Assessment for Learning: Using learners' test data for professional development

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JOHANNESBURG

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Citation

Sapire, I. Shalem, Y. and Reed, Y. (2015). *Assessment for Learning: Using learners' test data for professional development*. Johannesburg: University of Witwatersrand and *Saide*.

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COURSE GUIDE

Assessment for Learning: Using data from learners' mathematics tests in teacher professional development

Who is this course for?

This five unit course is intended for mathematics teachers who work / teach in primary schools. The approach taken in the course can also be used in secondary school mathematics teaching but the mathematics content in the course activities is for grades 3 to 7.

What can you learn from this course?

The purpose of this course is to help you as a mathematics teacher to use information (data) from the assessment of learners' work to improve teaching and learning in your classroom.

The five units show you how to become more aware of the errors your learners make, how to understand these errors and the misconceptions that may have caused them, as well as how to plan lessons and assessment with errors in mind.

What is the content and structure of the course?

The five units in this course provide not only key content, but also offer you points at which to pause for reflection, and activities that engage you in tasks relevant to your classroom practice. Each activity is followed by commentary, which may provide the 'answers' to the questions, but more importantly, will help you think further about the responses you may have given to the activity questions.

Each unit also has one or more readings which you can use independently of the units. In most cases the 'theory' is encapsulated in the readings. The readings are useful not only for the particular unit to which they are linked, but can also be referred to when you work through other units. We have deliberately kept the reading requirements to a minimum, and we hope that users of the course will add to the readings.

For each of Units 2, 3, and 4, there is an additional section with activities for further practice. Our view is that teachers need to make awareness of errors part of how they teach, so that they teach reflectively, keeping the learning needs of their learners in mind. This takes practice, and the additional activities provide a starting point for developing habits of reflection that are the hallmark (distinctive feature) of sound professional practice.

The content focus in each of the units is as follows:

Unit 1: Using learners' test data for professional development
The purpose of this unit is to introduce you to a data-based approach to teacher professional development. The unit aims to:

- introduce some of the specialised assessment terms used in this unit and throughout the module;
- describe a teacher development project (the Wits/Gauteng Department of Education Data-Informed Practice Improvement Project Phases 1&2 2007-2010) in which mathematics teachers used evidence (data) from learners' responses to tests to deepen their mathematical knowledge and to inform their classroom practice;
- use some data from this project to raise and suggest answers to questions about data-based professional development.

Unit 2: Curriculum mapping
This unit is in three parts. Firstly it introduces some theory about understanding curriculum and curriculum mapping; secondly, there are activities in which you will practise doing curriculum mapping; and lastly the unit provides opportunities for reflection on the value of curriculum mapping activities.

Unit 3: Analysing learner errors
The first part of this unit introduces you to what you have to know as a teacher in order to understand learners' misconceptions and errors. The second part introduces you to a process you can use to identify and reflect on learner errors, with plenty of activities for practice and opportunities for reflection.

Unit 4: Giving feedback to learners
This unit explores the role of feedback in learning and teaching, different ways to offer feedback, and principles for providing feedback that helps the learning process. Through a range of activities, it provides you with opportunities to practise and think about giving feedback in ways that meet learner needs and contribute to learning.

Unit 5: Using learners' error to inform lesson planning
The content and activities in this unit will support you in planning lessons that show awareness of errors and cater for a range of learners and learning needs. The unit presents transcripts of short episodes from mathematics lessons to situate the tasks within real mathematics lessons. Planning for lessons with errors in mind is critical so that teachers can be ready to address learners' misconceptions.

How should the course be used?

The course has been designed in such a way that an individual teacher can work through the units on his or her own - **independent study**. Each activity has a commentary attached to it, which provides independent learners with built–in feedback.

However, it would be much better for groups of mathematics teachers to work through and discuss the units together – as part of a **professional learning**

community. The course material can be used to structure meaningful PLC discussions.

The units, readings, video clips and additional activities can be downloaded and worked through **offline.** But they can easily be used as the building blocks for an **online** course.

How long will the course take to work through?

Unit 1 should take you 2 x two hour sessions to work through.
Units 2 through 5 will take longer: 3 or 4 x two hour sessions each, with additional time for the further activities, and for assessment tasks.

Unit 1: Using learners' test data for professional development

Introduction

Formative assessment is part of a teacher's daily work. This first unit outlines how data from learners' answers to test questions can be used not only summatively, but also for formative assessment. These and other assessment terms are explained. Activities based on data from learners' tests have been included so that readers can begin to engage with this approach to professional development.

Aims of the unit

- To introduce readers to a data-based approach to teacher professional development.
- To introduce some of the specialised assessment terms used in this unit and throughout the module.
- To describe a teacher development project in which mathematics teachers used evidence (data) from learners' responses to tests to deepen their mathematical knowledge and to inform their classroom practice.
- To use some data from this project to raise questions and to suggest answers to questions about data-based professional development.
- To provide an overview of the rest of units which focus on different aspects of the project.

Taking a data-based approach to teacher professional development

While it has been widely accepted that teachers have a key role to play in their own professional development, there is considerable debate about how this can be done most effectively. With reference to mathematics teachers, researchers have found that when these teachers focus on errors, as evidence of reasonable and interesting mathematical thinking on the part of learners, this can help them to understand their learners' thinking (Borasi, 1994; Nesher, 1987; Smith, DiSessa, & Roschelle, 1993). It can also help them to adjust the ways they engage with learners in the classroom and to revise their approaches to teaching. By examining examples of learners' responses to items in mathematics tests (data) teachers can develop greater understanding of learners' errors and of ways in which to develop learners' mathematical knowledge. These are important aspects of professional development.

Using summative assessment formatively

Formative assessment tasks are set when learners are still in the process of learning something. For example, learners' responses to a homework task can give teachers information (data/evidence) about the progress that learners are making towards the achievement of particular learning goals. However, the results of a summative assessment (see below) can also be used formatively. Whenever teachers give learners **feedback** (discussed in detail in Unit 4) to show them what they have understood, where they have made errors and what they need to do next, teachers are using information from summative assessment formatively.

Summative assessment is designed to assess what level of knowledge and skills learners have achieved by the end of a unit of work, a school term, a school year or phase of schooling.

Both internationally and locally, learners' performance is often assessed through standardised testing. These tests are an example of **summative assessment.** This unit and other units in this module focus on what teachers can learn by using data/evidence from summative assessment to understand learners' errors and to inform their classroom teaching. When teachers do this they are using summative assessment **formatively**.

Evidence

What counts as evidence?
When most people hear the word "evidence" they link it to court cases in which lawyers use evidence to argue their client's case. Lawyers offer evidence as proof of guilt or innocence. In this unit, evidence is all the information from learners' tests that can be used as "proof" of their current level of understanding – that is, the evidence provides information about what learners know and can do and also about what they still need to learn.

Evidence-based professional development
One project which used an evidence-based approach to teachers' professional development was the Data Informed Practice Improvement Project (DIPIP Phases 1&2). This project aimed to assist teachers to understand learners' errors, both generally and in terms of particular topics, with the overall goal of improving learners' performance. Information about DIPIP is given in the reading, *Introducing the Data-Informed Practice Improvement Project,* which follows.

Activity

Activity 1

Read the Reading for this Unit: *Introducing the Data-Informed Practice Improvement Project*

a) The reading makes several references to accountability. Schools are held externally accountable to the Department of Education and to the nation for the results that learners achieve on standardized assessments. However according to Elmore (Reading, page 1) "without internal accountability in schools, it is unlikely that measures for external accountability will make a significant difference". After you have completed the reading, explain what you think Elmore means by "internal accountability".

b) Summarise what the DIPIP project did to assist teachers to become more internally accountable.

c) Suggest how you and mathematics teaching colleagues could use test data to become more internally accountable.

Commentary on Activity 1

A criticism of external accountability measures, such as standardised test results, is that they do not necessarily lead to any improvements in teaching and learning. All they do is indicate which schools produce the best and worst results. For teachers to be truly accountable to learners, parents, the Department of Education and the nation, they need to take responsibility for improving learner performance on standardised tests. This involves working internally in their own classrooms and schools in order to be externally accountable.

The DIPIP project gave teachers the opportunity to work together with colleagues (in professional learning communities) to discuss data from a standardised test in order to reflect on learners' work and to inform their classroom practice.

Asking and answering questions about data-based professional development

For the three years of the DIPIP project teachers worked with mathematics test data at group meetings and in their own maths classrooms where they applied what they had learned at group meetings. In this unit, the activities for readers focus on mathematical ideas. As you work through them you may find that you can use some of these ideas in your own classroom practices. Whether or not you are able to do this, thinking about each of the questions below and completing each activity should help you to extend your mathematical knowledge.

Each of the questions was addressed in the DIPIP project. In this unit they are answered partly theoretically and partly with reference to some learner mathematical activity taken from one of the ICAS (International Competitions and Assessments for Schools) tests with which the DIPIP teachers worked.

How do I become curious about errors rather than judgemental?

It is well known that many learners in South Africa (although they are not the only ones!) perform very poorly in standardised mathematics test. But as already stated "naming and shaming" is not very helpful for learning how to change this situation. Instead of having a judgemental attitude towards the results achieved by learners on standardised tests, teachers can use data from these tests to access learners' thinking and to help them improve their performance.

Rather than putting a red mark of failure onto learners' errors, teachers in the DIPIP project used data from learners' tests to gain a better understanding of how learners think when they make certain errors. This type of activity might inspire you to start thinking about how you could look at the data you get in your classrooms from learners every day.

The following activity is based on a mathematical example from the content area of "number and operations", more specifically on the topic of *rate*.

Activity

Activity 2
To get started, think about the following example, taken from the Grade 7 ICAS (2006) test.

Sam and Kevin are bricklayers. Sam lays 150 bricks in 60 minutes. Kevin lays 20 bricks in 10 minutes. Working together, how many minutes will it take Sam and Kevin to lay 180 bricks?
(A) 25 (B) 40 (C) 70 (D) 100

a) Solve the question for yourself.
b) Think about the way in which learners would have solved the question correctly.

Commentary on Activity 2

Analysis of the correct answer (B) shows that learners have to have a clear understanding of rates and how and when they can be compared and used together. To arrive at the correct answer, they need to be able to write the rates in ways that enable them to combine them. To do this, they need to keep the time quantity the same.

They can do this by:

a) finding how many bricks each brick layer would lay per minute (2.5 for Sam and 2 for Kevin). They would then combine these to get 2.5 + 2 = 4.5 bricks per minute for the two bricklayers. This would then lead to 180 bricks being laid by the two men in 40 minutes.

OR

b) finding how long it takes each brick layer to lay one brick and obtain 0,4 minutes per brick for Sam and 0,5 minutes per brick for Kevin. They would then find how many bricks each of them would lay in any given length of time, and get, e.g. 0,5 bricks in 0,2 minutes for Sam and 0,4 bricks in 0,2 minutes for Kevin. They would then combine these to get 0,5 + 0,4 = 0,9 bricks in 0,2 minutes for both Sam and Kevin, which would give 180 bricks in 40 minutes.

There may also be other ways to arrive at the correct answer, but learners would still have to find how many bricks each bricklayer would lay in a given length of time before combining them. So, the conceptual issue here is *when and how to reduce as necessary and combine rates.* The error analysis described in Activity 3 confirms this.

Activity

Activity 3

Read the achievement data for the Grade 7 Rate item (Activity 2) given below.

a) What mathematical reasoning could learners have used in order to select the most highly selected distractor?

ICAS (2006) data

- Only 15% of learners chose the correct answer (B)
- 11% chose option A
- 46% (almost half) chose option C
- 25% chose option D

3% of learners did not answer the question

Commentary on Activity 3

For option C (the most highly selected distractor – 46% of learners chose this answer), it is possible that the learners only added the times and numbers of bricks laid as stated in the problem. This could be as a result of misinterpreting 'together' in the question as referring to addition. Sam's 160 bricks in 60 minutes and Kevin's 20 bricks in 10 minutes would combine to become 150 + 20 = 170 bricks in 60 + 10 = 70 minutes. The fact that the question asked about 180 bricks and their answer was 170 bricks may not have bothered them if they saw 170 as very close to 180 and thus they settled for 70 minutes.

Reflection

Think about the reasoning underlying the learners' choices for options A and D.

Activity

Activity 4

a) Give the test item in Activity 2 to your learners and ask them how they worked out the answer if they chose option C. Does this agree with the explanation given above? Do they give any other explanations?

b) Ask them what they did if they chose option A or D. Are the explanations different to those for the selection of option C? How are they different?

c) Did the learners find it easy or difficult to explain to you how they got their answers?

Commentary on Activity 4

The kind of investigation that you will have done in order to complete Activity 4 is likely to have given you insight into the way learners are thinking when they make errors. As a teacher of mathematics you probably mark learners' work every day. This means that every day you have an opportunity to think about the ways in which your learners are thinking and you can develop your own plans for teaching, based on ideas you get when you see the problems that your learners have.

Why should I bother to think about the errors learners make?

Research in mathematics classrooms has shown that learners' errors often make sense to them and are usually made consistently (Olivier, 1996). According to this research, errors also tend to be very resistant to attempts by the teacher to correct them. Errors can be useful to teachers because they can reveal incompleteness in learners' knowledge and thus enable the teacher to contribute additional knowledge, or better still, guide learners to realise for themselves where and /or how they are making an error.

Errors often show teachers the learners' **misconceptions**. Misconceptions are learners' conceptual ideas that explain why they (learners) might produce a particular error or set of errors. Note that misconceptions may also sometimes lead to *correct* answers, although the mathematical thinking that has produced these answers is incorrect.

Misconceptions are part of the knowledge that learners develop, and hence form part of their current knowledge. Teachers need to build on learners' current knowledge. This means that they need to listen to, work with and build on learners' misconceptions as well as on their correct conceptions. Learning always involves transforming current knowledge. Therefore, if misconceptions exist, the existing knowledge will need to be transformed and re-structured into new knowledge. If teachers listen to and work with learners' thinking they can learn about:

- the types of conceptions learners have;
- how to build onto these conceptions;
- how to use misconceptions to transform learners' thinking and to inform teaching.

Misconceptions are dealt with more fully in Unit 3, *Analysing Learners' Errors* which includes a reading about error analysis and other references.

Activity

Activity 5

This fraction item from the ICAS 2006 tests gives further insight into learners' thinking about equivalent fractions.

a) Solve the question for yourself.
b) Read the achievement data given.
c) What does the data show you about the way learners were applying the rule 'what is done to the numerator should be done to the denominator'?

Grade 8 Item: Equivalent fractions

Which of these expressions is equivalent to $\frac{5}{7}$?

A) $\frac{5}{7}+\frac{7}{5}$ B) $\frac{5\times5}{7\times7}$ C) $\frac{5+2}{7+2}$ D) $\frac{5\times7}{7\times7}$

ICAS item and data for activity

- Only 17% of learners chose the correct answer (D)
- 30% chose option A
- **33% chose option B**
- 17% chose option C

2% of learners did not answer the question

Commentary on Activity 5

The correct answer to this question is D, because in this case both the numerator and the denominator have been multiplied by the same number (7).

This most highly selected distractor (B) is evidence of learners applying the over-generalised rule for generating equivalent fractions: "what is done to the numerator should be done to the denominator" (or, more commonly stated as "what you do to the top you do to the bottom". Following this rule, learners may say $\frac{a}{b}$ is equal to $\frac{a\times2}{b\times2}$, which is correct, and at other times, $\frac{a}{b}$ is equal to, $\frac{a+2}{b+2}$ which is incorrect (as they have done when they chose option B above). The language commonly used to speak about finding equivalent fractions has enabled these learners to form this misconception. Teachers need to think carefully about how they speak about and introduce mathematical ideas. When speaking about equivalent fractions, it would be helpful to say "we can multiply (or divide) both the numerator and the denominator of a fraction to produce a fraction which is equivalent to the original fraction".

Reflection

What can you learn from thinking about the errors that your learners make?
Take some time to look at some errors your learners have made and think about whether these can give you any insight into the difficulty they were having when they made the errors.

Won't learners just grow out of the errors if I teach and explain properly?

There are three words that are used a great deal when teachers speak about learners' mathematical work. These are the words 'misconceptions', 'errors' and 'mistakes'. One of the aims of this material (based on learning from the DIPIP project), is to assist teachers to understand fully what these different words mean, because they have different concepts associated with them. The word 'misconception' is often used rather indiscriminately when teachers talk with colleagues, and more often than not, teachers are talking about learner errors or mistakes rather than misconceptions.

As mentioned above, 'misconceptions' is a term used to describe the alternative understandings that people may have about concepts. These understandings may be either incorrect or incomplete. Teachers are able to identify learners' misconceptions through understanding *why* they have made the errors that they see.

The correct use of the word, 'errors' is important. **Errors** are pervasive (found everywhere) and are often repeated in different contexts. Teachers shake their heads and wring their hands over learners who keep doing the same things wrong over and over again, even though these have been addressed and corrected repeatedly in class. Teachers need to realise that *understanding where these errors are coming from and* ***why*** *they are being made is a very valuable starting point for improving teaching and for improving learners' understanding of concepts*.

The reason why learners make errors is that their incorrect thinking (misconceptions), results in inaccurate or incomplete knowledge structures. **Mistakes**, on the other hand, are incorrect answers that have been obtained as a result of 'slips' such as pressing an incorrect calculator key or calculating $3 \times 3 = 6$. A mistake is quickly realised and corrected because there is no conceptual misunderstanding associated with the mistake. It is important that teachers are able to distinguish between mistakes (easily fixed) and errors (very difficult to correct), so that they do not become frustrated with their learners.

Activity

Activity 6

Subtraction of three digit numbers requires some understanding of place value. The following ICAS test item gives you an opportunity to think about errors, mistakes and misconceptions when learners perform such an operation.

a) Solve the question for yourself.
b) Read the achievement data given.
c) Decide which distractor(s) represent a misconception and why.

Grade 6 Item: Subtraction of three digit numbers

900 – 358 =

A) 542 B) 552 C) 642 D) 658

ICAS item and data for activity

- Only 34% of learners chose the correct answer (A)
- 11% chose option B
- 13% chose option C
- **39% chose option D**

1% of learners did not answer the question

Commentary on Activity 6

The correct answer to this question is option A. The solution requires an understanding of place value in order to break down the number 900 appropriately (using whichever algorithm the learner may have chosen) to facilitate the subtraction of the number 358 from 900.

The most highly selected distractor (D) represents a choice made based on the misconception that you "cannot subtract a bigger number from a smaller number". To get such an answer the learner loses track of which number is being subtracted from which number and simply subtracts convenient numbers (smaller from bigger in each case) from each other. Such activity also indicates a poor understanding of number concept since the learner treats individual digits in the numbers as independent numbers rather than as representative of certain values, based on their position in the actual number of which they are a part.

Reflection

Do you think that learners can grow out of errors? What would help them to do this?
Discuss the difference between mistakes, errors and misconceptions with your colleagues.

How can my colleagues and I talk productively about errors?

Identifying the misconceptions that lie behind the errors that learners make could serve as a productive discussion focus for teachers. It may be difficult and may require reading about the topics on the internet or in books, but such activity could enable teachers to broaden their understanding of the mathematics that they are teaching and help them to understand their learners and their learners' thinking more fully. One of the aims of this material is to assist readers to think about ways in which discussions of learners' errors could contribute to their professional development.

