>• explain what preceded the episode,<br>• what was the main mathematical issue the teacher dealt with in that episode and<br>• justify the selection (why the group thought the teacher dealt well/badly with learner issues and misconceptions)<br>Feedback on lesson presentation was coordinated by the project Maths leaders | Small and large groups |
| A guide for written lesson reflection | Written reflection on lesson taught.<br>In addition to small groups presenting their reflections, the teacher that taught the lesson was requested to submit a written reflection. The individual reflections included ideas such as:<br>• Did the lesson go as expected, or not? If not what made the teacher divert from the plan and how did s/he cope with that?<br>• Was the teacher nervous, excited at the start of the lesson and does s/he feel that the learners picked that up?<br>• How did the learners relate to the activities planned for the lesson? (explain and give examples)<br>• What happened when the learners tried the activities?, Were there any surprises or concerns? (explain and give examples).<br>• Were there any specific constraints that prevented the teacher from enacting the plan? | Individual teachers |
| Interview schedule | Focus group interview with all volunteer teachers.<br>The interview aimed to probe<br>• the teachers' understanding of the difficulties and successes of the lesson, with specific reference to questions asked and mathematical conversations in the lesson<br>• broader experiences regarding her/his participation in DIPIP and its influence on her/his teaching. | Small groups without group leaders |

[5] Generally, the teacher that taught the lesson did the presentation, but in some cases other teachers from the group and the group leader assisted in the presentation

**Activity Four: Round 2 of lesson planning and teaching on a central mathematical concept**

19 weeks: May 2009 to February 2010

The topic "problem solving and visualisation" was selected because many of the 2006 ICAS items presented mathematical problems to learners which included visual information. It is commonly assumed that "visuals" add to the given verbal information or help learners to access the mathematical problems addressed by the questions. Research shows however that learners maybe distracted by the visual or by contextual information. This was also noted by teachers' explanations of learners' errors on the 2006 test.

The lesson collaboratively planned and reflected on in round two was to focus on helping learners improve their problem solving abilities, specifically in interpreting visual material in relation to the verbal information in a question.

**Table 5: Activity Four – Round 2 Teaching**

| **Material** | **Tasks** | **Group type** |
|---|---|---|
| Two Readings[6] on problem-solving and visualisation<br>Guiding questions for the readings | The guiding questions to the readings included:<br>• Ways in which the readings informed teachers in the group to think about helping learners decode contextual information and visuals.<br>• Ways that the teachers in the group have tried to engage learners in their class, helping them to read visuals, making links between visual and verbal information.<br>• Examining the visuals in selected ICAS tests items in the grade, and using the readings to understand the problem and ways of improving learners' access to the item or develop a conceptual understanding. | 14 grade-specific small groups |
| Verbal instructions to groups. | Lesson planning.<br>The groups were asked to start planning a lesson by<br>• selecting two to three ICAS items and concepts associated with them (as gathered from the curriculum mapping activity)<br>• looking at the error analysis they conducted for the items to help them understand the misconceptions involved in the items selected.<br>The guidance for collaborative lesson planning was similar to that provided in the first round of teaching. | Small groups |
| Lesson plan presentation guide | Lesson plan presentations.<br>The plan for and the format of the presentation was | Small and large groups |

[6] Whitin, P and Whitin DJ, (2008). Learning to solve problems in Primary Grades, in *Teaching Children Mathematics*, Vol. 14, No. 7, March 2008;

Murrey, DL, (2008). Differentiating Instruction in Mathematics for the English Language Learner, in *Mathematics Teaching in the Middle School*, Vol. 14, No. 3, October 2008.

| Material | Tasks | Group type |
|---|---|---|
| | similar to Round 1 lesson plan presentations. | |
| Verbal instructions to groups. | Lesson plan revision.<br>Based on feedback received at these reflection presentations and ideas from the individual group reflection sessions, teachers made changes to their lesson plans. | Small groups |
| Verbal instructions to groups. | Teaching in schools.<br>As far as possible, a different teacher to the one volunteered in Round 1 was approached to teach in Round 2. There were also certain teachers who were asked to teach for a second time. This served the purpose of getting broad participation but also enabled seeing teacher learning across different sets of videotaped lessons. | Individual teachers |
| Guideline for video reflection | Small group reflections on lesson taught.<br>The groups received the same guide as in Round 1, this time directed at misconceptions related to problem solving with visuals. The guide included instructions on the group process of reflection and the preparation for the presentation. | Small groups |
| Guideline for presentation | Lesson presentation of two episodes.<br>The groups received the same guide as in Round 1, this time directed at misconceptions related to problem solving with visuals. The guide included instructions on the group process of reflection and the preparation for the presentation. | Small and large groups |
| A guide for written lesson reflection | Written reflection on lesson taught.<br>In the same way as in Round 1, any teachers who taught in Round 2 were requested to write an individual reflection. The teachers received the same guide as in Round 1, this time directed at misconceptions related to problem solving with visuals. | Individual teachers |

**Activity Five: Consolidation - between Round 2 teaching and final phase activities**

1 week: March 2010

The purpose of this activity was to give teachers an opportunity to engage with both theoretical ideas and practical experience. They used their experience in the project thus far, and were given an opportunity to reflect on this experience in the light of a reading that brought together many of the issues that they had been discovering through the previous activities.

**Table 6: Activity Five – Bridging between Round 2 and final phase**

| Material | Tasks | Group type |
|---|---|---|
| Reading on use of errors in teaching[7]<br>Guidelines for reading analysis. | Final reading on use of errors in teaching.<br>Teachers were asked to use classroom videos and gather examples of learner errors from their classrooms to study.<br>Teachers were asked to consider things such as:<br>• What do errors in the learning and teaching of mathematics tell us?<br>• Are errors an indication that something is horribly wrong? Elaborate in relation to teaching and in relation to learning.<br>• Are errors an indication that something is going right? Elaborate in relation to teaching and in relation to learning.<br>• Can errors be used to grow understandings of mathematics and the discipline itself? In what ways? Give an example.<br>Groups were also asked to give descriptions of the way in which they would work with different types of errors in their classrooms. | 14 grade-specific small groups |

**Activity Six: Setting of own test and interview of learners after test**

14 weeks: March to August 2010

The purpose of this activity was to give the teachers experience in designing their own assessments, which they could then analyse in terms of learners' errors and misconceptions. Instead of hypothesizing about the reasons for learners' errors evident in a large scale systemic test, the teachers could explore with their own learners the reasons for their errors in the test.

Groups were asked to choose to set their test on misconceptions associated with the equal sign OR with problem solving and visuals.

**Table 7: Activity Six – Setting own test and conducting learner interviews**

| Material | Tasks | Group type |
|---|---|---|
| Guideline on test setting | Setting a test<br>Groups were asked to<br>• design a 6 item test (not more than 3 items to be multiple choice).[8]<br>• explain the rationale for the test (the concepts and procedures selected as a test focus, their relevance to the curriculum and the misconceptions anticipated) | 11 small groups<br>(2 Grade 3<br>1 Grade 4<br>2 Grade 5<br>1 Grade 6<br>1 Grade 7 |

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[7] Borasi, R. (1987) Exploring Mathematics through the Analysis of Errors, in *For the Learning of Mathematics* 7, 3, November 1987

[8] All groups designed tests with 6 questions. Several of them had questions that were broken up into sub-questions. Only one group included a multiple choice item (one).