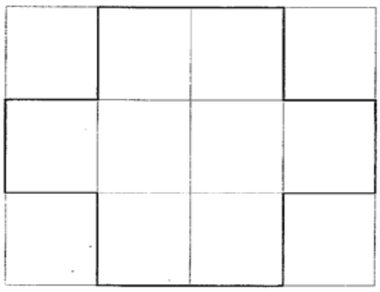
For Activity 7, read the following ICAS test item and the data related to it on the ICAS 2006 test.

Activity

Activity 7

a) Work out the correct answer to Gr 5, Item 9.
b) Read the achievement data given below the test item.
c) What do you think the learners were thinking when they chose the most popular distractor on this item?
d) Discuss your answer to c) with your colleagues. Do you agree or differ in your opinions on the ways in which learners were thinking when they chose the wrong answers to this question? Write about your discussion.

Gr 5 Item 9: Calculate the area of a shape based on squares
Holly drew this shape on 2 cm grid paper.



What is the area of Holly's shape?
A) 32 cm^2 B) 28 cm^2 C) 16 cm^2 D) 8 cm^2

ICAS (2006) data

- 14% of learners arrived at the correct answer (A)
- **34% chose option D**
- 23% chose option C
- 22% chose option B

5% of learners did not answer the question

Commentary on Activity 7
The correct answer to this question is (A). In order to answer correctly, learners had to notice that the area of one of the squares in the grid, on which the shape is drawn, is 4 cm^2.

The most popular distractor (D) would have been selected by learners who possibly did not read the information correctly and therefore assumed that the area of each block in the grid was only 1 cm^2. Based on this assumption if the shape is made up of

8 blocks, the area of the shape must be 8 cm^2. You and your colleagues may have discussed various ways in which you thought learners would have reached the different incorrect answers presented in the three distractors for this question.

Reflection

Is it productive for you and your colleagues to discuss questions such as the one given above? Explain what you could gain from such a discussion.

Isn't it a waste of time to talk endlessly about errors?

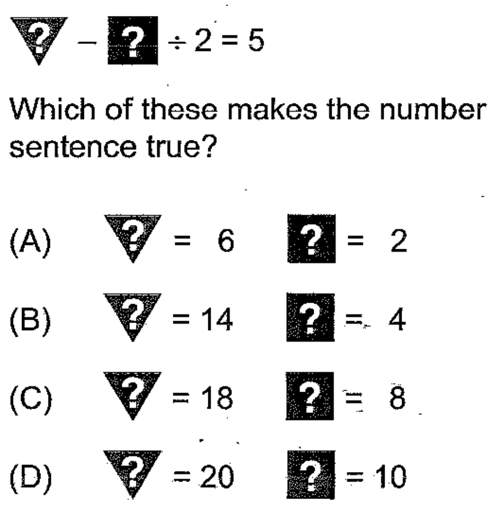
Sometimes, teachers believe that they '*have a syllabus to get through*', and they become concerned about spending time talking about errors with learners. However, Orton (2004) argues that *'so much time in mathematics lessons is currently spent on re-teaching and providing routine practice on ideas which do not seem to have been mastered however many times they are re-taught in the 'traditional' manner'* (p.202). Inadequate or faulty thinking is a normal and expected part of the learning process. Teachers need to help learners to recognise that some of their understandings of mathematical principles may be incomplete or inaccurate and to work with them to improve their understanding.

The next example from the DIPIP test analysis raises a problem commonly experienced in the classroom – that of teaching the solution of number sentences and equations using steps, procedures and rules. It is an approach which many teachers prefer, because it teaches learners *what to do*. However, it can lead to misconceptions and errors in solving equations. When the application of the rule is straightforward, there are no problems. But when it is not, the misconceptions begin to emerge.

Activity 8

a) Solve the question for yourself.
b) Read the achievement data given below the test item.
c) What was the error made by the learners when they chose the most popular distractor?
d) Do you think that this error is based on a misconception and if so, what is that misconception?

Grade 6: Supply the missing numbers to solve an order of operations problem



ICAS (2006) data

- 19% of learners arrived at the correct answer (A)
- 19% chose option B
- 15% chose option C
- **42% chose option D**

3% of learners did not answer the question

Commentary on Activity 8

The correct answer to this question (A) requires a sound understanding of the rules for the order of operations when operating on a string of numbers. The rules for the order of operations are standard and according to these rules, multiplication and division take precedence over addition and subtraction, when they appear in the same string of numbers to be operated on. Knowing this would enable a learner to identify that the operation of division (second in this string of numbers) takes priority over the first operation (subtraction) in the string. Applying this rule, the learner correctly decides that 6 – 2 ÷ 2 = 6 – 1 = 5.

The confusion that could result from a poor understanding of the rules for the order of operations could lead to selection of option (D). Here the learner decides that

20 – 10 ÷ 2 = 10 ÷ 2 = 5. In this instance the learner is applying the general rule "work from left to right" when working on a string of numbers without applying the

exceptions to this general rule. Here application of a more "straightforward rule" has resulted in an error.

Reflection

If there is a pattern to learners' errors what can a teacher learn from this?
What is a productive attitude towards learners' errors?

Conclusion

This unit has:

- introduced a data-based approach to teacher professional development;
- introduced some specialised assessment terms;
- described a teacher development project in which mathematics teachers used evidence (data) from learners' responses to tests to deepen their mathematical knowledge and to inform their classroom practice;
- used some of the data from this project to raise and suggest answers to questions about professional development that is based on analysis of data from teaching and assessment.

The writers hope that we have aroused your interest in using data (evidence) from learners' responses to mathematical tasks (classwork, homework or test items) to inform your teaching.

We encourage you to use what is offered in Units 2 to 5 of the module as part of your on-going professional development:

Unit 2: Curriculum mapping

This unit is in three parts. Firstly it introduces some theory about understanding curriculum and curriculum mapping; secondly, there are activities in which you will practise doing curriculum mapping; and lastly it gives you opportunities to reflect on the value of these activities.

Unit 3: Analysing learners' errors

The first part of this unit introduces you to what teachers need to know in order to understand learners' misconceptions and errors. The second part introduces you to a process you can use to identify and reflect on learners' errors, with plenty of activities for practice and opportunities for reflection.

Unit 4: Giving feedback to learners

This unit explores the role of feedback in learning and teaching, different ways to offer feedback, and principles for providing feedback that helps the learning process. Through a range of activities, it provides you with opportunities to practise and think about giving feedback in ways that meet learner needs and contribute to learning.

Unit 5: Using learners' error to inform lesson planning

The content and activities in this unit will support you in planning lessons that show awareness of errors and cater for a range of learners and learning needs. The unit presents transcripts of short episodes from mathematics lessons to situate the tasks within real mathematics lessons. Planning for lessons with errors in mind is critical so that teachers can be ready to address learners' misconceptions.

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Unit 1 Reading: Introducing the Data Informed Practice Improvement Project (DIPIP)

DIPIP was a three-year research and development programme (2007-2010) for 62 Grades 3 to 9 mathematics teachers from a range of schools in Johannesburg, South Africa. The project was a partnership between the School of Education at the University of Witwatersrand (WSoE) and the Gauteng Department of Education (GDE). Project staff members worked with teachers to design and to reflect on lessons, tasks and instructional practices, and to build professional learning communities (PLCs).

In South Africa, the Department of Basic Education's Delivery Agreement for Improved Basic Education notes that a "number of international testing programmes have demonstrated that South Africa's learner performance in reading, writing and mathematics is well below what it should be" (Department of Basic Education 2011, p.10). The Minister of Basic Education is accountable to South Africa's president for the improvement of the quality of teaching and learning, and specifically, improvement in the results of the National Senior Certificate and the Annual National Assessments (ANA) in Grades 3, 6, and 9. However, research confirms that without **internal accountability** in schools, it is unlikely that measures for **external accountability,** such as testing, will make a significant difference (Elmore, 2000).

One of the most important elements of internal accountability is systematic and collaborative reflection by teachers and school leaders on teaching and learning processes. To promote such internal accountability DIPIP provided an opportunity for collaborative enquiry in "professional learning communities" or PLCs. In these communities, teachers and "critical friends" (those with expertise and experience but not directly involved in classroom teaching) looked critically at the answers learners gave to multiple choice questions and at their worked solutions to other questions. Participants in the PLCs then considered how their reflections could be used to improve classroom practice.

In 2006, 2007 and 2008 a number of schools in Gauteng province used the ICAS[1] test in mathematics. The results of learners from these schools on the 2006 and 2007 tests provided the starting point for teacher engagement with learners' errors.

The project set up the following activities:

1. analysis of learners' results on the ICAS mathematics tests (with a focus on the multiple choice questions and the possible reasons behind the errors that led to learners' choices of the distractors[2]);
2. mapping of ICAS test items on to the South African mathematics curriculum;

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[1] International Competitions and Assessments for Schools (ICAS) is conducted by Educational Assessment Australia (EAA), University of New South Wales (UNSW) Global Pty Limited. Students from over 20 countries in Asia, Africa, Europe , the Pacific and the USA participate in ICAS each year. EAA produces ICAS papers that test students in a range of subjects including mathematics.

[2] "Distractors" are the three or four incorrect answers to multiple choice test items. They are designed to be close enough to the correct answer to 'distract' the person answering the question.

3. readings and discussions of texts about learners' errors in relation to two central mathematical concepts;
4. development of lesson plans for between one and five lessons (based on activities 1 to 3) which engaged with learners' errors in relation to central mathematical concepts;
5. reflections on videotaped lessons of some teachers teaching from the lesson plans;
6. design of a test of one of the central mathematical concepts, analysis of learners' errors on this test and an interview with one learner to probe his/her mathematical reasoning in relation to errors made on the test.

DIPIP as a response to increased concerns for accountability in education
Since 1994 the national Department of Education in South Africa has focused on a number of options to improve education. These have included redressing past social and economic inequalities, improving efficiency, increasing resource allocations, building capacity, and developing and phasing in three versions of a new curriculum. Once access and redress started to improve, there were expectations of improved quality of education and of improved learner achievement.

In order to compare the achievements of learners in South Africa with those in other countries, South African learners have participated in international tests such as Trends in International Mathematics and Science Study (TIMSS)[3] and the Progress in International Reading Literacy Study (PIRLS)[4]. However, not enough emphasis has been placed on the potential value of the data available from these international tests for informing teaching and learning practices. International research shows that merely having another set of data in the form of **benchmarking**[5], targets and progress reports that 'name and shame' schools leads to resentment and compliance but not to improvements in learning and teaching (McNeil, 2000; Earl and Fullan, 2003, and Fuhrman and Elmore, 2004).

Kanjee (2007, p. 493) sums up the challenge for South Africa:

> For national assessment studies to be effectively and efficiently applied to improve the performance of all learners, the active participation of teachers and schools is essential. … Teachers need relevant and timeous information from national (as well as international) assessment studies, as well as support on how to use this information to improve learning and teaching practice. Thus a critical challenge would be to introduce appropriate policies and systems to disseminate information to teachers. For example, teacher-support materials could be developed using test items administered in national assessments.

International research has begun to engage with the question of how to use test data for purposes other than benchmarking (Earl and Fullan, 2003; Earl and Katz, 2005; Katz et al., 2009). Katz et al (2005) draw an important distinction between two very

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[3] TIMSS was carried out in South Africa in 1995, 1999, 2003, 2011

[4] PIRLS was carried out in South Africa in 2006, 2011

[5] Benchmark testing measures students' learning against particular learning standards which are expected at a particular grade level.

different kinds of benchmarking practices. These are "accounting", which is the practice of gathering and organising data and "accountability", which refers to teacher-led educational conversations about what the data means and how it can inform teaching and learning. As noted by Elmore (2000), teachers can be held to account for their performance **only if** they have a deep understanding of the demands made upon them.

Curriculum statements about assessment standards, together with results of various standardised assessments, do not, in themselves, make standards clear to teachers. In themselves, they do not and cannot show what teachers' classroom practices should look like. As Katz et al argue

> Data don't "tell" us anything; they are **benign**[6]... The meaning that comes from data comes from interpretation, and interpretation is a human endeavour that involves a mix of insights from evidence and the **tacit knowledge**[7] that the group brings to the discussion. ... (2009, p.28)

For Hargreaves (2001)**,** working together on learning from data can assist teachers to build **collegiality**[8]. He argues that the future of collegiality may best be addressed by

> taking professional discussion and dialogue out of the privacy of the classroom and basing it on visible public evidence and data of teachers' performance and practices such as shared samples of student work or public presentations of student performance data (p.524)

A key issue to be addressed by those interested in teacher professional development is how to transform data collected from testing into structured learning opportunities for teachers. A number of questions need to be answered:

- In what ways can teachers be involved in analysing public evidence (test data)?
- What kinds of test data should be chosen for analysis?
- In what ways can analysis of data be integrated into the work that teachers do in schools?

Professional Learning Communities

Research on "Professional Learning Communities" (PLCs) and the related concept of "Networked Learning Communities", informed the approach used in the DIPIP project. The term 'professional learning communities' generally refers to structured professional groups, which are usually school-based and which provide teachers with opportunities for processing the implications of new learning (Timperley et al, 2007, p.201). Commonly, PLCs are created in a school and consist of school staff members or a cross section of staff members from different schools in a specific area of specialisation. "Networked Learning Communities", by contrast

> provide educators with opportunities to interact with each other within the boundaries of their own schools and boards or far beyond those traditional boundaries (Curriculum Services Canada, 2008, p.1).

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[6] Benign – in this context benign means neutral.

[7] Tacit knowledge – is unspoken, informal knowledge: something that a person "just knows" but cannot easily explain to someone else.

[8] Collegiality – a cooperative relationship among colleagues.

In the DIPIP project the groups had some elements of each:

- The small groups consisted of a group leader (a mathematics specialist – Wits School of Education staff member or post-graduate student who could contribute knowledge from outside the workplace), a Gauteng Department of Education (GDE) mathematics subject facilitator/advisor and two or three mathematics teachers (from the same grade but from different schools). This meant that the groups were structured to include different authorities and different kinds of knowledge bases. These were called **small grade-level groups** (or small groups). As PLCs the groups worked together for a long period of time (weekly meetings during term time at the Wits Education Campus for up to three years), sharing ideas and learning from and exposing their practice to each other. In these close knit communities, teachers worked collaboratively on curriculum mapping/error analysis, lesson and interview planning, test setting and reflection.
- For certain tasks (such as presenting lesson plans, video clips of lessons taught or video clips of learner interviews) the groups were asked to present to **large groups**. A large group consisted of the grade-level groups coming together into larger combined groups, each consisting of four to six small groups. This further expanded the opportunities for learning across traditional boundaries.

Evidence-based Learning
An "Evidence-based Learning" approach was adopted. This approach was used by the various groups to:

- identify (diagnose) learners' errors that occur during *teaching* and *assessment*;
- identify and develop strategies for addressing these errors in teaching and assessment (application of learning);

The ICAS tests provided the evidence base for error analysis that was then used to develop the teachers' diagnostic abilities. The test items as well as the results of Gauteng learners on the tests were available for analysis.

The main reasons for selecting the various activities that teachers worked on were the following:

- **Learners' needs inform what teachers need to learn.** The main argument emerging in recent literature in the field of teacher professional learning is that there is a positive (although not conclusive) relationship between teacher learning and student learning. In view of this, the project aimed to provide data for teachers on student learning (in the form of tests results, for example) and to structure activities for the teachers around this data. It was intended that they would draw on the data and activities to work out for themselves what they needed to learn.
- **Learners' mathematical errors are viewed as reasoned rather than random, and as such, error analysis can inform teaching and provide a focus for teacher development:** This idea comes from decades of research in mathematics education which suggests that many of learners' errors are underpinned by conceptual misunderstandings (or misconceptions). Mathematics researchers who view errors in this wayinsist that the role of teachers should not only be to

correct errors, but to understand the reasoning behind the errors that learners make. Then these errors can be used productively by teachers when they are teaching.

- **Reflection on assessment and classroom teaching data provides the evidence for learner errors and learner needs:** Working with teachers on classroom teaching is a very common activity in teacher development projects. Classroom teaching data is then used in a variety of ways to analyse and develop teachers' teaching competence. Less common is working with teachers on test data to diagnose learners' learning. In this project these two sets of data were combined sequentially into a series of activities to help teachers focus their classroom teaching (planning and enacting) on what the error analysis had shown them.

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Unit 2: Curriculum Mapping

Introduction
This unit is in three parts. Firstly it introduces some theory about understanding curriculum and curriculum mapping, then it presents activities in which you will practise doing curriculum mapping and lastly it gives you opportunities to reflect on the value of these activities.

Most dictionaries list two meanings of the word **map:**

- As a noun, it is a drawing of an area or country that shows key features such as roads, cities, rivers, mountains.
- As a verb, it means either to make a map of an area or to plan something carefully ('to map something out').

When teachers do curriculum mapping, they take notice of the key features of curriculum documents and use them to plan their teaching and their testing.

Aims of the unit

- To extend understanding of the concept 'curriculum'
- To introduce the concept of curriculum mapping
- To provide opportunities for comparing mathematics test items with curriculum requirements
- To provide activities that encourage reflection on mathematics teaching in relation to curriculum requirements
- To provide activities that focus on preparing mathematical tests with an awareness of curriculum requirements.

The relationship between the curriculum, teaching and assessment

A curriculum sets out what teachers are expected to teach in a particular subject. Yet many teachers never actually read curriculum documents or use them to inform their preparation – either for lesson planning or for test setting. This may be because they do not have access to a copy of these documents or because they do not realise the value of referring to them or because the documents are very difficult to read.
This Unit aims to assist you to understand your mathematics curriculum better by taking you through a series of **curriculum mapping** activities. To complete these you will need to refer to a mathematics curriculum and to reflect on your teaching of mathematics in relation to what the curriculum states.

Activity 1

Think about your teaching and the classroom tasks, tests or exams that you design.

d) How do you decide what to teach?
e) How do you know that the teaching methods you use are effective?
f) How do you decide what to assess?
g) Do you ever read the curriculum for the grades below and above the one you teach? If not why not? If you do, does this help you to teach and test? If it does, how does it help?

Commentary

Even though teachers are very busy, it is important to make time to think about the relationship between the curriculum, teaching and assessment. For example, in South Africa, learners have to write Annual National Assessments. The data collected from these assessments show how learners in particular schools and classrooms are performing in comparison to national and international standards for their grade. The department of education uses these test results to evaluate schools and has also introduced other forms of teacher evaluation.

These assessments are based on the curriculum for each grade. In order for the learners in your classes to perform well you will need to have 'covered' the curriculum and tested learners on sections of it as you go along. The minimum criteria for a **valid test** are that it covers what has been taught, and that what has been taught is aligned (is in line) with the curriculum for the grade.

The distinction between the intended, enacted and examined curriculum

In order to explain your assessment practices, for example during teacher evaluations, you need to be able to explain why particular content was selected for an assessment task and at what level of cognitive demand or complexity it was selected. Such explanations involve understanding the relationship between the intended, the enacted curriculum, and the examined curriculum. These concepts and the relationship between them are the focus of the reading, *The intended, enacted and examined curriculum,* which follows.

Activity 2

Please work through **Reading 1,** *The intended, enacted and examined curriculum***.**

a) What have you learnt from the reading about the distinction between the intended, the enacted, and the examined curriculum?
b) Can you think of an example from your own practice where the examined curriculum did not reflect the intended curriculum? If so, can you explain why this happened?