| Material | Tasks | Group type |
|---|---|---|
| | • specify marking criteria for the test. | 2 Grade 8<br>2 Grade 9) |
| Readings on assessment | Readings on assessment<br>Three chapters from a book on assessment[9] were given to all group members as a resource. The reading of these chapters was not structured and group members were asked to read them at home in order to familiarise themselves with the ideas and be able to apply them in their test setting activity. | At home |
| Verbal instructions to groups. | Prepare for presentation to large group.<br>Groups were asked to<br>• prepare their tests and rationales for a presentation to the large group<br>• at the presentations members of the groups were make notes of comments and feedback that applied to their group. | Large groups |
| Verbal instructions to groups | Revision of test.<br>On the basis of feedback the groups revised their tests. The groups were required to explain the changes they made to the test. | Small groups |
| Guideline on choosing a learner for an interview | Marking own test and selecting a learner for an interview.<br>Teachers were asked to:<br>• Administer the test to one of their classes<br>• Mark all of the written tests, looking out for interesting learner responses.<br>• Back in their groups, select three learners that they identified for a learner interview (see below).<br>• Motivate the choice of question for the interview (in relation to the learners' work) by describing:<br>  • the mathematical concept the question is addressing<br>  • the misconception evident in the learner's work<br>  • the way it is exemplified in the learner's work<br>  • what questions the learner could be asked to expose her/his thinking more fully. | At school and in small groups |
| Guidelines for learner interview | Preparation for learner interview.<br>The aim of the interview was not to fish for the right answer but for the teacher to ask probing questions and develop his/her understanding the learner's reasoning. The groups were to consider the following:<br>• ways of creating a conducive atmosphere for the interview<br>• ways of avoiding leading questions<br>• watching one's body language and allowing oneself to be puzzled by the learner's answer rather than being critical of it | Small groups |

[9] Linn, RL and Miller, MD (2005) Measurement and Assessment in Teaching (New Jersey, Pearson Prentice Hall), Chapters 6, 7. And 9

| Material | Tasks | Group type |
|---|---|---|
| | • types of questions that can be used to prompt a learner to talk | |
| Interview Protocol | Learners' interviews were in two parts:<br>1. The teacher and learner discuss the item, and the learner's reasoning in addressing the question.<br>2. The learner is asked to do a mathematical problem similar to the item already discussed.<br>The learners' interviews were videoed. | Small groups and teachers' classrooms. |
| Learner interview presentation guide | Reflections on learners' interviews and preparation for presentations.<br>Each group was asked to:<br>• watch the recordings of all the learner interviews conducted in the group<br>• select different episodes for different categories of episodes (e.g. a time that a teacher was impressed by, surprised by, unsure about learner's thinking during the interview, or when something planned/unplanned happened and how they handled it)<br>In addition to this, each volunteer interviewer was asked to:<br>• to select one episode (from her/his own interview) for presentation to the large group (an episode where it was difficult to understanding the learner's thinking)<br>• justify the selection (reasons for selection and why it was difficult to understand learner thinking) | Small groups and large groups |

### Activity Seven: Independent group development of leverage items (alternative[10] to Activity Six)

14 weeks: March to August 2010

The purpose of this activity was to give some of the groups the opportunity of broadening their understanding of typical errors by discussing leverage items. Leverage analysis of test items provides a way of selecting concepts to focus on that will make the most difference to the greatest number of learners. In other words, if analysis shows that a large number of learners have difficulties with a certain test item because of an underlying misconception, then it is worth addressing this misconception in whole class teaching. Similarly, if a particular misconception affects learners' ability to answer a variety of questions, then addressing the misconception will positively affect learner performance on a number of different questions.

[10] Alternative activities were implemented in order to undertake a greater number of tasks that would illuminate the process – while still keeping within the timeframe of the project.

**Table 8: Activity Seven – Development of leverage items**

| Material | Tasks | Group type |
|---|---|---|
| Leverage item discussion guideline | Groups were asked to identify hypothetical high leverage concepts for further investigation by revisiting:<br>• ICAS tests for their grade (2006 version)<br>• Achievement data for their grade on the ICAS 2006 test<br>• Error analysis templates (completed for their grade by their group)<br>• Curriculum mapping documents (completed by their group)<br>They were tasked to draw on their classroom experience to and use all of the DIPIP documentation listed above<br>• to formulate ideas on leverage concepts for their grade<br>• to record the steps in their identification of leverage concepts<br>• to do their own research (in the Wits Education library) to verify their selected leverage concepts and write a report on these. | Three small groups<br>A grade 4, grade 6 and grade 7 group |

### Activity Eight: Curriculum mapping and error analysis of ICAS 2007 items and own tests

5 weeks: September to October 2010

The purpose of this activity was for groups to repeat the error analysis and curriculum mapping activities on selected ICAS 2007 items or on their own tests, but without the facilitation of the group leader. This was important from a research point of view – to establish how effectively the experiences during the course of the project had enabled teachers to work independently.

Due to time constraints, as well as to the fact that a further round of teaching was not going to follow this exercise, the revised template did not include a question on whether and how teachers taught the concept underlying the test item.

**Table 9: Activity Eight – Round 2 curriculum mapping and error analysis**

| Material | Tasks | Group type |
|---|---|---|
| ICAS 2007 test items<br>Error analysis template incorporating shortened curriculum mapping template (only questions on alignment, not on teaching of items) | Error analysis of ICAS 2007 tests.<br>Groups were assigned 12 items for analysis but did not all complete the analysis of these items[11].<br>They conducted the analysis as they did for the ICAS 2006 test items. | 6 small groups (One of each of Grades 3, 4, 5, 6, 7, and 9) |

[11] Grade 3 (10 items); Grade 4 (11 items); Grade 5 (4 items); Grade 6 (11); Grade 7 (8 items); Grade 9 (11 items).

| Material | Tasks | Group type |
| --- | --- | --- |
| Error analysis template including curriculum mapping, but modified to accommodate their own tests | Error analysis of own tests.<br>The groups were asked<br>• to pool together the results of all the learners to work out the achievement statistics on the test<br>• to place the results on a scale and on that basis judge the difficulty of each item<br>• to analyse the ways in which learners got the correct answer to each of the questions as well as the ways the learners got each of the items wrong.<br>As for the first round of error analysis, the emphasis in the group's analysis was on the ways of thinking that the learner might have used to get the answer correct or incorrect. | 5 small groups on own test items<br>(One of each of Grades 3, 4, 5, 6 and 8) |
| Verbal instructions to groups | Error analysis presentation.<br>All groups were requested to present the error analysis of one item to the larger groups.<br>Group leaders were invited for the presentation. | Three Large groups<br>Grades 3 and 4, Grades 5 and 6 and Grades 7 and 9. |

**Activity Nine: Round 3 of lesson planning and teaching on a central mathematical concept (alternative[12] to Activity Eight)**

6weeks: August to October 2010

The purpose of this activity was to build on learning from Rounds 1 and 2 lesson planning and teaching. Groups could select either the Equal Sign or Problem-solving and visualisation for their lessons. No additional readings were given to the groups and the process was very similar to Rounds 1 and 2.

**Table 10: Activity Nine – Round 3 teaching**

| Material | Tasks | Group type |
| --- | --- | --- |
| Verbal instructions to groups | Lesson planning.<br>As for Rounds 1 and 2, the groups went through the process of collaborative lesson planning, and one member of each group volunteered to teach the lesson, which was videoed. | Three small groups<br>A grade 7, grade 8 and grade 9 group |
| Verbal instructions to groups | Small group reflections on lesson taught.<br>Groups reflected on their videoed lessons and prepared for large group presentations as in the previous two rounds.<br>However, group members were also asked to reflect in their journals<br>• on what went well or badly in the lesson, and | Small groups |

[12] This enabled two strands of research to be followed: one strand to study teachers' ability to build on what had been learned through teaching; and the other, teachers' ability to work on error analysis without the guidance of the group leaders.

| Material | Tasks | Group type |
|---|---|---|
| | • changes in the teaching that they noticed from earlier rounds. | |
| Verbal instructions to groups | Lesson presentation of two episodes.<br>The groups were asked to present as they had done for Rounds 1 and 2.<br>Since this round was truncated there was no lesson plan revision built into the cycle. | Large group (One group consisting of the three small groups that did this activity.) |
| A guide for written lesson reflection | Written reflection on lesson taught.<br>In the same way as in Rounds 1 and 2, any teachers who taught in Round 3 were requested to write an individual reflection. The teachers received the same guide. | Individual teachers |

## Data analysis process

As can be seen from the activities above, a large amount and variety of data was produced in the course of this project. The project team is in the process of conducting extensive evaluative research on effectiveness of teachers' use of data to inform their practice, their own learning, and their understanding of the learning in their own classrooms. This section of the report presents the criteria for analysis, and tables the data sources analysed.

### Criteria for analysis of teacher's explanation of and engagement with learners errors

All the project activities described above were analysed with the same set of criteria. This is in line with the aim of the project of designing activities for teachers' engagement with learners' errors across different teaching contexts.