Curriculum alignment – coverage and level

A useful term for describing the relation between the intended, the enacted and the examined curriculum is "*curriculum alignment*". It was first used by John Biggs (2003) to indicate a way of thinking about what counts as a thorough approach to learning and teaching. In simple terms, to align something to something else means to arrange both 'things' so that they are in the same line. For example, in a car the wheels need to be aligned with each other so that the car can be driven in a straight line.

In curriculum terms, the concept of curriculum alignment provides "criteria" for what counts as a "productive educational environment". These criteria can help teachers with the main challenge they face when interpreting any curriculum document, i.e. when they need to identify "what" has to be covered (we call this curriculum coverage) as well as "the level" at which the selected content needs to be taught (we call this the level of cognitive demand).

Content and concepts

Curriculum coverage is never just about completing the learning programme designed by a teacher or school. Teachers need to think carefully about the levels of understanding of content that are appropriate to the age-level of the learners.

This means that it is important to think about the relation between content and concepts. The content of a topic consists of concepts, which organise the information embodied in the content and in this way control what the content means. To say that concepts organise the information embodied in the content, means that concepts impose order on the information, and they generate levels of complexity. Concepts also regulate what can and should be connected with what, and they give a form or a shape to the content. Concepts give a shape to the information by showing which piece of information is more central than another, which should come before the other, which is bigger and which is smaller etc. In order to know content well, teachers and learners need to understand the concepts that inform or shape it.

See for example
http://arb.nzcer.org.nz/supportmaterials/maths/concept_map_fractions.php

Figure 1: Fractional thinking concept map

<table>
<tr><td colspan="2">Partitioning and dividing quantities</td><td colspan="2">Part-whole fractions</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="2">Introduction</td><td colspan="2">Equivalence</td></tr>
<tr><td></td><td colspan="2">Fractions</td><td></td></tr>
<tr><td colspan="2">Fractions as operators</td><td colspan="2">Fractions and number lines</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="2">Unitising with fractions</td><td colspan="2">Adding and subtracting fractions</td></tr>
</table>

What the concept map about fractional thinking shows is that concepts structure the content into a network of ideas.

Activity 3

Study this ICAS test item for Grade 8.

10. Bella bought a surfboard that is one-third taller than her height.

    Bella is 156 cm tall.

    What is the height of her surfboard?

    (A) 208 cm (B) 189 cm

    (C) 104 cm (D) 52 cm

a) What is the correct answer?
b) What conceptual knowledge does the learner need in order to get to the correct answer?
c) Does this conceptual knowledge 'fit' into the concept map above? If so, where?

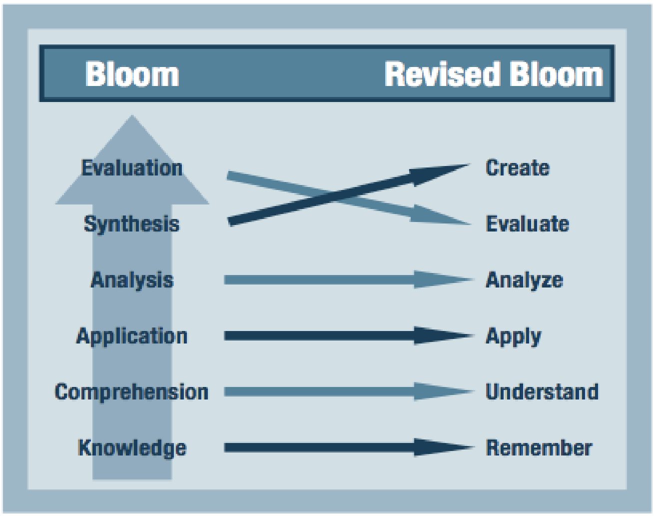
Commentary

To get the correct answer of 208 cm, learners need to have an understanding of how to calculate a third of a number. They would also have to know that 'one third taller than her height' means that the third is added to the figure of 156cm for her height. Finding a third of a number involves recognising that numbers can be partitioned or divided into equal quantities. This means that it would fit into the concept map above under 'partitioning or dividing quantities'.

Level

The next question that arises is how learners engage with these ideas. The most widely used taxonomy that categorises tasks in terms of their level of cognitive demand is Bloom's taxonomy. In the diagram below, Bloom's original taxonomy is on the left. Knowledge is at the lowest or first level and evaluation of knowledge at the highest. In the revised version on the right, new terms have been used for each level, beginning with 'remembering' knowledge. And the positions of synthesis (and creation of new knowledge) and evaluation have been reversed. The level of cognitive demand refers to the level of thinking that is required to address a question about a concept.

[for more information on Bloom's Taxonomy, please see http://www.unco.edu/cetl/sir/stating_outcome/documents/Krathwohl.pdf



Source: A Taxonomy for Learning, Teaching and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives, 2001.

Which of the following questions is at a higher cognitive level? Why?

a. Add 20 marbles to 450 marbles. How many marbles do you have?
b. If Lumkile gets pocket money of R12 each day for 14 days, and he decides to save a third of it, how much does he have available for spending?

Commentary

(a) Ask learners to remember the operation that is referred to by the word 'add' and then to apply it in response to a specific mathematical problem.
(b) Ask learners to identify which of the operations to use in order to solve a problem and then to apply those operations correctly in order to arrive at the correct answer. These are two very different levels of cognitive demand.

Mathematics educators argue that learners need to understand both of these levels well. In mathematical language, the distinction between these levels relates to a distinction between procedural and conceptual understanding of operations. Question (a) indicates which operation is required, there is only one operation and the numbers on which to operate are given. To find the solution learners could follow the procedure for addition. Question (b) is a more complex problem in which information about savings over a period of time is given. Learners would need to decide which operation to use to calculate the total amount of pocket money accumulated (multiplication) and which fraction of that will be saved (fraction of an amount; division). To find the solution they first need to think conceptually as they make the decisions about which operations to use after which they would perform the necessary procedures.

Biggs also discusses different levels: learners can do activities with concepts at different 'levels', for example,

1. “describe, identify, memorize”
2. “reflect, hypothesise, solve unseen complex problems, generate new alternatives” (Biggs 2008: 2).

The cognitive demand in the first set is “low level” - less difficult for learners to respond to as it asks them to remember a procedure they were taught.

The cognitive demand in the second set is “high level” because the question demands far more complex understanding. Learners need to understand the concepts that make up the content and the relation between them and in what mathematical ways they can be applied.

When a teacher covers the content of an intended curriculum at the appropriate cognitive level of demand (“enacted curriculum”) and her learners perform well on high quality tests (“examined curriculum”), it can be said that this teacher has created a productive learning environment, aligned to the demands intended by the curriculum. In Biggs’ words: this teacher’s learning environment “maximises the likelihood that students will engage in activities designed to achieve the intended outcomes” (Biggs 2008:2).

If the quality of a learning environment is judged by the results of the learners (from the perspective of the examined curriculum), all things being equal, it can be said that the results of the learners demonstrate that they have studied key content of the subject (“curriculum coverage”) and that they are able to use the content to answer a range of questions (“cognitive level of demand”).

If the quality of a learning environment is judged from the perspective of classroom practices (“the enacted curriculum”), all things being equal, it can be said that the methods that the teacher used to teach the content were useful, the questions he or she asked during the lesson or for homework were clear and appropriate to the content taught and to range of levels of cognitive demand, and to the age of the learners, and therefore aligned to the demands of the intended curriculum.

The word ‘alignment’ refers to how well the following elements of the learning environment created by a teacher are ‘connected’:

- the content selected for the topic;
- the cognitive level of demand expected from the learners;
- the methods of teaching chosen to teach the content;
- the cognitive level of understanding demanded in the various assessment tasks;
- the learners’ competence and motivation.

Together, these elements form a productive learning environment. In Biggs’ words:

> In setting up an aligned system, we specify the desired outcomes of our teaching, in terms not only of topic content, *but in the level of understanding* we want our students to achieve. We then set up an environment that maximises the likelihood that students will engage in the activities designed to achieve the intended outcomes.

> Finally, we choose assessment tasks that will tell us how well individuals have attained these outcomes, in terms of graded levels of acceptability. (Biggs 2008: 2)

Although all the different elements mentioned above contribute to the creation of a productive learning environment, “level of understanding” (or level of cognitive demand), is the key element. *'Outcomes'* is similar in meaning to curriculum requirements. If the level of understanding that a teacher works with is inappropriate to the content and the age of the learners, the learning environment in his or her classroom will not enable this teacher to achieve the curriculum requirements. For example, the appropriate learning is not likely to be achieved if a teacher designs assessment tasks that are not sufficiently demanding, or skips complex content when he or she teaches, or chooses methods of teaching that simplify the content too much.

Activity 4

Think about the last examination paper you designed.

a) How do you know that the questions you included covered the important content?

b) How do you judge that the questions matched the level of cognitive demand the curriculum required?

As stated above, teachers’ understanding of the level at which a topic should be taught depends on the quality of their subject matter knowledge. But it also depends on the degree to which various intended curriculum documents (e.g. the curriculum policy document, the textbook and the learning programme) make the range of levels explicit. Below we provide you with an example from a Grade 5 History Curriculum, where the differences between the two forms of the curriculum are very clear. If you have two different forms of a mathematics curriculum, you could ask yourself the same questions in relation to those documents. The example includes two extracts, each taken from a different form of curriculum. Extract 1 is taken from an outcomes-based curriculum and Extract 2 is taken from a content-based curriculum.

Activity 5

Use the table below to compare two extracts from two different types of curriculum for Grade 5 History/Social Sciences. The first extract comes from Curriculum 2005 (an outcomes-based form of curriculum). The second comes from CAPs (Curriculum and Assessment Policy Statements), (a content based form of curriculum).

How is the content of the curriculum specified?

a) How clear is the topic?

b) In how much detail is the content presented?

c) Are there any textbook or sources specified?

d) What detail does the extract give about assessment?

e) What indications are given about how the topic should be taught?

f) How is time on task specified?

Question	Extract 1: Curriculum 2005	Extract 2: CAPS
How clear is the topic?		
In how much detail is the content presented?		
Are there any textbook or sources specified?		
What detail does the extract give about assessment?		
What indications are given about how the topic should be taught?		
How is time on task specified?		

Extract 1 from Curriculum 2005 (C2005, South Africa)

Grade 5 Social Sciences (History)

The learning outcome 1 for Grade 5 history defines history enquiry as follows:

The learner will be able to use enquiry skills to investigate the past and present.

Important enquiry processes for this Learning Outcome include:

- finding sources;
- working with sources – asking questions, finding information, and organising, analysing and synthesising information;
- writing a piece of history (answering a question); and
- communicating historical knowledge and understanding (communicating an answer).

The assessment criteria for this outcome are as follows:

The learner asks questions about aspects of the past, present and future, using objects, pictures, written sources, buildings, museum displays and people (oral history).

We know this when the learner:

- with guidance selects sources useful for finding information on the past (e.g. oral, written and visual sources, including maps, graphs and tables, objects, buildings, monuments, museums) [finds sources]
- records and categorises information from a variety of sources (e.g. oral, written and visual sources, including maps, graphs and tables, objects, buildings, monuments, museums) [works with sources].
- continues to use information from sources to answer questions about people, events, objects, and places in the past [answers the question].
- communicates knowledge and understanding in a variety of ways, including presenting historical information in short paragraphs, simple graphs, maps, diagrams, creating artwork, posters, music, drama and dance; uses information technology where available and appropriate [communicates the answer].

List of topics:

- early civilisations: -- an early African civilisation: Egypt/Nubia; and one example from the rest of the world (Mesopotamia, Indus River Valley, China, and the Americas.
- why these civilisations occurred where they did; the key characteristics of these societies (e.g. the role of the environment in shaping the societies, use of resources, farming, the development of cities, technology, trade, communication, belief systems).
- early Southern African societies until 1600: how the environment shaped these societies, social organisation, appropriate technologies, stories exploring systems of belief, co-operation and conflict: hunter-gatherer societies; herders; African farmers.
- provincial histories: heritage and identity; tradition and indigenous knowledge of the significance of place names, rivers, mountains and other land-marks, including indigenous environmental practices; provincial government and symbols; role of democratically-elected leaders; how to participate in a democracy.

Extract 2 from the Curriculum and Assessment Policy Statement (South Africa)

<table>
<tr><th colspan="2">GRADE 5: INTERMEDIATE PHASE HISTORY - TERM 1</th></tr>
<tr><td>Topic: Hunter-gatherers and herders in southern Africa</td><td>Suggested contact time
One term/15 hours</td></tr>
<tr><td colspan="2">This content must be integrated with the historical aims and skills and the associated concepts listed in Section 2</td></tr>
<tr><td colspan="2">Background information: The content listed below applies to the last 10 000 years of the Later Stone Age. Older Stone Age periods go back over hundreds of thousands of years. Farmers entered southern Africa about 1 700 years ago. Hunter-gatherers were not marginalised or out-competed, but shared the southern African farming landscape with farmers over much of the last 1 700 years.

Focus: The way of life of the hunter-gatherers and herders, the earliest inhabitants of southern Africa, and how we find out about them.

Content and concepts
South Africa from 10 000 thousand years ago: people of the Later Stone Age
• How we find out about hunter-gatherers and herders (2 hours):
  • Stories, objects, rock paintings, books; in the present, by observing living societies (ethnography).
• San hunter-gatherer society in the Later Stone Age (8 hours)
  • Lived off the environment (A deep knowledge of the environment meant the San knew when wild resources were seasonally available. They moved to coincide with that availability.)
  • The invention of the bow and arrow, which contributed to hunting effectiveness
  • Social organisation: all things were meant to be shared equally within a group
  • Plant medicines
  • San beliefs and religion
  • Rock art
    o Where, when, how and why it was created
    o Interpretations of rock art
    o South African Coat of Arms and the Linton Rock Art Panel
• Khoikhoi herder society in the Later Stone Age (2 hours)
  • Pastoral way of life
  • How San and Khoikhoi shared the same landscape

Revision, assessment (formal and informal) and feedback should take place on an ongoing basis - 3 hours.
Learners should read and write for part of every lesson.
Evidence of learner’s work, including assessments, should be kept in the learner’s notebook.
Note: LTSM writers should not include detail on modern San in the Kgalagadi or in Namibia.</td></tr>
</table>

This above format of content specification is provided for all the topics to be covered in each term and must be integrated with the aims and skills listed in the following table.

The specific aims of History	Examples of the skills involved
• Finding a variety of kinds of information about the past.	Being able to bring together information, for example, from text, visual material (including pictures, cartoons, television and movies), songs, poems and interviews with people; using more than one kind of written information (books, magazines, newspapers, websites).
• Selecting relevant information.	Being able to decide about what is important information to use. This might be choosing information for a particular history topic, or, more specifically, to answer a question that is asked. Some information that is found will not be relevant to the question, and some information, although relevant, will not be as important or as useful as other information.
• Deciding about whether information can be trusted.	Being able to investigate where the information came from: who wrote or created the information and why did they do it? It also involves checking to see if the information is accurate – comparing where the information came from with other information. Much information represents one point of view only.
• Seeing something that happened in the past from more than one point of view.	Being able to contrast what information would be like if it was seen or used from another point of view. It also requires being able to compare two or more different points of view about the same person or event.
• Explaining why events in the past are often interpreted differently.	Being able to see how historians, textbook writers, journalists, or producers and others come to differing conclusions from each other and being able to give a reason(s) for why this is so in a particular topic of history.
• Debating about what happened in the past on the basis of the available evidence.	Being able to take part in discussions or debates and developing points of view about aspects of history, based on the evidence that comes from the information available.
• Writing history in an organised way, with a logical line of argument.	Being able to write a piece of history which has an introduction, sets out the relevant information in a logical way and in chronological order, and comes to a conclusion that answers the question asked in a coherent way.
• Understanding the importance of heritage and conservation.	Being able to explain how and why people and events are publicly remembered in a community, town or city, province and the country. It also involves investigating how people and events in the past are commemorated in ceremonies, celebrations, museums and monuments.

Commentary

Question	**Extract 1: Curriculum 2005**	**Extract 2: CAPS**
How clear is the topic?	Topic is not stated, the learning outcome is stated: 'Investigate the past and present'.	Topic is stated: 'Hunter-gatherers and herders in southern Africa'
In how much detail is the content presented?	A list of broad topics (e.g. the characteristic of societies) is provided in relation to a broad historical period such as 'early civilization', 'Southern African societies' and 'provincial history'.	The content and concepts of this topic are listed in a logical order. They include broad content such as social organisation and more specific ones such as 'The invention of the bow and arrow, which contributed to hunting effectiveness'.
Are there any textbook or sources specified?	No	No. In some topics a note is added to learning and teaching study material (LTSM) writers.
What detail does the extract give about assessment?	The curriculum provides learning outcomes and assessment criteria and expects the teachers to design assessment activities aligned with those.	The curriculum states the focus of the topic, provides a list of the content and concepts to be covered and expects the teachers to design assessment activities to cover these and include these in the learner's notebook.
What indications are given about how the topic should be taught?	Teachers are asked to draw on variety of sources and communicate what they know in variety of ways, some of which are specific to the subject of History (historical information, graphs and maps) and others are not (e.g. artwork	Not in the extract. The aims of history included in the extract form part of the introduction to the curriculum of this Grade and it includes examples of the skilled expected to be taught. It is expected that teachers will find appropriate opportunities

Question	Extract 1: Curriculum 2005	Extract 2: CAPS
	music, drama and dance). **The emphasis is on variation**	to apply those skills in relation to the topics, the coverage of which forms the bulk of the documents. **The emphasis is on content coverage**
How is time on task specified?	None	The total amount of time to be spent on teaching the topic and the time allocated to the contents and concepts of the topic are specified.

Outcomes-based curricula, such as C2005, intend to make the range of levels of understanding explicit. They use assessment standards to do this, but because they under-specify the concepts and content to be taught, the levels may remain abstract and vague. None of the assessment criteria cited in Extract 1 explain what it means to analyse and synthesise information in history or communicate historical knowledge and understanding and how these are different from doing so in a different school subject (e.g. Natural Science). These 'doing verbs' are under-specified, which makes lesson planning very difficult and may leave teachers feeling unguided.

When the curriculum provides content in a list of topics without any specific content attached to the topics, it opens up the possibility for too much variation (low reliability) in the types of textbooks that can be produced for teachers, in the criteria used by the school to select textbooks, in teachers' professional judgement of what counts as good achievement and most importantly in the kind of tasks teachers design, in the pacing and coverage of the curriculum. In this kind of curriculum it is much more difficult to gauge the concepts on which a topic is based and needs to cover. This would make curriculum mapping and drawing up a concept map, such as the one shown in Figure 1 of this Unit, almost impossible.

CAPS, the current curriculum in South Africa has gone a long way forward in specifying its requirements. The extract included in this Unit (Extract 2) shows how the new curriculum attempts to specify content coverage. The curriculum is arranged around content topics and not by assessment standards. The contact time is specified, and more importantly for curriculum mapping, the focus, the content and key concepts (e.g. social organisation, rock art, pastoral way of life) are listed. The aims of History and relevant skills are listed at the beginning of the curriculum document. Unlike an outcomes-based curricula which foreground learning outcomes and assessment criteria, a content-based curricula try to do both.

The above analysis shows that even a content-based curriculum, such as CAPS, does not always give enough guidance about the levels of understanding intended in it. In response to this lack of specification of content levels, departments of education have begun doing large-scale learner assessments in order to establish whether curriculum requirements are being met, and if so, to what extent. Together with textbooks and learning and teaching guides for teachers (including things such as lesson plans), these assessments are intended to mediate the conceptual requirements of the curriculum or to make it more explicit.

These large-scale learner assessments are a very useful tool for making curriculum requirements explicit as they provide teachers with a range of assessments at different levels of cognitive demand in relation to key subject matter content. However, if teachers do not have opportunities to take part in analysing the results of these learner assessments they will not be able to fully gauge the requirements of the intended curriculum. Taking part in curriculum mapping activities provides such an opportunity. In doing these activities you will use test data ("the examined curriculum"), the content and concepts specified in the curriculum ("the intended curriculum"), and your professional knowledge and experience ("the enacted curriculum").

Curriculum mapping can be defined as a "tool for establishing congruence between what is taught in the classroom and what is expected in state or national standards and assessments" (Crawford Burns 2001:1). *Congruence* is similar in meaning to alignment. Curriculum mapping enables teachers to record what content they teach in the classroom and to evaluate whether what they teach is aligned with the levels of cognitive demand expected in the examined curriculum.

After working through the activities in this unit you should be able to evaluate the quality of your assessment tasks, be clearer about the expectations you have of your learners, and be confident that you have a procedure to identify concretely, in your assessment tasks, the curriculum requirements they represent (or misrepresent!). To return to Biggs' idea of alignment:

> When strongly aligned, standards and assessments bring clarity to the education systems by providing a coherent set of expectations for students and educators. The assessments concretely represent the standards [curriculum requirements], providing the target upon which teachers can focus their instruction and students can focus their studies… Alignment to the standards also ensures that the assessment is trustworthy source of data. A study of an assessment's degree of alignment to the standards can serve as evidence of validity. (Case and Zucker, 2008: 3)

Think about a recent test that you or other teachers in your school designed.
Was it well aligned with the part of the curriculum on which it was based? Give reasons for your response.

A process for curriculum mapping

Before you embark on a series of activities that will help you to map test items onto the curriculum in a systematic way, you might find it interesting to read an account of the benefits of this process in the DIPIP project.

Activity 6

Go through **Reading 2** for this unit. While you are reading, think about this question:

According to the writers, what were the chief benefits for the teachers in engaging in the process of mapping test items against the curriculum?

Commentary

You might have noticed that the DIPIP teachers became more aware of the difference between the intended curriculum for the grade they teach as described in the official policy documents, and the enacted curriculum they actually taught in their school (Shalem, Sapire, Huntley, 2013). They became more aware of the extent to which they covered the curriculum, and whether or not they were providing sufficient cognitive challenge to the learners. These benefits were felt particularly because the teachers worked in groups that were well-facilitated, and they could talk through their responses with each other.

Although it is possible to work through the curriculum mapping process individually, to get the most benefit out of the exercise, we recommend that you work with fellow teachers in applying the questions in the following template to the test items in a systemic test.

		Teacher input
1	Grade level of item	
2	Test item (describe or write out)	
3	What are the mathematical concepts needed to successfully answer the question?	
4	In which grade are the concepts covered in the curriculum and how does the curriculum define them?	
5	Explain your reasons for mapping the item content with this curriculum reference.	
6	Do you teach this/these concepts?	
7	In what grade and during which term do you directly teach this/these concepts?	
8	Do you link this/these concepts with others? If so, which ones?	

- The first two rows require the identification of the grade and particular test item under discussion.
- The third row requires the identification of the mathematical concepts that a learner would need to understand in order to answer the test item. There may be more than one concept required.
- The fourth row requires a reading of the mathematics curriculum and identification of where the mathematical concept/s tested in the particular item is/are found in the curriculum. The identification of the curriculum specification

of the concepts should be given according to grade level and curriculum wording.

- The fifth row requires the teacher to explain his/her own reasons for mapping the test item concept/s onto the particular curriculum reference which has been noted.
- The sixth row begins the reflection on the teachers' practice. Here teachers are required to indicate whether or not they teach the mathematical concept/s tested in the particular item.
- The seventh row requires a more specific indication of the grade and term in which these mathematical concept/s is/are taught.
- The eighth row requires a closer look at the mathematical concept/s under discussion – teachers record in this row any concept which they teach in conjunction with the test item concept/s under discussion.

The activities that follow require you to use this template row by row, slowly building up understanding of all of the aspects involved in curriculum mapping.

When a systemic test, such as the Annual National Assessment test, is designed it is generally done in such a way that it covers the core curriculum concepts for the grade. Teachers of mathematics can analyse any test that their learners do to see whether or not it is indeed aligned with what they could be expected to know. The curriculum mapping activities that follow deal with the mathematical content represented in one item selected from the ICAS 2006 test.

NOTE: ICAS 2006 Grade 5 Item 9 is used in Unit 2 several times and again in Units 3 and 4. Choosing the same item across the units enabled us to use the same mathematical content to illustrate the three main activities covered in this learning material - curriculum mapping, error analysis and feedback to learners.

Analysing a test item

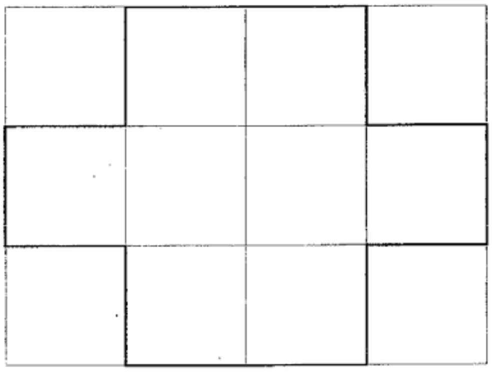
Any mathematical test item will call on learners to do some mathematics that he/she should know how to do.

When a learner reads a test item in order to answer it, he/she is actually decoding the question for mathematical meaning. As a teacher, if you did not set the test, you want to identify exactly what is being called for by a test item. To do this you need to identify the mathematical concepts tested in the particular item.

Activity 7

Study the test item below:

9. Holly drew this shape on 2 cm grid paper.



What is the area of Holly's shape?

(A) 32 cm^2
(B) 28 cm^2
(C) 16 cm^2
(D) 8 cm^2

(ICAS 2006 Grade 5 item 9)

a) Solve the item yourself.
b) Does this test item call on mathematical concepts from only one or from more than one area of mathematical knowledge? Identify the area(s).
c) What mathematical concepts would a learner need to know in order to answer the question?

Commentary

This item requires learners to calculate the area of shape which is made up of eight 2 cm by 2 cm squares which are drawn on 2 cm grid paper. They could do so by drawing in the grid lines and counting the squares which make up the shape, or by using the formula for area of a square (in this case 2 cm by 2 cm which is 4 cm^2) and then finding out the total area covered by the shape which is made up of eight squares altogether. The area of Holly's shape is thus 4 $cm^2 \times 8 = 32$ cm^2.
The mathematical concepts involved are area, calculation of area (using a formula or grid), units of area and operations (multiplication and addition). So there are concepts from mathematical knowledge about numbers and operations as well as shapes and measurement. This item tests learners' broad mathematical understanding.

What can teachers learn through analysing the mathematical content present in a systemic test that learners have written?

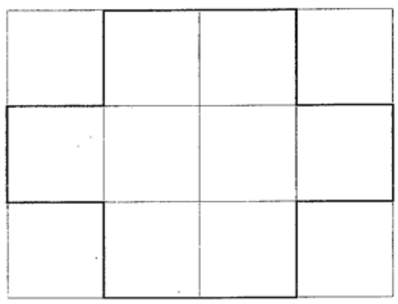
Reading the curriculum document

To be considered rigorous [careful and thorough], of high quality and valid [reasonable, acceptable], tests need to be shown to be aligned to the curriculum (McGehee & Griffith, 2001, Case, Jorgensen & Zucker, 2008; Brookhart, 2009). Teachers are not usually part of the process of curriculum alignment which is generally left to policy makers, designers of large scale assessment and educational researchers. However, as a teacher, you can learn a lot through being involved in curriculum alignment activities. The next activity gives you an opportunity to study the curriculum document and to align test content with curriculum content.

Activity 8

Refer again to the test item below:

9. Holly drew this shape on 2 cm grid paper.



What is the area of Holly's shape?

(A) 32 cm^2
(B) 28 cm^2
(C) 16 cm^2
(D) 8 cm^2

(ICAS 2006 Grade 5 item 9)

a) In which grade(s) in the mathematics curriculum is the mathematical content, that you identified as covered by this test item, to be found?
b) Write out the specific curriculum wording for the identified content.
c) Draw up a list or a mind map in which you write about as many concepts as you can think of that are related to the concept of area as it is presented in this test item.

Commentary

The concept of area as it is presented in this item goes beyond what is specified for grade 5 in the South African Curriculum and Assessment Policy Statement (CAPS). It would be covered by the grade 6 curriculum specification (where learners are expected to develop the formulae for calculation of area) and also the grade 7 specification (where learners should be using the formulae for the calculation of area).

Appropriate curriculum wording from CAPS:

Grade 5: *Area of polygons (using square grids and tiling) in order to develop an understanding of square units* (p.38).
Grade 6: *Area of polygons (using square grids) in order to develop rules for calculating the area of squares and rectangles* (p.51); and *Investigates relationships between the perimeter and area of rectangles and squares* (p.61).
Grade 7: *Area of triangles, rectangles and squares* and *describes inter-relationships between perimeter and area of geometric figures* (p.56).

A concept map for area could bring in concepts that are built on in this activity as well as concepts that can be built on to what is being done here. Here are some of the necessary and related concepts called upon in the completion of this activity:

1. The use of operations (in this case, basic multiplication and addition).
2. Understanding of the concept of area – to read and understand the question learners need to know what "area" means (the amount of surface covered by a shape).
3. Calculation of area of a shape – either using a formula or a grid. This is necessary since learners have to answer the question "what is the area?".
4. Building on the knowledge required to work out the answer to this question, learners would be able to move on to the related activities of investigating and describing the relationship between the area and perimeter of a shape.

Do you ever teach mathematical concepts from more than one area in the same lesson?
If so, explain why you do this, and how you do it.
If not, would you consider doing it? What do you think might be the advantages? Could there be any disadvantages?

Reflecting on mapping choice

When teachers reflect on their mapping choices they complete the cycle of activities from the test to the curriculum and back – and through it they justify the choice(s) made in identifying curriculum references for the mathematical concepts specified as "covered in the test item".

Activity 9

Look back at ICAS 2006 Grade 5 item 9 in Activity 8 above.

a) Explain your reasons for mapping the item content to your chosen curriculum reference(s).

b) Do you notice any differences between the concepts you first identified and the curriculum references that you chose? If so, what are these differences?

c) Do you think you might change the concepts you initially identified as necessary to solve the item correctly? If so, how?

d) Do you think you might change the curriculum references you initially identified as those which specify the concepts required to successfully work out the answer to the question? If so, how?

Commentary

This activity calls on teachers to re-think both aspects of curriculum mapping introduced in the two activities that came before it. In so doing they may change their minds about some of the concepts that seemed necessary to answer the question correctly and/or about the grade level at which the question was set. Reflecting on the decisions made in curriculum mapping enables teachers to refine and clarify their mapping. Especially at first, it is important to reflect on decisions in this way.

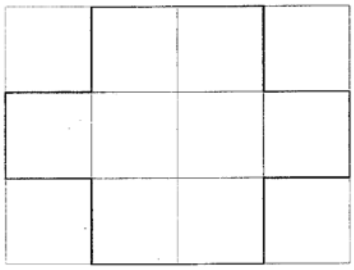
Reflecting on teaching

Building on the previous content items introduced, this activity provides an opportunity for reflection on the teaching of concepts related to the test item under discussion.

Activity 10

Refer again to the test item below:

9. Holly drew this shape on 2 cm grid paper.



What is the area of Holly's shape?

(A) 32 cm^2
(B) 28 cm^2
(C) 16 cm^2
(D) 8 cm^2

(ICAS 2006 Grade 5 item 9)

a) Do you teach this/these concepts in your mathematics class?
b) In what grade and during which term do you directly teach this/these concepts?
c) Do you link this/these concepts with others? If so, which ones?
d) If you don't teach these concepts in your grade, explain why not.

Commentary

This activity has called on teachers to reflect on the teaching of the maths content presented in the test item under discussion. Each teacher's responses will be personal but may be shared by others. Notice that one question asked about teaching concepts directly or indirectly. Direct teaching occurs when a teacher explains things to a class using words, diagrams or algorithms that relate specifically to the concept being taught.

Indirect teaching occurs when a teacher provides an opportunity for learners to work on or think about the concept being taught without a direct explanation being given. Sometimes teachers take an indirect approach when learners already know how to do something and so they build it into another exercise to consolidate or apply what has been learned. It is important for teachers to be aware of how they teach concepts. If teachers make links between related concepts this will enable learners to better understand the concepts under discussion.

What do you notice about your lessons when you teach mathematical content with which you are familiar?
Reflect on the spread of the mathematics that you teach your learners over a year. Do you sometimes teach mathematical content with which you are not familiar? If so, discuss what happened in a lesson when you taught such content.

Further curriculum mapping activities
In order to provide you with more practice in curriculum mapping across all mathematical content areas, we suggest you work through the items in **Unit 2: Further Curriculum Mapping Activities.**

The items have been selected in order to raise discussion of the mathematical content they present. There is one activity per grade, from grade 3 to grade 7, so you can choose the grade level or levels at which you work. The activities do not represent an entire mathematics curriculum since this is not possible within the scope of the unit, but they do present material which is often misunderstood by learners and about which teachers would benefit from a deep understanding.

The commentary given does not relate to the full range of questions you need to work through when you do these activities, but it pinpoints the curriculum requirements for the given tasks. You should discuss your activity responses with a colleague if possible to gain the full benefit from these activities.

Preparing mathematical test content with an awareness of the curriculum requirements

When you prepare a test for your learners it is useful to do so with an awareness of the curriculum requirements for the topics which the test will cover. You should do this developmentally (in small tests that do not necessarily cover the full curriculum content on a particular topic) or summatively (after you think you have completed the teaching on a particular topic). If you are constantly thinking about curriculum requirements you are more likely to make sure that your learners are given the opportunity to show their full understanding of the content that you have covered in class. It will also help you to identify possible gaps in your own teaching which you could then fill before moving on to the next topic. The next activity gives you an opportunity to “set a test” on a particular topic – starting from the curriculum document and then moving on to designing the questions.

The following table gives a summary of the number ranges for the teaching of place value from Grades 2 to 6 in the South African Curriculum.

Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
Recognises the place value of digits in whole numbers to at least 2-digit numbers.	Recognises the place value of digits in whole numbers to at least 3-digit numbers.	Recognises the place value of digits in whole numbers to at least 4-digit numbers.	Recognises the place value of digits in whole numbers to at least 6-digit numbers.	Recognises the place value of digits in: whole numbers to at least 9-digit numbers decimal fractions to at least 2 decimal places

Activity 11

Study the curriculum information given above which relates to the teaching of place value. If you are a grade 7 teacher, use the grade 6 requirements to set a “revision test” on place value concept.

Design a short test with six questions for the grade which you teach.

a) First examine your curriculum specification and then write out in your own words what you think it requires of learners in your grade.

b) Explain how you would be able to know (by observing your learners’ work) that they have understood the concepts you have written about in (a).

c) Design a six question test so that it would enable learners to show evidence of their understanding or reveal gaps in their knowledge that you could address after setting the test.

Commentary

Test setting, if it is done with the curriculum in mind, can be directed to exposing gaps in the learners’ knowledge which would be useful for you to address. It can also enable you to check that your learners have adequately grasped the knowledge you have taught so that you can move forward with confidence to the next section of your teaching. In order for you to be able to set tests and use them in this way you need to refer to your curriculum documents often and think carefully about what they say about the concepts and skills that you are required to teach.

Have you ever set a test after first referring to your curriculum document? How did you find this experience?

Conclusion

This unit has:

- introduced some specialized curriculum terms;
- explained the relationship between the intended, enacted and the examined curriculum;
- introduced the activity of Curriculum Mapping;
- showed a way for learning the conceptual demands of a curriculum by identifying the concepts that underpin the test items of a large-scale assessment, their place in the curriculum, and the level of thinking that is required to address a question about a concept;
- explored the usefulness of curriculum mapping as a means for learning about your teaching, specifically about aligning it more closely to the curriculum for your grade.

The idea of curriculum mapping is based on an understanding that there are differences between the intended, enacted and assessed curriculum and following from John Biggs's concept of curriculum 'alignment', this Unit provided you with the process developed in DIPIP 1&2 to teach teachers how to compare mathematics test items with curriculum requirements.

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Unit 2 Reading 1: The intended, enacted and examined curriculum

This Reading was prepared by Yael Shalem especially for this module. It draws on the *Saide*/Oxford publication, *Curriculum: Organising knowledge for the classroom* (third edition) authored by Ursula Hoadley and edited by Yvonne Reed (December 2012)

In order to explain your assessment practices, for example during teacher evaluations, you need to be able to explain why particular content was selected for an assessment task and at what level of cognitive demand or complexity it was selected. Such explanations involve understanding the relationship **between the intended curriculum, the enacted curriculum, and the examined curriculum**. These terms will be explained below.

Which of the items below would you call a curriculum?

a) a policy document on educational principles and grade specific content put out by the National Department of Education;
b) the timetable for a school term, including extra- mural activities;
c) a learning programme created by a teacher for her class for a term;
d) a mathematics textbook for grade 6.

All of the above have something in common. They prescribe to the teacher, in one form or another, what is to be learned, how and when:

- *The time table for a school term* specifies when the content of a specific subject is to be taught. In curriculum terms, the time-table specifies pacing and sequence – how long a teacher should spend on specific content and when this content should be taught.
- *A policy document on educational principles and grade specific content* (henceforth the policy document) provides schools with the official curriculum plan prescribed by the department of education. It outlines the values and principles that underpin the curriculum, and specifies the content that should be covered for each grade and the ways the grade-specific content should be assessed. In curriculum terms it provides "a curriculum framework" that broadly conveys the standards that learners should achieve.
- A *learning programme created by a teacher* and *a text book* also provide a plan, but a more detailed plan. Both are official documents produced by people with educational authority (a teacher, a textbook writer), and therefore both form a part of the official curriculum.

The main difference between the first two documents and the last two is in the detail with which they describe what is to be covered. The last two are far more specific. They rely on the policy document but produce a more specific plan for the teacher. A textbook writer reads the curriculum specifications of the policy document and on the basis of these, elaborates and explains key content, selects educational material on specific topics and designs tasks that teachers can use when they are teaching that material. A teacher's learning programme is based on the curriculum specified in the policy document, on the time table of a school term and on the textbook. A teacher uses all of these to produce a plan for covering subject matter topics each week of the term, and for deciding what teaching and assessment activities to use. In this way a learning programme puts into practice the intended curriculum standards.

Together, all of these documents provide teachers with an official curriculum plan of what the department of education expects learners in a particular grade to know by the end of the year. The use of the word 'plan' is important. It suggests that curriculum organises content in a particular way, conveying what teachers are expected to cover and how they are expected to do this. The idea of a **plan** is central to curriculum. Elliot Eisner defines curriculum as

> a series of planned events that are intended to have educational consequences for one or more students (Eisner 1985: 61)

According to this definition, a curriculum is a planned programme for teaching learners new content with the goal of extending their knowledge and skills.