Each criterion has four category descriptors which are designed to capture vertical progression. The vertical progression reflects four levels of achievement moving from "full" explanations to "explanations that were "not present" (lowest). For example:

**Table 11: Example of four levels of achievement for one criterion applied to one activity**

| Criteria | Category descriptors: Error analysis | | | |
|---|---|---|---|---|
| | **Full** | **Partial** | **Inaccurate** | **Not present** |
| **Awareness of mathematical error**<br>Teachers' explanation of the actual mathematical error and not on learners' reasoning. This is a first step in error analysis. | Teachers explain the error. The explanation is mathematically specific and links to common misconceptions or errors | Teachers explain the error. The explanation is broad; it does not link to a common misconception or error | Teachers explain the error. The explanation is flawed | No explanation is given of the mathematical error |

The criteria and the category descriptors are adapted to each activity. For example, criteria for analysis of teacher presentations are further specified so that it can be noted whether the voice is that of the presenters or other members of the audience. In this way the criteria capture the specific demands of each activity.[13] In analysing the teachers' understanding of learners' errors we focus on teacher explanations of and their mode of engagement with learners' errors. The analysis of teachers' explanations and mode of engagement with learners' errors is based on:

- teacher's analysis of learners' errors in the 2006 and 2007 ICAS tests and in the tests they designed,
- selected teachers' classroom teaching,
- selected small group reflection discussions on a lesson,
- selected interviews which teachers conducted with learners, and lastly
- selected presentations by small groups to a large group.

Six criteria are used to analyse the quality of the teachers' explanations and engagement with learners' errors.

Separate reports on each data set will be written as well as a final concluding summary that will integrate all of the findings across the data sets.

### 1. Procedural explanation

The literature emphasises that the quality of teachers' explanations depends on the balance they achieve between explaining the procedure required for addressing a mathematical question and the mathematical concepts underlying the procedure. Teaching mathematics involves a great deal of procedural explanation which should be done fully and accurately for the learners to grasp and become competent in working with the procedures themselves. This criterion aims to grade the quality of the teachers' procedural explanations. The emphasis in the criterion is on the quality of the teachers' procedural explanations when discussing the solution to a mathematical problem, when engaging with learners' errors through analysis of learner test data, during whole class teaching or during a one-on-one learner interview, when discussing a lesson in small groups or when small groups present to a large group.

The criterion was adapted to each of the activities. The four category descriptors for this criterion, which capture the quality of the procedural explanation demonstrated by a teacher/group, are presented in the table below. Table 12 includes the highest level expected in each of the activities.

**Table 12: Category descriptors for "full procedural explanation"**

| **Error analysis** | **Classroom teaching** | **Reflections on teaching**<br>**(Small group discussion)** | **Interviews** | **Presentations**<br>**(Large group discussion)** |
|---|---|---|---|---|

[13] This excludes Curriculum mapping which does not contain analysis of the learner error component.

| | | | | |
|---|---|---|---|---|
| Teachers' explanation of the learners' mathematical reasoning behind the solution includes demonstration of procedure.<br><br>The procedure is accurate and includes all of the key steps in the procedure. | When the teacher explains the solution to the learner/ probes further, the teacher demonstrates a procedure.<br><br>The procedure is accurate and includes all of the key steps in the procedure. | When small groups reflect on a lesson and explain the error, there is discussion of a procedure.<br><br>The procedure is accurate and includes all of the key steps in the procedure. | When the teacher explains the error to the learner/ probes further, the teacher demonstrates a procedure.<br><br>The procedure is accurate and includes all of the key steps in the procedure. | When the presenter/a member of the audience explains the error, there is discussion of a procedure.<br><br>The procedure is accurate and includes all of the key steps in the procedure. |

## 2. Conceptual explanation

The emphasis in this criterion is on the conceptual links made by the teachers in their explanations of the learners' mathematical reasoning in relation to the errors. The conceptual explanations may or may not be linked to a procedure. Mathematical procedures need to be unpacked and linked to the concepts to which they relate in order for learners to understand the mathematics embedded in the procedure. The emphasis of the criterion is on the quality of the teachers' conceptual links made in their explanations when discussing the solution to a mathematical problem, when engaging with learners' errors through analysis of learner test data, during whole class teaching or during a one-on-one learner interview, when discussing a lesson in small groups or when small groups present to a large group.

The criterion was adapted to each of the activities. The four category descriptors for this criterion, which capture the quality of the conceptual explanation demonstrated by a teacher/group, are presented in the table below. Table 13 includes the highest level expected in each of the activities.

**Table 13: Category descriptors for "full conceptual explanation"**

| **Error analysis** | **Classroom teaching** | **Reflections on teaching**<br>**(Small group discussion)** | **Interviews** | **Presentations**<br>**(Large group discussion)** |
|---|---|---|---|---|
| Teachers' explanation of the learners' mathematical reasoning behind the solution includes conceptual links. | When the teacher explains the solution to the learner/ probes further, the teacher includes conceptual links. | When small groups reflect on a lesson, their explanations include conceptual links. | When the teacher explains the error to the learner/ probes further, the teacher includes conceptual links. | When the presenter/a member of the audience explains the error, the speaker includes conceptual link. |

| | | | | |
|---|---|---|---|---|
| The explanation illuminates conceptually the background and process of the activity. | The explanation illuminates conceptually the background and process of the procedure. | The explanation illuminates conceptually the background and process of the procedure. | The explanation illuminates conceptually the background and process of the procedure. | The explanation illuminates conceptually the background and process of the procedure. |

### 3. Awareness of mathematical error

The emphasis in this criterion is on the teachers' explanations of the actual mathematical error (and not on the learners' reasoning)**.** The emphasis in the criterion is on the mathematical quality of the teachers' explanations of the actual mathematical error when discussing the solution to a mathematical problem, when engaging with learners' errors through analysis of learner test data, during whole class teaching or during a one-on-one learner interview, when discussing a lesson in small groups or when small groups present to a large group.

The criterion was adapted to each of the activities. The four category descriptors for this criterion, which capture the quality of the awareness of error demonstrated by a teacher/group, are presented in the table below. Table 14 includes the highest level expected in each of the activities.

**Table 14: Category descriptors for "full awareness of mathematical error"**

| **Error analysis** | **Classroom teaching** | **Reflections on teaching (Small group discussion)** | **Interviews** | **Presentations (Large group discussion)** |
|---|---|---|---|---|
| Teachers explain the mathematical error made by the learner. | Teacher explains the mathematical error to the learner. | When small groups reflect on a lesson, they explain the actual mathematical error. | Teacher explains the mathematical error to the learner. | When the presenter/a member of the audience explains the error, he/she explains the actual mathematical error. |
| The explanation of the particular error is mathematically sound and suggests links to common misconceptions or errors. | The explanation is mathematically sound and suggests links to common misconceptions or errors. | The explanation is mathematically sound and suggests links to common misconceptions or errors. | The explanation is mathematically sound and suggests links to common misconceptions or errors. | The explanation is mathematically sound and suggests links to common misconceptions or errors. |

### 4. Diagnostic reasoning

The idea of error analysis goes beyond identifying the actual mathematical error. The idea is to understand how teachers go beyond the mathematical error and follow the way learners were reasoning when they made the error. The emphasis in the criterion is on the quality of the teachers' attempt to provide a rationale for how learners were reasoning mathematically when they chose a distractor. This was looked at in different contexts: when teachers discuss the solution to a mathematical problem, when they engage with learners' errors through analysis of learner test data, during whole class teaching or during a one-on-one learner interview, when they discuss a lesson in small groups or when small groups present to a large group.

The criterion was adapted to the different activities. The four category descriptors for this criterion, which capture the quality of the diagnostic reasoning demonstrated by a teacher/group, are presented in the table below. Table 15 includes the highest level expected in each of the activities.