Before we continue with the discussion of the curriculum process, we would like you to consider a few definitions of curriculum. These are taken from three readings that we strongly suggest you read carefully. Below we list three definitions. We would like you to reflect on each one and explain its specific contribution to the understanding of curriculum. You are not requested to compare and evaluate the definitions. You are requested to single out the different ideas about curriculum that each of the definitions brings out:

- A curriculum is an attempt to communicate the essential principles and features of an educational proposal in such a form that it is open to critical scrutiny and capable of effective translation in practice (Stenhouse, 1975: 4)
- The curriculum …comprises all the opportunities for learning provided by a school. It includes the formal programme of lessons in the timetable…and the climate of relationships , attitudes, styles of behaviour and the general quality of life established in the school community as a whole (The former Department of Education and Science for England and Wales quoted by Graham-Jolly in Hoadley 2012: 219)
- Curriculum…can be seen to be interconnected at different levels of scales. Indeed, it is the translation from one level to another which often produces gaps between intentions and what actually occurs in the classrooms (Lovat and Smith, 1995: 18).

Curriculum theorists have developed the term *the intended curriculum* to emphasise that at the top of the curriculum process is a plan that is designed with a view to meeting a set of carefully selected aims and to satisfy the educational meanings behind these aims.

The first of the four curriculum documents described above - a *policy document on educational principles and grade specific content* - provides this kind of plan. It is the big plan, the intent, the curriculum requirements that should be adhered to when a school plans for a specific school subject for a specific grade.

Each of the other three documents makes the intended plan more specific. The time-table interprets the government's total time allocation for a school subject for the grade, into a weekly organised time-table. This plan influences how a teacher or a head of department will then plan a learning programme. Whoever designs the learning programme in the school will need to take into account the total content coverage demanded by the official curriculum policy document and then devise a plan that will divide the total content of a topic across the different time sessions allocated by the school timetable. The learning programme planner may use textbook/s and other educational material to select and sequence learning activities suitable for the content at a particular grade level. He or she will also need to specify the assessment activities for which teachers should prepare their learners so that the content is taught at the expected standard.

Until this point two main ideas about curriculum have been emphasized. The first is that there are different educational materials that make up the curriculum. This means that the curriculum is not summed up in one document but is the sum is informed by all sorts of documents. Some of these are created externally to the school and others are created by teachers at the school in response to the specific needs of their learners. The second idea is the level of specificity. As already explained, there are some differences among curriculum documents in terms of how specific / comprehensive they are, their levels of prescription, their degree of influence, who designs them, and the way each of them specifies what is to be covered, in what depth, and at what pace. For example, curriculum documents that are designed by the department of education (such as a policy document) are official and influence any curriculum documents produced at any school. While some are more prescriptive than others even the most prescriptive cannot fully guide the day to day work of the teacher. This is why other educational experts such as textbook writers, a head of department or a principal produce other curriculum documents (a textbook, a learning programme, a timetable), which are more prescriptive although they may not have as wide an influence as a department of education policy document.

What does all of this mean for you? First, that the educational aims that underpin your subject matter and your aspirations for your learners require deep reflection and occasionally an analysis of the curriculum. We hope that the curriculum mapping activity (more on this below) will show you one useful way of analysing what you want your learners to achieve. The second implication of thinking about curriculum as being made of several documents of different degrees of specificity, is that there is a gap between educational aims and goals as they are prescribed in the intended curriculum and what, in fact, happens in the classroom. It is to this idea of a gap between the intended and the enacted curriculum that we now turn. Understanding the gap is important for all teachers.

A useful curriculum theorist to read is Lawrence Stenhouse. If you would like to read his views in more detail, you can consult chapter 1 of his book *An Introduction to Curriculum Research and Development* (published in 1975 by Heinemann).

Stenhouse draws a distinction between two perspectives on curriculum. The first perspective (what is now called the intended curriculum) emphasises the prescriptive dimension of curriculum. The emphasis in this perspective is on the documented intentions of the school, on what the school and more specifically the teachers aspire(hope) for learners to achieve. The second perspective (what is now called the enacted curriculum) draws attention to what actually happens in the classroom as a result of what the teachers do and to the reality of everyday school life (p2). **When you think about curriculum on which perspective do you focus more?**

Stenhouse argues that neither of the perspectives on its own is sufficient, that they need to be viewed together rather than in opposition. Although Stenhouse maintains that curriculum is both 'aspiration' (intended curriculum) and the 'existing state of affairs' (enacted curriculum), he also claims that "the gap between aspiration and practice is a real and frustrating one" (p3).

How do you understand the gap between the intended curriculum and the ways it is operationalized or enacted in the classroom?

There are many reasons why the intended curriculum is not always enacted exactly as it was planned. Firstly, classrooms differ in the amount of physical and learning resources available to teachers and learners and lack of resources means that certain expectations specified in the intended curriculum, by curriculum policy documents and/or by textbooks, cannot be met by teachers who work in poorly resourced schools. Many of these schools do not have access to important resources (e.g. finances, computers, laboratory, library) and to learning material (e.g. textbooks and other learning material often referred to in the literature as cognitive resources). The official curriculum plan is designed for learners who have access to many cognitive resources (such as books, the internet and conversations in the home on abstract ideas). Research shows that learners who have the advantage of well-resourced classrooms, homes and communities achieve better results. Secondly, teachers' morale and motivation may vary. For example, there are times at which labour issues may have a negative effect on the morale of teachers and this may affect the quality of their teaching. Thirdly, the subject matter knowledge of teachers varies. Teachers with weak subject matter knowledge (for whatever reasons) are unlikely to be able to cover all the content that is specified by the curriculum, and may also select inappropriate methods for teaching it. Limited subject matter knowledge is likely to affect their capacity to enact the official curriculum plan (the intended curriculum).

Learners in their classes can be expected to have serious gaps in what they are supposed to know and be able to do for their grade level. Lack of subject knowledge is also likely to make it difficult for teachers to adjust the curriculum in ways that will benefit their learners. Put in different words, whether or not teachers

> have a thorough knowledge of their subject, have teaching and assessment skills, are motivated to learn more as they go along, care for their students, have self-confidence, or feel enthusiastic about their jobs all make a difference to the way in which they work with the curriculum plan and to the way in which students experience the curriculum. Bialobrzeska, & Allais, 2005, p. 129)

Finally, official curriculum policy documents are not always written very clearly, which means that it cannot be taken for granted that teachers understand what they mean. There are many factors that make an official curriculum policy or the intended curriculum more or less clear, but the main one is their form. There are two main forms of intended curriculum: outcomes-based and content-based. Outcomes-based curricula tend to specify the assessment criteria and the tasks associated with them. They underspecify (give little detail about) the content that teachers are required to cover. The idea behind an outcomes-based curriculum, which more and more research shows is incorrect, is that learners need to be taught skills and that the skills can be transmitted by using any subject matter content, chosen and selected by teachers. A content-based curriculum, on the other hand, is based on a very different principle. It assumes that subjects have different knowledge structures. For example, subjects such as physics and biology and to a great extent mathematics, have a vertical knowledge structure. This means that what is taught in higher grades builds on what was taught in lower grades. If for any reason learners were not taught what they should have been taught in a lower grade, the 'knowledge gap' that they bring to a higher grade will make it very difficult for them to understand the new content introduced in this grade. A content-based curriculum, specifies what content teachers need to cover, in what sequence, and what skills they should emphasise when they teach it. Research shows (Jansen, 1998 a and b; Hoadley, 2011) that teachers prefer a content-based curriculum because they feel more confident about what it requires them to cover. Textbook writers, time-table planners and learning programme planners also feel that this type of curriculum offers them more guidance.

Curriculum theorists have developed the terms "**curriculum as praxis**" (Grundy, 1987), "**curriculum as process**" (Stenhouse, 1975) "**curriculum in context**" (Cornbleth, 1988) to signal that there may be gaps between what is intended by an official curriculum plan and what is **enacted** in the classroom. From the perspective of studying curriculum, teachers need to understand that curriculum is more than its intended aims and standards. It is more than its intended principles and it is more than its intended plan. The curriculum also involves actual classroom practices that teachers design for their lessons, learners' experiences, teachers' attitude and morale, parents' involvement, and of course teachers' and learners' access to physical and learning resources.

The **examined curriculum** is usually part of the enacted curriculum. The examined curriculum, which involves classroom or school-based assessment and large-scale learner assessment, is not often discussed as a learning opportunity for teachers. It is considered an obvious educational practice, through which learners are tested on what they have learned or teachers and schools are evaluated on whether or not they have delivered a service of quality.

One of the aims of this unit is to demonstrate that the examined curriculum can be a vehicle through which teachers can learn about the intended and the enacted curriculum. For example, assessment (the examined curriculum) can make the knowledge demands of the intended curriculum much more explicit to teachers (and to learners) than a learning programme is able to do. Analysis of tests and large-scale examinations can help teachers not only to understand the curriculum better, in particular its breadth and depth, but also to identify the knowledge gaps in their learners' and their own practices.

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Mapping onto the mathematics curriculum – an opportunity for teachers to learn

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Dates:
Received: 07 Sept. 2012
Accepted: 22 Mar. 2013
Published: 22 May 2013

How to cite this article:
Shalem, Y., Sapire, I., & Huntley, B. (2013). Mapping onto the mathematics curriculum – an opportunity for teachers to learn. *Pythagoras, 34*(1), Art. #195, 10 pages. http://dx.doi.org/10.4102/pythagoras.v34i1.195

Curriculum mapping is a common practice amongst test designers but not amongst teachers. As part of the Data Informed Practice Improvement Project's (DIPIP) attempt to de-fetishise accountability assessment, teachers were tasked to investigate the alignment of a large-scale assessment with the South African mathematics curriculum. About 50 mathematics teachers from Grade 3–9 worked in groups together with subject facilitators from the Gauteng Department of Education and a university postgraduate student or lecturer who acted as group leader. The first project activity, curriculum mapping, provided a professional development opportunity in which groups mapped mathematical assessment items to the assessment standards of the curriculum. The items were taken from three sources: the 2006 and 2007 International Competitions and Assessments for Schools tests and from 'own tests' developed by the groups in the last term of the project. Groups were required to analyse the knowledge base underlying test items and to reflect on what they teach in relation to what the curriculum intends them to teach. They used a protocol (mapping template) to record their responses. This article deals with the question of how to transform data collected from large-scale learner assessments into structured learning opportunities for teachers. The findings were that through the curriculum mapping activity, groups became more aware of what is intended by the curriculum and how this differs from what is enacted in their classes. The findings were also that the capacity of groups to align content was better when they worked with leaders and that with more experience they gained confidence in mapping test items against the curriculum and made better judgments in relation to curriculum alignment. Involving teachers in the interpretation of both public assessment data and data from their own classroom activities can build their own understanding of the knowledge base of test items and of the curriculum.

Introduction

As a policy lever for benchmarking standards and for monitoring performance, the South African Department of Basic Education has embarked on a number of national initiatives to collect learner assessment data. A variety of international and local large-scale systemic assessments have been conducted in the country. To date the data from these systemic assessments, the test items as well as the test results, have been used by mathematical and language experts, economists and statisticians at a systemic level and predominantly for benchmarking. Teachers have not participated in the production of this evidence nor has the opportunity for developing teachers' interpretive skills of such data been taken up. The question is how to transform data collected from large-scale learner assessments into structured learning opportunities for teachers. This article deals with this question.

Merely having another set of data in the form of benchmarking, targets and progress reports that 'name and shame' schools leads to resentment and compliance but not to improvement of learning and teaching (Earl & Fullan, 2003; McNeil, 2000). In South Africa, Kanjee (2007) sums up the challenge:

> For national assessment studies to be effectively and efficiently applied to improve the performance of all learners, the active participation of teachers and schools is essential. … Teachers need relevant and timeous information from national (as well as international) assessment studies, as well as support on how to use this information to improve learning and teaching practice. Thus a critical challenge would be to introduce appropriate policies and systems to disseminate information to teachers. For example, teacher-support materials could be developed using test items administered in national assessments. (p. 493)

It appears that in using Annual National Assessments the South African Department of Basic Education is aiming to provide teachers with timeous information from national assessments to guide planning and monitor progress (Department of Basic Education, 2010). What is not clear is how the department is planning to support teachers on how to use this information to improve learning and teaching practice. Very little attempt has been made to involve teachers in data interpretation and not enough emphasis has been placed on the potential value of the data available from these systemic evaluations for informing teaching and learning practices. International research has engaged with the question of how to use assessment data beyond benchmarking (Earl & Fullan, 2003; Earl & Katz, 2005; Katz, Earl & Ben Jaafar, 2009). In thinking about this question, Katz, Sutherland and Earl (2005) drew an important distinction between two very different kinds of practices in benchmarking: 'accounting', which is the practice of gathering and organising of data, and 'accountability', which refers to teacher-led educational conversations about what the data means and how it can inform teaching and learning. Katz et al.'s (2005) distinction is very important and is in line with Elmore's (2002) and Hargreaves's (2001) important arguments. Hargreaves (2001, p. 524) argues that the future of collegiality may best be addressed by (*inter alia*) taking professional discussion and dialogue out of the privacy of the classroom and basing it on visible public evidence and data of teachers' performance and practices, such as shared samples of student work or public presentations of student performance data. Elmore (2002) claims that teachers can be held accountable for their performance only if they have a deep sense of the demands made upon them. Although this may seem obvious, the challenge lies in identifying what counts as making accountability standards explicit.

Literature on professional development programmes for teachers shows that piecemeal forms of intervention are not effective (Borko, 2004; Cohen & Ball, 1999; Earl & Katz, 2005; Elmore & Burney, 1997; Katz et al., 2009). A broad consensus seems to emerge around the following claims: firstly, that teachers require continuous interactive support over a substantial period of time. Secondly, that teacher learning should be focused on specific (and few in number) educational objects and guided by an expert who is acting as a critical friend. Thirdly, that within the current emphasis on accountability, professional conversations by teachers, in support networks (broadly referred to as 'professional learning communities'), can provide teachers with a productive opportunity to cultivate a sense of ownership of what the data means, specifically in relation to their current practices.

The Data Informed Practice Improvement Project

Working with teachers on interpretation of learner assessment data was the central goal of the Data Informed Practice Improvement Project (DIPIP), Phase 1 and Phase 2 (Shalem, Sapire, Welch, Bialobrzeska & Hellman, 2011). The DIPIP project provided a context for professional conversations in which mathematics teachers, together with university academics, graduate students and department-based subject advisors, discussed assessment data. In these discussions, groups were dealing with information from the assessment data that could be used to think about reasons for learners' errors, map the test items to the National Curriculum Statements (NCS), read and discuss academic texts about mathematical concepts (e.g. the equals sign) and learner errors related to these, develop lesson plans, and reflect on videotaped lessons of some teachers teaching from the lesson plans.

The positive outcomes of research done on the efficacy of professional learning communities served to inform the approach used in this project (Brodie & Shalem, 2011). The term 'professional learning communities' generally refers to structured professional groups, usually school-based, providing teachers with opportunities for processing the implications of new learning (Timperley & Alton-Lee, 2008). Commonly, professional learning communities are created in a school and consist of school staff members or a cross section of staff members from different schools in a specific area of specialisation. The groups in the DIPIP project were structured differently and included teachers and practitioners with different knowledge bases and role specialisations (see below). As professional learning communities, the groups worked together for a long period of time (weekly meetings during term time at the Wits Education campus for up to three years from 2007–2010), sharing ideas, learning from and exposing their practices to each other. In these close-knit communities, teachers worked collaboratively on curriculum mapping and error analysis, lesson and interview planning, test setting and reflection.

To provide the basis for a systematic analysis of learners' errors, test items and learner achievement data of an international standardised multiple-choice test, the 2006 and 2007 International Competitions and Assessments for Schools (ICAS), was used.[1] For the curriculum mapping activity the groups were tasked with investigating the alignment of the ICAS tests items (not learner achievement data) with the curriculum at the time, that is, the NCS for Mathematics (Department of Education, 2002). This article focuses on the nature and outcome of the curriculum mapping activity, presenting findings on how the curriculum mapping activity provided groups of practitioners an opportunity to engage with and reflect on the curriculum, and discussing in what ways and to what extent this activity succeeded.

Curriculum standards – teacher knowledge and interpretation

There are two main different forms of curriculum. The first is skills based and presents a collection of statements (outcomes and assessment standards). The second is content based and its form foregrounds the conceptual structure of the intellectual field from which it selects specific subject matter. Research in South Africa has shown that an outcomes-based curriculum provides weak signals to teachers about coverage, sequence and progression. Upon the findings of several

1. The ICAS test is designed and conducted by Educational Assessment Australia (EAA). In the Gauteng province of South Africa, 55 000 learners across Grade 3–11 in both private and public schools (3000 in total) wrote the ICAS tests in 2006, 2007 and 2008.

national investigations, the NCS was replaced with a content-based curriculum (Department of Basic Education, 2011). There is hope that the provision of a curriculum that gives better signals on content and forms of learning will enable teachers to implement that curriculum more effectively. We argue that it is one thing to design a better curriculum, but it is a very different matter to achieve teachers' understanding of what standard is required for the grade they are teaching and what content they should focus on in their teaching.

Teachers' understanding of accountability demands, their consent and their readiness to accept change are interrelated processes (Shalem, 2003), but policymakers often assume that curriculum standards make policy requirements sufficiently clear to teachers. In practice, 'reading' the curriculum requires an application of teacher knowledge. Shulman (1986) refers to three categories of teacher knowledge, namely, pedagogic knowledge, content knowledge and pedagogic content knowledge. In order to properly interpret the curriculum, teachers are expected to draw on their subject matter knowledge and to contextualise curriculum standards within their learning environment, taking into account the needs of their learners. In terms of Shulman's categories of knowledge, this means that teachers need to draw on both their content knowledge and pedagogic content knowledge in order to interpret and apply the curriculum. More specifically, Ball's sixth domain of teacher knowledge, 'horizon knowledge', is useful here. It refers to teacher knowledge 'of how mathematical topics are related over the span of mathematics included in the curriculum' (Ball, Thames & Phelps, 2008, p. 403). Following the work by Ball et al., one can say that in order to set high expectations for their learners, in addition to their specialised mathematical knowledge which straddles six domains of mathematical knowledge for teaching, teachers need to understand the sequence and progression of the mathematics they teach. Teachers need to understand what the curriculum aims to achieve in an earlier grade and in what ways the topics they teach connect to the conceptual development of the same concept in a later grade. In this way, teachers could better understand the standards required of the curriculum.

International empirical research shows that curriculum statements about assessment standards, together with results of various standardised assessments, do not, in themselves, make standards clear (Darling-Hammond, 2004; Katz et al., 2009). Empirical research in South Africa has identified misalignment between the demands of the curriculum, teaching and assessment (Reeves & Muller, 2005). Classroom research suggests that many teachers simply ignore important aspects of the NCS and continue to teach poorly what they taught before (Brodie, Jina & Modau, 2009; Chisholm et al., 2000; Fleisch, 2007; Jansen, 1999). There are a variety of reasons for this overarching finding. Research in South Africa gives primacy to two inter-related explanations: poor teacher knowledge, in particular subject matter knowledge, and poor signalling of the (NCS) curriculum (Taylor, Muller, & Vinjevold, 2003). The argument is that the outcomes-based curriculum provided teachers with very weak signals as to what content should be made available to learners and how this should be done. Many teachers in South Africa lack strong content knowledge to 'design down'[2] tasks, activities and assessments from the outcomes specified in the curriculum (South African Qualifications Authority, 2005). NCS curriculum standards were generally weak, both in content and progression, and therefore provided a weak guide for teachers (Muller, 2006; Reeves & Muller, 2005; Shalem, 2010). Reeves and McAuliffe (2012) found in their study of 'topic sequence' and 'content area spread' in mathematics lessons that 'for most learners mathematics was not presented in a coherent and composite manner over the school year' (p. 28). When the curriculum specifies content as a list of topics, it does not elaborate sufficiently on the topics conceptually, and does not relate the topics to one another adequately. Furthermore, lists of skills of what learners must do without any specific content attached to these skills allow for too much variation (low reliability) in the types of textbooks that are produced, in the criteria used by schools to select textbooks, in teachers' professional judgement of what counts as an achievement, in the kind of tasks teachers design and in 'curriculum coverage'. The main point here is that 'lists of statements' do not necessarily show what concepts are key to a field, what activities are worthwhile and what texts are worthwhile (Shalem, 2010, p. 91). Taken together, these explanations suggest that teachers struggle to interpret the curriculum (Brodie, Shalem, Sapire & Manson, 2010).