**Table 15: Category descriptors for "full diagnostic reasoning"**

| **Error analysis** | **Classroom teaching** | **Reflections on teaching (Small group discussion)** | **Interviews** | **Presentations (Large group discussion)** |
|---|---|---|---|---|
| Teachers describe learners' mathematical reasoning behind the error. | The teacher seeks to find out the learner's mathematical reasoning behind error. In response to the error the teacher probes further and asks the learner to explain the steps of her/his reasoning. | When small groups reflect on a lesson, they reflect on the learner's reasoning behind the error. | The teacher seeks to find out the learner's mathematical reasoning behind error. In response to the error the teacher probes further and asks the learner to explain the steps of her/his reasoning. | When the presenter/a member of the audience explains the learner's error, he/she reflects on the learner's reasoning behind the error. |
| Teachers describe the steps of learners' mathematical reasoning systematically and hone in on the particular error. | Probing engages with learner's reasoning and is open or directed. Probing hones in on the mathematical error. | The explanation describes the steps of learners' mathematical reasoning systematically and hones in on the particular error. | Probing engages with learner's reasoning and is open. Probing hones in on the mathematical error. | The explanation describes the steps of learners' mathematical reasoning systematically and hones in on the particular error. |

## 5. Use of everyday knowledge

Teachers often explain why learners make mathematical errors by appealing to everyday experiences that learners draw on and confuse with the mathematical context of the question. The emphasis in this criterion is on the quality of the use of everyday knowledge, judged by the links made to the mathematical understanding that the teachers attempt to advance. This was looked at in different contexts: teachers discussing the solution to a mathematical problem, engaging with learners' errors through analysis of learner test data, whole class teaching or one-on-one learner interviews, discussing a lesson in small groups or small groups presenting to a large group.

The criterion was adapted to the different activities. The four category descriptors for this criterion, which capture the quality of the use of everyday knowledge demonstrated by a teacher/group, are presented in the table below. Table 16 includes the highest level expected in each of the activities.

**Table 16: Category descriptors for "full use of everyday knowledge"**

| Error analysis | Classroom teaching | Reflections on teaching (Small group discussion) | Interviews | Presentations (Large group discussion) |
|---|---|---|---|---|
| Teachers' explanation of the learners' mathematical reasoning behind the error appeals to everyday knowledge.<br><br>Teachers' use of the 'everyday' enables mathematical understanding by making the link between the everyday and the mathematical clear. | When the teacher explains the error to the learner/probes further, the teacher appeals to everyday knowledge.<br><br>The teacher's use of the 'everyday' enables mathematical understanding by making the link between the everyday and the mathematical clear. | When small groups reflect on a lesson and a speaker explains the learner's error, he/she appeals to everyday knowledge.<br><br>The speaker's use of the 'everyday' enables mathematical understanding by making the link between the everyday and the mathematical clear. | When the teacher explains the error to the learner/probes further, the teacher appeals to everyday knowledge.<br><br>The teacher's use of the 'everyday' enables mathematical understanding by making the link between the everyday and the mathematical clear. | When the presenter/a member of the audience explains the learner's error he/she appeals to everyday knowledge.<br><br>The speaker's use of the 'everyday' enables mathematical understanding by making the link between the everyday and the mathematical clear. |

## 6. Multiple explanations of error

One of the challenges in the teaching of mathematics is that learners need to hear more than one explanation of the error. This is because some explanations are more accurate or more accessible than others and errors need to be explained in different ways for different learners. This criterion examines the teachers' ability to offer clear and correct explanations as well as alternative explanations of the error when they are engaging with learners' errors through analysis or during teaching, when they discuss a lesson in small groups or when small groups present to a large group. In addition, the criterion examines the ways in which the teacher probes for multiple explanations of errors from the same learner or from others in the class (during teaching and during an interview situation).

The criterion was adapted for the different activities. The four category descriptors for this criterion, which capture the quality of the multiple explanations of error demonstrated by a teacher/group, are presented in the table below. Table 17 includes the highest level expected in each of the activities.

**Table 17: Category descriptors for “full multiple explanations of error”**

| Error analysis | Classroom teaching | Reflections on teaching (Small group discussion) | Interviews | Presentations (Large group discussion) |
|---|---|---|---|---|
| Multiple mathematical explanations are provided. | Multiple mathematical tracks are offered/probed by the teacher. | When small groups reflect on a lesson, the group offers multiple mathematical tracks. | Multiple mathematical tracks are probed/directed by the teacher. | When the presenter explains to the big group the learner's error and /or the learner's reasoning behind the error, he/she provides multiple explanations. |
| All of the explanations (two or more) are mathematically feasible/ convincing. (No general track is used.) | At least two mathematically feasible/convincin g tracks are followed by the learner in response to the teacher's probing or directing. (No general track is used.) | All of the explanations (two or more) are mathematically feasible/ convincing. (No general track is used.) | At least two mathematically feasible/ convincing tracks are followed by the learner in response to the teacher's probing or directing. (No general track is used.) | All of the explanations (two or more) are mathematically feasible/ convincing. (No general track is used.) |

## Data sources analysed in the project evaluation

The project evaluation was the catalyst for the development of the criteria used to analyse the quality of the teachers' explanations and engagement with learners' errors. An overview of the sampled data used in the evaluation is given in Table 18 below.

**Table 18: Evaluation data sources**

| Activity | Data to be considered | Quantity | Tasks | Templates/ Protocols |
|---|---|---|---|---|
| 1. Curriculum mapping | 2006 curriculum mapping templates<br><br>2007 mapping aspect from 2007 EA templates | 2006: All groups all templates (14 groups) 20 items per group. Selected to represent items mapped early and late in the activity and items across all mathematical content areas.<br><br>2007: All groups all templates (11 groups) – 82 items altogether. | 1) Coding of curriculum mapping. | Coding template for curriculum mapping: quality and links to practice. |
| 2. Error analysis | 2006 error analysis templates | 2006: All groups all templates (14 groups) 20 items per group, one distractor per item. Selected to cover item content across all | 1) Selection of distractors to be coded according to item stats. | Coding template for error analysis. 6 criteria, 4 |

| Activity | Data to be considered | Quantity | Tasks | Templates/ Protocols |
|---|---|---|---|---|
| | 2007 error analysis templates | mathematical content areas.<br><br>2007: All groups all templates (11 groups) one distractor/explanation per item – 82 items. | 2) Coding of distractors according to criteria | levels. |
| 3. Classroom teaching | Classroom teaching videos<br><br>Focus group interviews (round 1)<br><br>Focus group interviews (round 2) | One teacher per grade (Rounds 1, 2 and 3) One video per teacher (17 videos). Lesson videos 30 – 60 min in length.<br><br>Focus group interviews to match selected classroom teaching videos. | 1) Time segment analysis of the videos using criteria<br>2) Thematic content analysis of transcriptions of audios. | Coding template for classroom teaching.<br>6 criteria, 4 levels. |
| 4. Learner interviews | Learner interview videos | Same teachers as for classroom teaching. (17 interview videos)<br>Interview videos 20 – 30 min in length. | 1) Time segment analysis of the videos using criteria | Coding template for learner interviews.<br>6 criteria, 4 levels. |
| 5. Group reflection on classroom teaching | Audio of group discussions reflecting on videos<br><br>Written reflections (individual and group assisted) | Same teachers as for classroom teaching. (17 teachers' group discussions) | 1) Group discussions – thematic content analysis guided by criteria.<br>2) Thematic content analysis of individual reflection discussions and written notes. | Coding template for reflection discussions.<br>6 criteria, 4 levels. |
| 6. Presentations of teaching | Presentation videos | Same teachers as for classroom teaching. (17 presentation videos)<br>Presentation videos generally about 60 min in length. | 1) Time segment analysis of the videos using criteria | Coding template for large group presentations.<br>6 criteria, 4 levels. |

## Report writing and external validation

The data analysis was written up in the form of an internal evaluation research report for each of:

1. Curriculum mapping
2. Error analysis
3. Learner interviews.

Each of these reports was critically reviewed by the external evaluation team. The external evaluators' comments were discussed and addressed as appropriate. A revised version of each of the internal evaluation research reports was then provided for the validation process.

The validation process consisted of the following:

1. Additional data collected in 2010.
2. Expert consultants appointed to prepare reports on curriculum mapping and error analysis – reports prepared in 2011.
3. Validation criteria developed and applied to the three internal evaluation reports in 2012.
4. Final evaluation report prepared with two parts, the first part analysing additional data and integrating the expert consultant reports, and the second part concerned with the validation of three internal evaluation reports, but drawing on the analysis of the additional data in the first part.

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