In this article we add a third explanation. We propose that even if standards are sequenced and well explicated by examples, they do not disclose to teachers what instructional practice should look like or what constitutes acceptable coverage and cognitive demand of curriculum content. Curriculum standards intend to transmit criteria to teachers of what, when and how to teach mathematical content, but transmission this through telling teachers is not enough. Teachers need to be involved, we argue, in a practice that will require them to use the curriculum standards so that they understand what they mean and how they are related to their existing practice. Criteria, says Cavell (1979), are embedded in practice. We 'find' them in the way we do and say things. It is in the way we speak or in the way we do things that we make relevant connections, and thereby show that we understand the way a concept is related to other concepts, or its criteria (Shalem & Slonimsky, 1999). Put differently, by doing knowledge-based professional work, teachers, we argue, are given an 'epistemological access' (Morrow, 1994) to the form in which the curriculum is designed, and more specifically to the content that it privileges. According to Ford and Forman (2006), this kind of professional development work requires a relational framework between three fundamental constitutive disciplinary resources: disciplinary material (working with 'the material aspects' of a specific intellectual field or with a set of propositional knowledge limited to the field), collectivity (using the norms of the intellectual field to produce proofs and grounds for judgement), and disciplinary

2.'Design down' was one of the imperatives of the NCS. This meant that teachers had to start with curriculum specifications and design lesson plans through which these specifications would be delivered in their classes.

procedure (following procedures to evaluate claims made about the natural or the social world) (p. 4). Taking part in curriculum mapping activities, we argue, provides such an opportunity for teachers' professional development.

Curriculum mapping

Curriculum literature distinguishes between the intended, enacted and examined curriculum (e.g. Stenhouse, 1975). In broad terms, this distinction refers to the differences and connections between what the official curriculum document intends, including the academic literature teachers use to decide what to emphasise when they teach a mathematical concept in a specific grade (the intended curriculum), what teachers do in their classrooms (the enacted curriculum) and what is assessed in order to determine achievement and progress (the examined curriculum). 'Curriculum alignment', the idea that informs 'curriculum mapping', describes what counts as a productive educational environment. Biggs's (2003) premise is that when a teacher covers the content of an 'intended curriculum' at the appropriate cognitive level of demand and their learners perform well on high quality tests (the examined curriculum), they have created a productive learning environment (the enacted curriculum), aligned to the demands intended by the curriculum. The corollary of this is that if the quality of a learning environment is judged from the high results of the learners, all things being equal, it can be said that the results of the learners demonstrate that they have studied key content of the subject (curriculum coverage) and that they are able to use the content to answer a range of questions (cognitive level of demand). This is an important insight for understanding the role of curriculum knowledge in teacher practice and the significance of having experience in curriculum alignment.

Teachers can become more familiar with the requirements of the curriculum and in this way improve the conditions for achieving curriculum alignment (Burns, 2001; Jacobs, 1997) through their involvement in curriculum mapping activities. Curriculum mapping is defined as a 'tool for establishing congruence between what is taught in the classroom and what is expected in state or national standards and assessments' (Burns, 2001). The idea was formulated by English in the 1980s (in Burns) and in its common form includes a type of calendar on which teachers, in grade-level groups, record time-on-task in each of the topics they teach and the order in which they teach the topics. Over time this practice developed to include teachers' records of the ways they taught and assessed the topic. In this common form, curriculum mapping is focused on the relation between the enacted curriculum and the intended curriculum.

Accounting – an opportunity to learn from the examined curriculum

It is current practice by policymakers to strengthen accountability by using large-scale assessments (the examined curriculum). The policy idea here is that providing teachers with a range of assessments at different levels of cognitive demand in relation to key subject matter content, education departments hope to use the examined curriculum to make curriculum standards explicit. To be considered rigorous, of high quality and valid, large-scale assessments need to be shown to be aligned to the curriculum (Brookhart, 2009; Case, Jorgensen & Zucker, 2008; McGehee & Griffith, 2001). However, and this is the argument of this article, if teachers do not have opportunities to participate in analysing the content of these learner assessments, more specifically to profile the test items or to examine the curriculum standards that they articulate, their mathematical content and its alignment with the curriculum standards of the grade they are teaching, and which mathematical concepts or skills are needed in order to find the solution to a test item, teachers will not be able to fully gauge the requirements of the intended curriculum. We believe that an opportunity for teachers to learn is missed here.

By working with test items (the examined curriculum) and thinking about the links between the content present in the test items in relation to the content present in the curriculum standards (the intended curriculum), the teachers in the DIPIP project were doing a different and more unusual form of curriculum mapping. The use of test items as an artefact to focus teachers' thinking when they interpret the curriculum addresses the main challenge faced when interpreting any curriculum document, that is, to identify 'what' has to be covered as well as 'the level' at which the selected content needs to be taught. In curriculum terms this refers to curriculum coverage in a specific intellectual field at levels of cognitive demand appropriate for specific grades. By structuring professional conversations around curriculum mapping of test items, the mapping activity intended to provide the teachers with a relational framework, one in which they enact dimensions of expertise that are commonly excluded from them (curriculum mapping). Through this we hoped to enable the teachers to gain a deeper and more meaningful understanding of the curriculum (the NCS), which, as we have shown above, is a skills-based curriculum that is, in best case scenario, opaque, especially in the case of teachers with weak subject matter knowledge. It is to the design of the activity that we turn next.

The project process

There were two rounds of the mapping activity in our project: the first in February – May 2008 (Round 1) and the second in August – mid-September 2010 (Round 2). About 50 mathematics teachers from Grade 3 to 9 worked in groups. The initial selection of teachers for participation in the project was guided by the Gauteng Department of Education. In particular, teachers from 'better performing schools' that had participated in the ICAS tests were selected. However, as a few teachers dropped out of the project, they were replaced by other mathematics teachers selected from schools with easy access to the Wits Education campus (Shalem et al., 2011). The group membership was highly stable and over the three year period, a total of 62 teachers participated. The teachers were divided into 14 groups, two groups for each grade. Each group consisted of 3–4 teachers, a subject facilitator from the Gauteng Department of Education and a graduate student or a university staff member as a group leader. Two points must be emphasised here. Firstly, the group leaders were selected for their mathematics classroom experience or alternatively for their involvement in initial teacher education and in-service teacher development. At different points of time during the duration of the project, before the introduction of a new activity, the leaders were trained by a mathematical education expert. Their role was very important in the project, which is borne out by the findings (see below). Secondly, since all the activities were conducted in groups, reporting on DIPIP activities relates to the groups and not to individual teachers. Although results about individual teachers' mapping would be more desirable, from a research perspective, the idea of professional learning communities and this methodological criterion are in conflict. Notwithstanding, the results reported in this article are statements arrived at through group discussion in relation to specific activities and reflect the consensual decision made by the group.

The ICAS tests were not designed especially for the South African curriculum. These tests, which are used in many countries in the world, were used in South Africa in good faith that they represented an 'international' mathematics curriculum. The mapping activity that the groups were given to complete was not done on this test by any expert or department official. The Gauteng Department of Education treated the test as generally valid for the mathematical content of the grades tested. Our curriculum mapping confirmed this assumption. Table 1 shows our analysis of the content coverage in the ICAS 2006.[3] The table lists the number of items per curriculum content area for each of the grades studied in the project (consistent with the NCS topic weighting).

The curriculum mapping activity

In Round 1, all 14 groups mapped the 2006 ICAS test items. The groups met once a week for about two hours for 14 weeks, working with their group leaders. Groups were expected to map a minimum of 20 items in Round 1. In Round 2, 11 of the groups mapped ICAS 2007 items or items from tests that they had set themselves (hereafter referred to as 'own tests'). Groups worked without group leaders in Round 2. This was done in order to see the extent to which they could manage the task on their own.

3.Coverage is designed in a similar way in the ICAS 2007 tests.

A modified curriculum document was prepared for use by the groups (Scheiber, 2005). The tabulated curriculum enabled the groups to navigate and refer to the NCS document more easily, to look at and compare the content and contexts across the different grades. The tabulated curriculum has a landscape page setup and matches the assessment standards for each grade, across the page, using numbers. This makes it easy to compare the assessment standards across grades and to see at a glance how concepts are built up in each grade. Figure 1 gives an illustrative example of the mental arithmetic assessment standard strand from Grade 1 to 5. In the full tabulated document, assessment standard strands from Grade 1 to 9 are given across the page.

The groups were given a template (see Figure 2), which structured their conversations and guided the process by which they arrived at a consensus, which was recorded in the template as the 'group response'. The template was given to the groups in order to focus the conversation around what the ICAS test assessment data (the examined curriculum) means, how it aligns with the conceptual demands of the NCS (the intended curriculum), and how it fits with teachers' professional knowledge and experience (the enacted curriculum) (Brodie et al., 2010).

For each test item the groups needed to:

- identify the mathematical concept or concepts being tested by the ICAS item
- find the relevant assessment standards relating to the concepts
- justify the choice of the assessment standard
- state when or if the content is taught and whether it is taught directly or indirectly.[4]

4.'Indirectly' means through an assignment, a project or homework task, or linked to another content area.

TABLE 1: Number of items per curriculum content area for each grade in the ICAS 2006 test.

Content area	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Total
Number	11	11	11	11	10	11	7	72
Pattern or Algebra	6	6	6	6	7	7	9	47
Shape	10	10	10	10	10	8	8	76
Measurement	8	8	8	8	8	9	8	57
Data	5	5	5	5	5	5	8	43

ICAS, International Competitions and Assessments for Schools.

1.1.12	2.1.12	3.1.12	4.1.12	5.1.12
Performs mental calculations involving addition and subtraction for numbers to at least 10	Performs mental calculations involving: • addition and subtraction for numbers to at least 20 • multiplication of whole numbers with solutions to at least 20.	Performs mental calculations involving: • addition and subtraction for numbers to at least 50 • multiplication of whole numbers with solutions to at least 50.	Performs mental calculations involving: • addition and subtraction • multiplication of whole numbers to at least 10 × 10.	Performs mental calculations involving: • addition and subtraction • multiplication of whole numbers to at least 10 × 10.

FIGURE 1: Exemplar assessment strands (mental arithmetic, Grade 1–5) in tabulated format.

Grade	We know this when the learner	ICAS item	Concepts needed to complete the problem	Reasons for mapping the item with this assessment standard	When do you <u>actually</u> teach it? Do you teach this at all? What other areas do you link it with, if any?

FIGURE 2: Curriculum mapping activity template.

The template required the groups to think about and decide which mathematical concepts or skills are needed in order to find the solution to a test item. The group's decision was to be based on which Assessment Standard(s) its members linked the test items to. The groups needed to be sure that their selection was appropriate, and to do this they were asked to give strong motivation for their decision. This selection was not confined to the NCS of the grade they were analysing, but was related to several grades. This was possible as the group worked with all the assessment standards across Grade R–9. The last section of the activity gave the groups an opportunity to examine the alignment between the intended and the enacted curriculum, comparing the content coverage assumptions made by the test designers and what, in fact, they cover in the classroom. In the last column of the template (Figure 2), the groups needed to report on their teaching practices. This allowed the gap and the congruence between the intended and the enacted curriculum (based on groups' reporting) to be made explicit to the teachers in their groups. In this way, the ICAS test was used as an *artefact* (a concrete textual item), which mediated between the intended curriculum and the groups' professional knowledge and experience of the enacted curriculum. The upshot of this is that by being involved in professional work (the work that normally mathematics and curriculum experts do when they align the examined with the intended curriculum), teachers, in their respective groups, reported that they came to understand the demands of the NCS curriculum for the first time (Brodie et al., 2010).

Sample of items

Of the 402 ICAS items that groups mapped in Round 1, 140 items were selected for analysis. From the first half of each grade level test 10 items were chosen and 10 items were chosen from the second half of the test. This gave 20 items per grade. The selected items included all the content areas tested. All 82 of the items mapped by the groups in Round 2 were selected for analysis.

Data analysis

A mathematics education expert was employed to map the sample of the ICAS test items. The expert's mapping was validated by a project manager and based on this agreed mapping, groups were considered to have 'misaligned' items with the curriculum standards when their mapping was different from the validated mapping. Coding of the alignment was recorded in spreadsheets. Coding of the remaining data involved recording (using spreadsheets) groups' comments on content taught 'directly', 'indirectly' or 'not at all'. Coded data was analysed quantitatively, finding observable trends and relationships evident in the sample. Examples of groups' explanations from the template were recorded to exemplify quantitative findings. We refer to some of these examples in the findings. For the purposes of reporting on the analysis, we combined the following sets of groups: Grade 3–6 and Grade 7–9.

Validity and reliability

A protocol (the mapping template shown in Figure 2) was used for the recording of the groups' responses in the curriculum mapping activity. The protocol was discussed amongst colleagues in the project management team. Findings were reported on at local and international conferences where these could be discussed to enhance quality. The following are points to be noted as a possible validity threat:

- Since only one group of Grade 7–9 mapped items in Round 2, comparisons cannot be made for these grades with Round 1 mapping.
- One of the groups (Grade 7) attained full matching with the expert. This may have skewed the data.
- Round 2 was a slightly abridged version of the curriculum mapping, due to time constraints, in which the teachers only mapped curriculum content and did not report on when and how they taught this content.

Ethical considerations

Approval for this study was granted by the Department of Education and at an institutional level by the university ethics committee. Informed consent was obtained from all of the teachers, university staff and students who participated in the professional development project meetings. Confidentiality and anonymity of participants was maintained through the use of classified group names (e.g. Grade 3 Group A, which is denoted as G3gA).

Findings

In total, in Round 1, the Grade 3–6 group mapped 246 and the Grade 7–9 group mapped 156 ICAS 2006 items. In Round 2, five of the 11 groups mapped their own tests. Altogether these five groups mapped 27 items. The other six groups mapped 55 ICAS 2007 items. In total, in Round 2, the Grade 3–6 group mapped 57 items and the Grade 7–9 groups mapped 25 items. In sum, 402 items were mapped in Round 1 and 82 items were mapped in Round 2. The group responses recorded on the mapping templates formed the data set on which the analysis presented in this article is based.

We first present the overall results of the curriculum mapping, in which we look at the alignment quality of the groups' mapping. We then discuss the groups' reporting on content ‘taught’ (directly and indirectly) and content ‘not taught’, which yields insight into the relationship between the intended and the enacted curriculum.

Overall alignment results

Two overall results can be noted. Firstly, the overall agreement between the experts’ (indicated as ‘alignment’) and the groups’ mapping was relatively high: 83% in Round 1 and 67% in Round 2. These percentages represent the average of the groups’ correctly aligned assessment standards to test items compared to the experts’ agreed curriculum alignment of the test items. This is an indication that the curriculum mapping was generally successful, particularly in Round 1. The mean misalignment was 26% higher in Round 2. It is important to remember that in Round 1 groups worked with group leaders whilst in Round 2 they worked without group leaders, which may have contributed to the increased misalignment in the Round 1. Secondly, the mapping alignment of Grade 7–9 groups was found to be stronger than Grade 3–6 groups. It stayed at the same level of accuracy (around 80%) in both rounds.

Increased confidence and improved judgement

We investigated what specifically in Round 1 may have contributed to differences between groups’ strength in mapping in Round 2. We compared the means of the groups who had mapped different numbers of items in the two rounds**.** We found that groups that had mapped more items in Round 1 achieved higher mean alignment in Round 2 than groups that had mapped fewer items. Put differently, groups that gained more experience of mapping in Round 1, as measured by their mapping more items, consistently showed higher alignment percentages in Round 2 (see Figure 3). For this comparison we could only use data from the Grade 3 to 6 group and we compared each paired grade group individually. Since only one group for each of the grades in Grade 7–9 did the curriculum mapping activity in Round 2 (see the discussion on validity), comparisons were not possible here.

This finding suggests that with more experience groups gain confidence in mapping test items against the curriculum and make better judgments in relation to curriculum alignment.

Mapping ICAS items and own test items

Higher misalignment was found in Round 2 of ‘own test’ items than of ICAS 2007 test items in the Grade 3–6 group. This finding is counter-intuitive: one would have thought that groups would be more familiar with content in a test that they had drawn up themselves. This might suggest that the groups designed tests that included mathematical content about which they were not entirely confident. Alternatively, it could be that the ‘own test’ items required a different way of thinking when aligning to the assessment standards of the curriculum, so the groups’ familiarity with the task made it easier in Round 2 to align ICAS test items but not ‘own test’ items. A third explanation may be that when groups

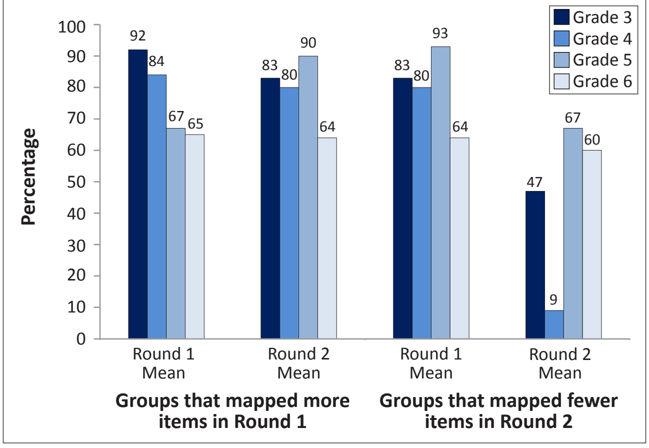


FIGURE 3: Comparison between Round 1 and Round 2 in terms of mean alignment for grade groups that mapped more items or mapped fewer items.

designed their own tests, because they were expected to select a misconception and to design the test items around it, they may have focused their attention on the misconception rather than on the level of content required. It seems they had difficulty embedding the concepts at the appropriate level.

Taking all these findings together, we suggest that the curriculum mapping activity gave the practitioners (we are especially interested in the experience of the teachers in the group) an opportunity to understand the selection of the mathematical content for the ICAS tests, per grade, as well as its level of cognitive demand. Since groups were working with curriculum standards ranging across several grades, teachers were given an opportunity to analyse what assessment standards (and related mathematical conceptual knowledge) in the South African curriculum learners would need to have achieved in order to answer a test item correctly. Analysing the items conceptually, which groups needed to do in order to identify the mathematical concepts being tested by the ICAS item, alerts the groups to conceptual progression.

Gaps between the intended and the enacted curriculum

The data about the intended curriculum and the groups’ reflections on practice are drawn from the groups’ reporting in Round 1, specifically from the information the group included in the last column of the curriculum mapping template (see Figure 2). The findings for this section refer only to the mapping activity of the ICAS 2006 test items (see the discussion on validity). Teachers identified the mathematical content in an ICAS 2006 test item and after aligning it to the NCS they needed to consider it in relation to their own teaching. This served to make explicit to the teachers the difference between the intended and the enacted curriculum. The highest percentage (46%) of the ICAS test related content was reported to be taught ‘directly’. Next highest (26%) of the ICAS test related content was reported as ‘not taught at all’. The lowest percentage (24%) was reported as being taught indirectly.[5]

5. It should be noted that the Grade 3–6 group reported certain items taught both directly and linked (for example) so the total of the ‘when I teach it’ percentages for this group goes slightly over 100%. This was not the case in Grade 7–9.

Quotes taken from grade groups' recorded responses are given as examples of teachers' reporting on:

- Direct teaching: 'When we teach bonds' (G4gA); 'Taught in term 1, as mental calculations' (G5gA); 'Decimal fractions are taught in the first term of Grade 7' (G7gA).
- Indirect/linked teaching: 'Should be covered in all problem-solving activities across all grades' (G6gA, G8gB); 'Place value is taught in the first term. Link it with money, mass and capacity' (G5gB); 'Should be done specifically as the concept of symmetry but it can be done via the theme e.g. Special me (body parts-left and right sides of the body)' (G3gB).
- Content not taught at all: 'Number patterns – Grade 1 onwards. Not focused on in Grade 8. Knowledge is assumed' (G8gB); 'The concept of odd and even numbers – Assume that it has been taught in earlier grades' (G5gB); '… pictorial representation of patterns is a neglected area, thus making it difficult for them to conceptualise what is required' (G3gB); 'Rotation is not taught in Grade 4, but is in the curriculum for Grade 5' (G4gA, G4gB).

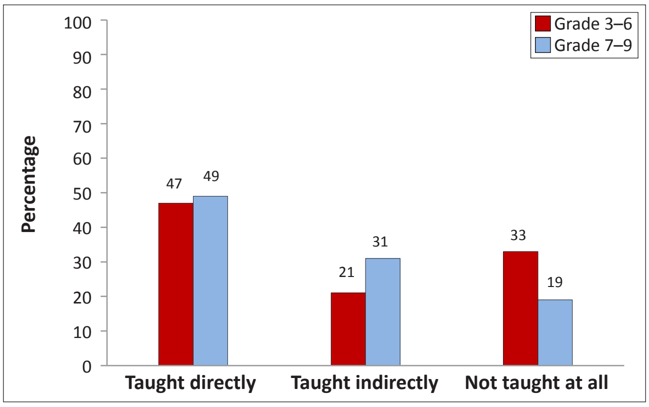
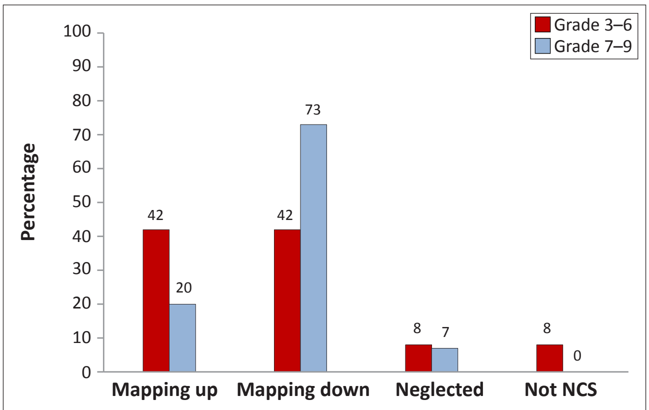


FIGURE 4: Grouped grades' reporting on 'when I teach it' (Round 1).



NCS, National Curriculum Statements.

FIGURE 5: Grouped grades' reasons given for content not taught.

It is interesting to note (see Figure 4) that the Grade 3–6 group identified more content in the tests which they said they did not teach because it was at a higher level than they were expected to teach (topics included irregular shapes, rotational symmetry, tessellations, reflections, rotations and probability). The opposite was true for the Grade 7–9 group.

We further investigated the reasons given by the groups for content 'not taught' (see Figure 5) in their completed mapping templates. Our investigation gave rise to two categories of mapping referred to as 'mapping downwards' and 'mapping upwards'. Mathematical content of an item that according to the curriculum should be covered in a lower grade than in the test was classified as 'mapped downwards' – reported as: 'This [*different perspectives of geometric solids*] is not specifically covered in Grade 8, it is formally required to be taught first in Grade 6' (G8gB). Or, pointing to the problem-based nature of an item, groups reported: 'Although the number range falls within the scope of the Phase, we do not teach this, because the way in which the problem is presented is beyond the scope of the Phase' (G3gA). Mathematical content of an item that according to the curriculum should be covered in a higher grade than in the test was classified as 'mapped upwards – reported as: 'We would use this task [*modelling involving area*] as an extension for the stronger learners' (G8gA). Most commonly, explanations for 'not teaching' fell into one of these categories.

It is also interesting to note that more content in the primary school (Grade 3–6) than in the secondary school (Grade 7–9) was classified as 'not taught' for reasons of being at a higher than expected level for the grade (according to the intended curriculum identified in the NCS).

Table 2 shows that most items reported as 'not taught' were at the grade level of the test, according to the intended curriculum. Few of the items reported as 'not taught' were mapped up or down, relative to the grade level according to the intended curriculum.

In the Grade 3–6 group, 44% of the content reported as 'not taught' was at the expected grade level. In the Grade 7–9 group, 76% of the content reported as 'not taught' was at the expected grade level. In both grade groups, the content reported 'not taught' at own grade level included mathematical content across all five of the NCS content areas. The Grade 3–6 group mapped up some data, measurement and geometry items and they mapped down some number and data items. The Grade 7–9 group did not map down any items but they mapped up some number, geometry and measurement items.

Some of the specific neglected areas reported by the teachers include (quotes are taken from grade groups' recorded responses):

TABLE 2: Summary of mathematical content reported 'not taught'.

Grade level	Number of content items 'not taught' mapped to own grade level	Number of content items 'not taught' mapped up	Number of content items 'not taught' mapped down	Total number of content items indicated 'not taught'
Grade 3–6	20	7	18	45
Grade 7–9	13	0	4	17

- ICAS items that included irregular shapes, rotational symmetry, tessellations, reflections, rotations and probability were reported as beyond the scope of Grade 3 (G3gA, G3gB).
- The pictorial representation of a pattern was reported as not taught at the Grade 3 level. Patterns are usually taught as a horizontal sequence of numbers: '… pictorial representation of patterns is a neglected area, thus making it difficult for them to conceptualise what is required' (G3gB).
- Reasoning logic was reported as not taught in Grade 8: 'Never! Not mathematics' (G8gA).
- Finding fractional parts of whole numbers was reported as not taught using learning aids in Grade 5: 'We never use manipulatives to determine fractional parts of whole numbers' (G5gA).

Our data analysis shows a direct relationship between the teachers' perception of the enacted curriculum – content taught 'directly' – and the degree of success in aligning the international test item to the curriculum. Figure 6 shows higher percentages of misalignment of item content reported as 'not taught' (compared to content reported as ' taught') for both groups, but particularly in the lower grades. Content reported as 'taught directly' is, for the best part, aligned better. This difference in alignment could be an indication of teacher content knowledge. It could be that in the higher grades teachers are teaching more work with which they are not sufficiently familiar, yet they do teach it because it is required of them. It is also possible that the teachers in the lower grades may have reported more openly on content 'not taught'. The teachers in the higher grades may leave out content with which they are not familiar but they do not report on it. Alternatively, the items may include content that the teachers expect should be taught earlier and thus do not report on it.

In conclusion, by allowing them to analyse if and when the content specifications of the intended curriculum are covered in practice, the mapping activity gave the teachers an opportunity to reflect on their practices. We argue that the mapping activity enabled them to develop their pedagogical content knowledge (Shulman, 1986). By examining curriculum coverage, cognitive demands and alignment, the teachers were involved in ascertaining the match between an international test, the South African curriculum and their professional knowledge and experience. This should have, at least to some extent, developed their understanding of the sequence and progression of the mathematics they teach and hence their grasp of Ball's sixth mathematical knowledge domain (Ball et al., 2008).

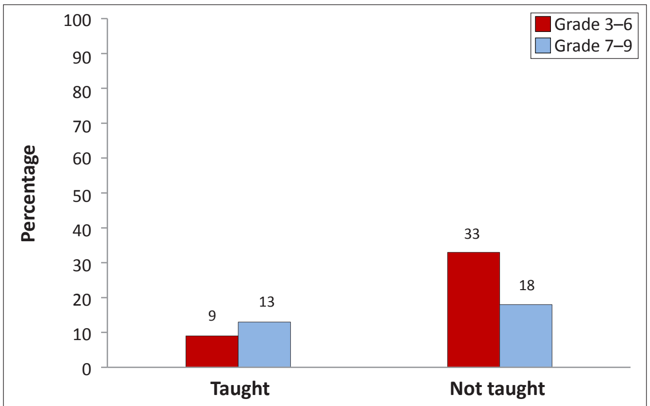


FIGURE 6: Misalignment of content taught and content not taught.

Conclusion

The findings of this research suggest that through the curriculum mapping activity, teachers became more aware of what is intended by the curriculum, in particular of the discrepancy between what they understand is intended by the curriculum and report is enacted in their classes. In relation to the intended curriculum, the teachers were able to report on when they teach and do not teach content and on 'neglected content area in schools', which shows that such activities can build teachers' awareness of the presence or absence of curriculum content in their classes. The findings also suggest that teachers' curriculum mapping ability is stronger when they are more familiar with and hence have greater confidence in doing the activity. When content was reported as 'not taught', a higher level of misalignment was generally seen, which indicates familiarity with mathematical content does affect the quality of curriculum mapping. The differences between alignment in Round 1 and Round 2 indicate that teachers' capacity to align content was better when they worked with well-selected and trained group leaders.

This research supports the claim that teachers can benefit a great deal from being involved in interpreting large-scale assessment tests. Curriculum mapping, using the format of a structured interface, creates a 'defensible focus' (Katz et al., 2009) for this kind of professional development. The analysis above shows that the structured process of interface enabled teachers to actively engage in curriculum translation of the disciplinary material embedded in curriculum statements. Working in groups, the teachers learned about the form of the curriculum by using an *artefact* (the test items) to engage the curriculum content and standards. Involving teachers in the interpretation of both public assessment data and data from their own classroom activities can build their understanding of the knowledge base of test items and of the curriculum.

Acknowledgements

We acknowledge the funding received from the Gauteng Department of Education and in particular would like to thank Reena Rampersad and Prem Govender for their support of the project. We would also like to acknowledge the pivotal role played by Karin Brodie (project leader, Phase 1 and Phase 2) in the conceptualisation of the project. The views expressed in this article are those of the authors.

Competing interests

The authors declare that we have no financial or personal relationship(s) which might have inappropriately influenced our writing of this article.

Authors' contributions

Y.S. (University of the Witwatersrand) was the project director, developed the theoretical framework for the article and contributed to the analytical part of the article. I.S. (University of the Witwatersrand) was a project coordinator, was responsible for the data analysis and contributed to the theoretical and analytical parts of the article. B.H. (St John's College), a mathematics education expert, was a group leader on the project and was involved in the data coding; she also contributed to the writing and development of the article.

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Unit 2 Further Curriculum Mapping Activities

Introduction

The following sets of activities, grouped according to mathematical content areas, will give you the opportunity to map test items to the curriculum document across a range of topics.

The items have been selected in order to raise discussion of the mathematical content they present. There is one activity per grade, from grade 3 to grade 7, so you can choose the grade level at which to work. But you can also work through the activities for other grade levels if you wish.

If you would like to improve your understanding of the relevant content areas in the curriculum, we suggest that you consult *Mathematics for Primary School Teachers*, an openly licensed module digitally published by *Saide* and the University of the Witwatersrand (Wits), downloadable from OER Africa: http://www.oerafrica.org/ResourceResults/tabid/1562/mctl/Details/id/39030/Default.aspx.

The activities below present material which is often misunderstood by learners and about which teachers would benefit from a deeper understanding. The commentary given does not relate to the full range of questions you need to work through when you do these activities, but it pinpoints the curriculum requirements for the given tasks. You should discuss your full activity responses with a colleague if possible to gain the full benefit from these activities.

Place value

Learners' understanding of place value as used in our base ten numeration system is developed from the moment they start counting and recording numbers. They extend this understanding of one-by-one counting to knowledge of numbers with bigger value until they are able to read, represent and work with numbers of any size.

Activities 10 – 14 Place value

The next five activities present questions based on learner work from Grades 3 to7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the place value activities in order to learn more about how learners work with place value.

For an understanding of the content that underpins place value consult Unit 2 pp. 53-92 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

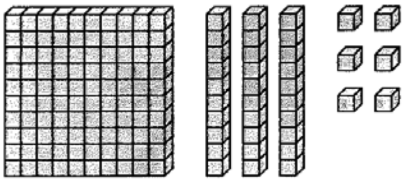
Grade 3 – Activity 10
Grade 4 – Activity 11
Grade 5 – Activity 12
Grade 6 – Activity 13
Grade 7 – Activity 14

For each activity you should answer the following questions:

a) In which grade(s) is the mathematical content that you identified as covered by this test item to be found in the mathematics curriculum?
b) Write out the specific curriculum wording for the identified content.
c) Draw up a list or a mind map in which you write about as many concepts as you can think of that are related to the concept of place value as it is presented in this test item.
d) Explain your reasons for mapping the item content to your chosen curriculum reference(s).
e) Do you notice any differences between the concepts you first identified and the curricular references that you chose?
f) Do you think you might change the concepts you initially identified as necessary to solve the item correctly? If so, how?
g) Do you think you might change the curriculum references you initially identified as those which specify the concepts required to successfully work out the answer to the question? If so, how?
h) Do you teach this/these concepts in your mathematics class?
i) In what grade and during which term do you directly teach this/these concepts?
j) Do you link this/these concepts with others? If so, which ones?
k) If you don't teach these concepts in your grade, explain why not.

Activity 10
Grade 3 Place value

12.



What number is shown by this model?

(A) 136
(B) 163
(C) 631
(D) 1036

Commentary
The blocks used in this concrete representation of a number are called Dienes blocks. They are used in the teaching of place value because the different sized blocks give a concrete, visual display of the size of the number. In order to answer this question correctly a grade 3 learner needs to be able to work with place value up to 3-digit numbers.

How would you use Dienes' blocks with a Grade 3 class? Describe an activity you could work through with your learners and what you think they would learn through doing this activity.

Activity 11
Grade 4 Place value

13. 186 – 47 = ?

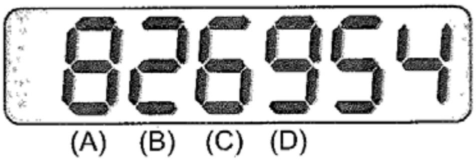
(A) 149
(B) 141
(C) 139
(D) 39

Commentary
In order to answer this question correctly a grade 4 learner needs to be able to subtract a 2-digit number from a 3-digit number. This requires prior understanding of place value of numbers up to 3-digit numbers. In this question there is an *impasse* – this is when the subtraction requires breaking down and regrouping in order to subtract.

Describe an activity that you would set for a grade 4 class to help learners to understand the correct use of place value when subtracting numbers from each other where an *impasse* is involved.

Activity 12
Grade 5 Place value

5. Which digit shows the number in the thousands column on this calculator display?

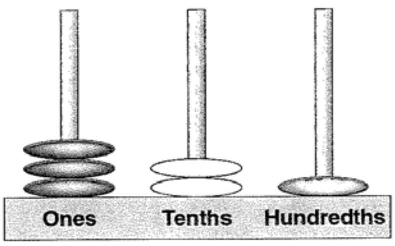


Commentary
In order to answer this question correctly a grade 5 learner needs to be able to identify the thousands digit in a large number. This requires understanding of place value in numbers with up to 6 digits. .

Describe an activity that you would set for a grade 5 class to help learners to understand how to identify the place values of the digits in large numbers with up to six digits..

Activity 13
Grade 6 Place value

24. This is a bead counter.



Which number is shown on the bead counter?

(A) 1.23 (B) 3.21

(C) 123 (D) 321

Commentary
In order to answer this question correctly a grade 6 learner needs to be able to work with place values including tenths and hundredths (in other words, decimal fractions). In this question the learner also needs a prior knowledge of the use of bead counters to represent numbers.

Describe an activity that you would use with a grade 6 class using a bead counter (or abacus) to consolidate their understanding of decimal fractions up to hundredths.

Activity 14

Grade 7 Place value

13. Which of these numbers is smallest?

(A)	0.1	(B)	0.09
(C)	0.109	(D)	0.0999

Commentary

In order to answer this question correctly a grade 7 learner needs to be able to work with decimal fractions up to ten-thousandths.

Describe an activity which you could set for a grade 7 class which would enable learners to consolidate their ability to compare decimal fractions of up to ten-thousandths.

Operations

Learners' understanding of operations (addition, subtraction, multiplication and division) begins when they have adequate number concept knowledge which allows them to operate on pairs of numbers. Early operation strategies should be closely linked to concrete examples but as learners develop this understanding they should start to use abstract procedures and not rely on counting to do their computations.

Activities 15 – 19 Operations

The next five activities present questions based on learner work from Grades 3 to 7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the operations activities in order to learn more about how learners work with operations.

For an understanding of the content that underpins Operations consult Unit 3 pp. 93-124 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

Grade 3 – Activity 15
Grade 4 – Activity 16
Grade 5 – Activity 17
Grade 6 – Activity 18
Grade 7 – Activity 19

For each activity you should answer the following questions:

a) In which grade(s) is the mathematical content that you identified as covered by this test item to be found in the mathematics curriculum?
b) Write out the specific curriculum wording for the identified content.
c) Draw up a list or a mind map in which you write about as many concepts as you can think of that are related to the concept of operations as it is presented in this test item.
d) Explain your reasons for mapping the item content to your chosen curriculum reference(s).
e) Do you notice any differences between the concepts you first identified and the curricular references that you chose?
f) Do you think you might change the concepts you initially identified as necessary to solve the item correctly? If so, how?
g) Do you think you might change the curriculum references you initially identified as those which specify the concepts required to successfully work out the answer to the question? If so, how?
h) Do you teach this/these concepts in your mathematics class?
i) In what grade and during which term do you directly teach this/these concepts?
j) Do you link this/these concepts with others? If so, which ones?
k) If you don't teach these concepts in your grade, explain why not.

Activity 15

Grade 3 Operations

26. Here is a pattern.

Each row of three cards follows the same pattern.

6	10	18
8	12	20
12	?	24

What is the missing number card from the third row?

14 (A)	16 (B)
18 (C)	20 (D)

Commentary

In order to answer this question correctly a grade 3 learner needs to be able to add numbers up to 2-digit numbers. The question also requires pattern recognition. The learners would use their operations skills to identify the missing number.

Reflect on the general usefulness of basic numbers bonds in mathematical pattern recognition.

Activity 16
Grade 4 Operations

22. $945 \div 9 =$?

(A) 15 (B) 100

(C) 105 (D) 150

Commentary
In order to answer this question correctly a grade 4 learner needs to be able to divide a 3-digit number by a 1-digit number. The division question presented here could be done mentally if the learner has a good concept of division and knowledge of his/her basic division facts.

How does a well-developed skill of mental operations help learners to cope with a division question such as the one in Activity 16 above?

Activity 17
Grade 5 Operations

31. $32.80 × 15 = ?

(A) $19 680 (B) $196.80

(C) $49 200 (D) $492

Commentary
In order to answer this question correctly a grade 5 learner needs to be able to multiply a number with two decimal places by a whole number (with two digits).

Explain the multiplication strategy that you think would be most useful in completing the solution to Activity 17 above.

Activity 18

Grade 6 Operations

9. 7.35 – 5.85 = ?

(A) 1.20 (B) 1.50

(C) 2.00 (D) 2.50

Commentary

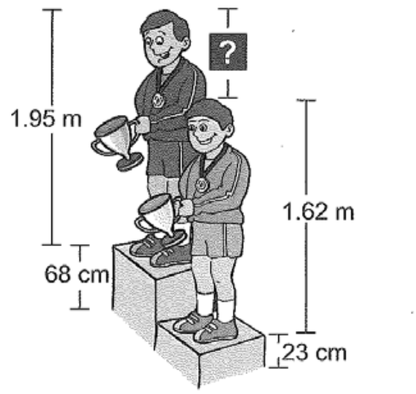
In order to answer this question correctly a grade 6 learner needs to be able to subtract numbers with two places after a decimal comma. This subtraction requires an adequate understanding of place value in order to work with the numbers presented in the question.

Explain the subtraction strategy that you think would be most useful in completing the solution to Activity 18 above.

Activity 19

Grade 7 Operations

24. Calvin and Steven are standing on steps of different heights.



NOT TO SCALE

What is the distance between the top of Calvin's head and the top of Steven's head?

(A) 33 cm (B) 45.33 cm

(C) 78 cm (D) 91.33 cm

Commentary

In order to answer this question correctly a grade 7 learner needs to be able to add and subtract denominate numbers (units of measurement, in this case m and cm). This question also requires that learners are able to convert between these units of measurement (cm and m) since in order to subtract/add, the numbers need to be written as the same kind of unit.

How would you help a learner to interpret the question given in Activity 19 above?

Fractions

Learners' understanding of fractions begins at an early age, almost at the same time as their concept of whole numbers is being developed. Initially it is based very much on concrete examples of fractional parts which can help them to visualise and then generalise the idea of a part relative to a whole. The fraction concept needs to be sufficiently well generalised until it is an abstract number concept which is not reliant on concrete images or objects.

Activities 20 – 24 Fractions

The next five activities present questions based on learner work from Grades 3 to7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the fractions activities in order to learn more about how learners work with fractions.

For an understanding of the content that underpins Fractions consult Unit 4 pp. 125-174 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

Grade 3 – Activity 20
Grade 4 – Activity 21
Grade 5 – Activity 22
Grade 6 – Activity 23
Grade 7 – Activity 24

For each activity you should answer the following questions:

a) In which grade(s) is the mathematical content that you identified as covered by this test item to be found in the mathematics curriculum?
b) Write out the specific curriculum wording for the identified content.
c) Draw up a list or a mind map in which you write about as many concepts as you can think of that are related to the concept of fractions as it is presented in this test item.
d) Explain your reasons for mapping the item content to your chosen curriculum reference(s).
e) Do you notice any differences between the concepts you first identified and the curricular references that you chose?
f) Do you think you might change the concepts you initially identified as necessary to solve the item correctly? If so, how?
g) Do you think you might change the curriculum references you initially identified as those which specify the concepts required to successfully work out the answer to the question? If so, how?
h) Do you teach this/these concepts in your mathematics class?
i) In what grade and during which term do you directly teach this/these concepts?
j) Do you link this/these concepts with others? If so, which ones?
k) If you don't teach these concepts in your grade, explain why not.

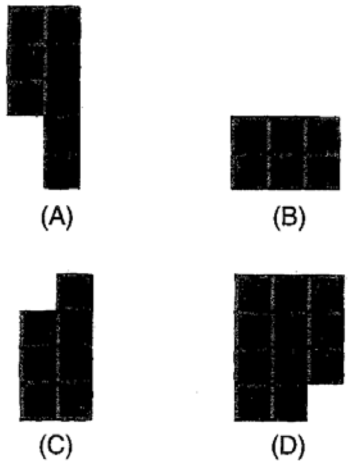
Activity 20

Grade 3 Fractions

22. Jill had a block of chocolate with 32 pieces.

She broke off a quarter of the block.

Which of these shows the quarter she broke off?



Commentary

In order to answer this question correctly a grade 3 learner needs to be able to identify a fractional part of a whole using diagrammatic representations.

The diagrams in Activity 20 above are not all "familiar" shapes. (For example, only one of them is a simple rectangle.) Discuss the value of varying the shapes that can be used as parts of the whole when developing fraction number concept.

Activity 21

Grade 4 Fractions

12. Jay had a melon.

He ate $\frac{1}{4}$ of the melon.

Then Jay ate **another** $\frac{1}{4}$ of the melon.

What fraction of the melon did he eat in total?

(A) $\frac{1}{8}$ (B) $\frac{2}{8}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Commentary

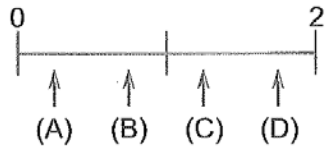
In order to answer this question correctly a grade 4 learner needs to be able to add two fractions with the same denominator.

What activities could you set for a grade 4 class to enable them to begin to add fractions?

Activity 22

Grade 5 Fractions

25. Which arrow points to $1\frac{3}{4}$?



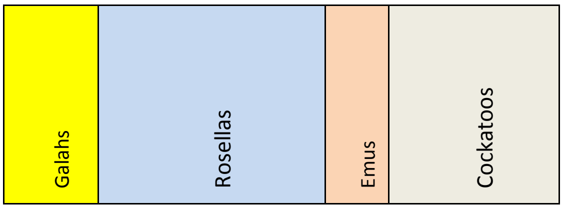
Commentary

In order to answer this question correctly a grade 5 learner needs to be able to recognise a fraction number greater than one represented on a number line.

In what way does a number line help to consolidate a learner's fraction number concept?

Activity 23
Grade 6 Fractions

14. Bill went on a walk in an Australian national park.
He drew a graph to show the proportion of different birds he saw on his walk.



(A) $\frac{1}{4}$ (B) $\frac{2}{5}$

(C) $\frac{2}{4}$ (D) $\frac{3}{5}$

Commentary
In order to answer this question correctly a grade 6 learner needs to be able to use his/her knowledge of fractional parts to work out the proportion of birds.

The task in Activity 23 above is set in an Australian context. Design a similar task which is set in a South African context and discuss whether or not you think the context would be helpful or not to learners answering the question.

Activity 24

Grade 7 Fractions

13. Which of these expressions is equivalent to $\frac{5}{7}$?

(A) $\frac{5}{7}+\frac{7}{5}$ (B) $\frac{5\times5}{7\times7}$

(C) $\frac{5+2}{7+2}$ (D) $\frac{5\times7}{7\times7}$

Commentary

In order to answer this question correctly a grade 7 learner needs to be able to recognise equivalent forms of fractions.

In which other contexts are learners called on to use their knowledge of equivalent fractions when they do mathematical calculations?

Ratio

Learners' developing understanding of ratio should also be based on concrete representations and activities. This concept needs to be sufficiently well generalised until it is an abstract number concept which is not reliant on concrete images or objects.

Activities 25 – 29 Ratio

The next five activities present questions based on learner work from Grades 3 to 7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the ratio activities in order to learn more about how learners work with ratio.

Grade 3 – Activity 25
Grade 4 – Activity 26
Grade 5 – Activity 27
Grade 6 – Activity 28
Grade 7 – Activity 29

For each activity you should answer the following questions:

a) In which grade(s) is the mathematical content that you identified as covered by this test item to be found in the mathematics curriculum?
b) Write out the specific curriculum wording for the identified content.
c) Draw up a list or a mind map in which you write about as many concepts as you can think of that are related to the concept of ratio as it is presented in this test item.
d) Explain your reasons for mapping the item content to your chosen curriculum reference(s).
e) Do you notice any differences between the concepts you first identified and the curricular references that you chose?
f) Do you think you might change the concepts you initially identified as necessary to solve the item correctly? If so, how?
g) Do you think you might change the curriculum references you initially identified as those which specify the concepts required to successfully work out the answer to the question? If so, how?
h) Do you teach this/these concepts in your mathematics class?
i) In what grade and during which term do you directly teach this/these concepts?
j) Do you link this/these concepts with others? If so, which ones?
k) If you don't teach these concepts in your grade, explain why not.

Activity 25

Grade 3 Ratio

33. Students at Happyville School can earn Bronze, Silver and Gold awards.

Students with 6 Bronze awards receive a Silver award.

Students with 4 Silver awards receive a Gold award.

How many Bronze awards are needed to receive a Gold award?

(A) 6×4
(B) $6 + 4$
(C) $6 \times 4 + 1$
(D) $6 + 4 + 1$

Commentary

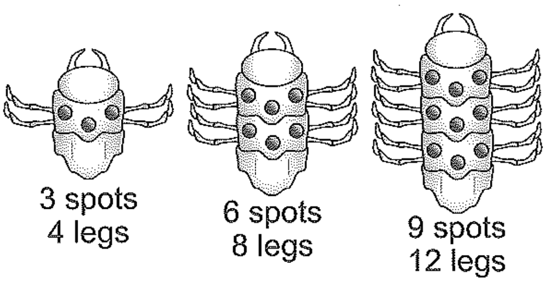
In order to answer this question correctly a grade 3 learner needs to be able to use ratios to work out the number of bronze awards needed in order to receive a gold award.

Describe a concrete activity (using for example cans of cool drink) which would enable learners to develop their understanding of basic ratios.

Activity 26

Grade 4 Ratio

33. Sarah made these model bugs.



She then made a model bug that had 36 legs.

How many spots did that model bug have?

(A) 9
(B) 12
(C) 27
(D) 48

Commentary

In order to answer this question correctly a grade 4 learner needs to be able to use ratios to work out unknown terms in a number pattern.

Describe how the visual representation of the bugs in Activity 26 enables the learners to identify the ratio involved in the pattern that governs their growth.

Activity 27

Grade 5 Ratio/Rate

37. Maria was training for a race.

She ran 3 km each day except on Sundays when she ran 5 km.

She ran every day for 31 days, and started her first run on a Friday.

How many kilometres did she run in total?

(A) 90
(B) 101
(C) 103
(D) 155

Commentary

In order to answer this question correctly a grade 5 learner needs to be able to use rate (km per day) to work out a total distance run. The distance is not entirely dependent on the ratio since she does not run the same distance on all of the days.

Explain in what way an understanding of rate would help a learner to find the solution to Activity 27 more efficiently.

Activity 28

Grade 6 Ratio/Rate

30. Sam and Kevin are bricklayers.

Sam lays 150 bricks in 60 minutes.
Kevin lays 20 bricks in 10 minutes.

Working together, how many minutes will it take Sam and Kevin to lay 180 bricks?

(A) 25 (B) 40

(C) 70 (D) 100

Commentary

In order to answer this question correctly a grade 6 learner needs to be able to use rate (bricks/minute) to work out the total time taken for a certain number of bricks to be laid.

Explain in what way an understanding of rate would help a learner to find the solution to Activity 28 more efficiently.

Activity 29

Grade 7 Ratio/Rate

3. This machine prints 119 copies of a book every 7 minutes.

How many copies does it print in 1 minute?

(A) 17
(B) 19
(C) 112
(D) 833

Commentary

In order to answer this question correctly a grade 7 learner needs to be able to use rate (copies/minute) to work out the time taken to print one copy of a book. This is called the unit rate.

Explain in what way an understanding of rate would help a learner to find the solution to Activity 29 more efficiently.

Geometry

Geometry often involves visualisation of shapes when learners work under test conditions since they are not given concrete shapes in tests. Leaners should be given ample opportunities to work with concrete shapes and do visualisation activities so that they can develop the necessary skills to work with abstract representations of shapes.

Activities 30 – 34 Geometry

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the geometry activities in order to learn more about how learners work with geometry.

For an understanding of the content that underpins place value consult Unit 1 pp. 5-52 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

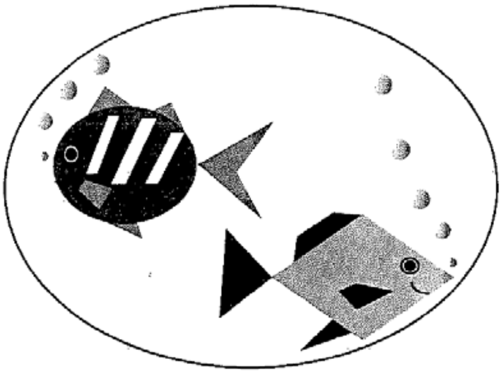
Grade 3 – Activity 30
Grade 4 – Activity 31
Grade 5 – Activity 32
Grade 6 – Activity 33
Grade 7 – Activity 34

For each activity you should answer the following questions:

a) In which grade(s) is the mathematical content that you identified as covered by this test item to be found in the mathematics curriculum?
b) Write out the specific curriculum wording for the identified content.
c) Draw up a list or a mind map in which you write about as many concepts as you can think of that are related to the concept of geometry as it is presented in this test item.
d) Explain your reasons for mapping the item content to your chosen curriculum reference(s).
e) Do you notice any differences between the concepts you first identified and the curricular references that you chose?
f) Do you think you might change the concepts you initially identified as necessary to solve the item correctly? If so, how?
g) Do you think you might change the curriculum references you initially identified as those which specify the concepts required to successfully work out the answer to the question? If so, how?
h) Do you teach this/these concepts in your mathematics class?
i) In what grade and during which term do you directly teach this/these concepts?
j) Do you link this/these concepts with others? If so, which ones?
k) If you don't teach these concepts in your grade, explain why not.

Activity 30
Grade 3 Geometry

18.



How many four-sided shapes are in this picture?

(A) 5
(B) 6
(C) 7
(D) 8

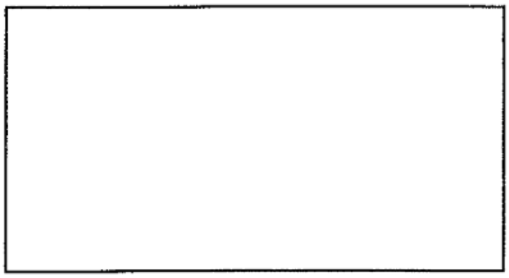
Commentary
In order to answer this question correctly a grade 3 learner needs to be able to recognise shapes which have four sides in the context of an image which has many different shapes.

Drawings of shapes are more abstract than real objects that learners know about from everyday life. Describe how you can help young learners to become more familiar with abstract representations of geometric shapes.

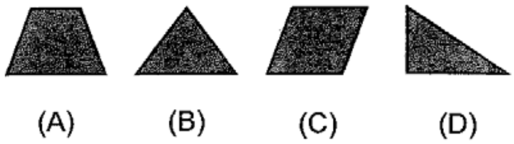
Activity 31

Grade 4 Geometry

20. Julia wants to tile this area of her bathroom wall.



Which of these tiles can she use to cover this area with no cutting, overlapping or gaps?



Commentary

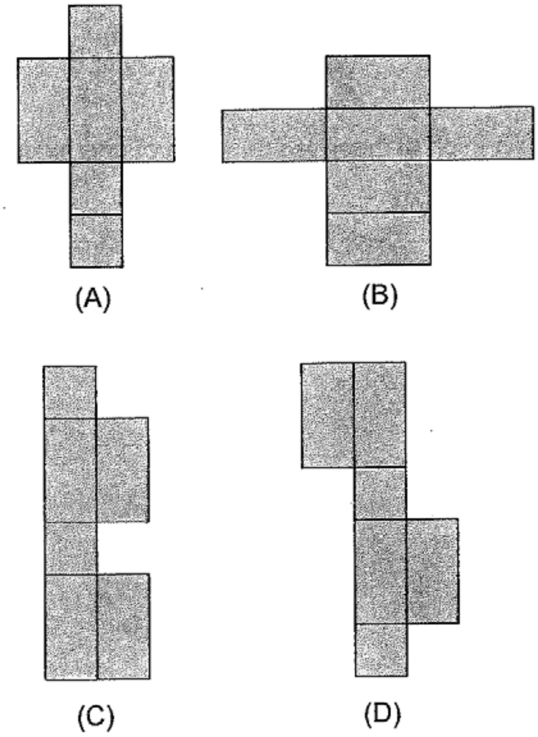
In order to answer this question correctly a grade 4 learner needs to be able to identify which shape can be used to tessellate and cover the entire surface of the given rectangle.

Discuss the kinds of spatial awareness that tessellation develops in learners.

Activity 32

Grade 5 Geometry

33. Which of the following is a net of a rectangular prism?



(A) (B) (C) (D)

Commentary

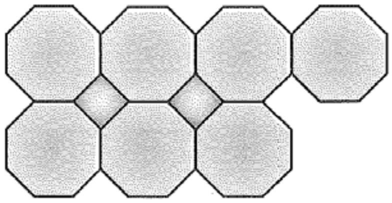
In order to answer this question correctly a grade 5 learner needs to be able to identify which net (2-D fold-out) can be used to make a 3-D shape.

Discuss the ways in which nets assist learners to learn about the characteristics of 3-D shapes.

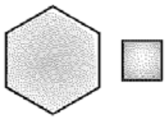
Activity 33

Grade 6 Geometry

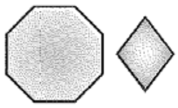
15. This pattern is made out of tiles.



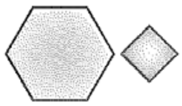
Which two tiles are needed to complete the pattern?



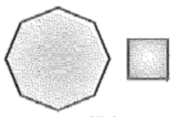
(A)



(B)



(C)



(D)

Commentary

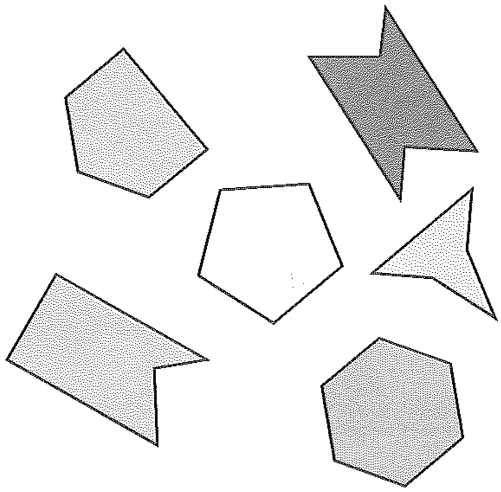
In order to answer this question correctly grade 6 learners need to be able to use their understanding of tessellation to identify which shapes are needed to complete a pattern.

Discuss the ways in which multiple-shape tessellations are different from single-shape tessellations.

Activity 34

Grade 7 Geometry

1. How many of these shapes are pentagons?



(A) 4 (B) 3

(C) 2 (D) 1

Commentary

In order to answer this question correctly a grade 7 learner needs to be able to identify a pentagon.

Naming of 2-D shapes is fundamental geometric knowledge. Discuss some activities that you could set for a grade 7 class to help learners to learn the names of 2-D shapes.

Measurement

Measurement is all about quantifying characteristics of shapes. Only certain characteristics can be measured – those to which a "number" can be assigned. Knowledge of "what" can be measured incorporates an understanding of how this measurement is made. Too often teachers expect learners to measure and find quantities before they know what these quantities represent. It is important to use appropriate arbitrary units of measurement (such as steps to measure the length of the classroom) to develop the concept of each quantifiable characteristic before moving on to formal measurement and the use of standard units of measurement (such as metres, for the length of the classroom).

Activities 35 – 39 Measurement

The next five activities present questions based on learner work from Grades 3 to 7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the measurement activities in order to learn more about how learners work with measurement .

For an understanding of the content that underpins place value consult Unit 6 pp. 197-210 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

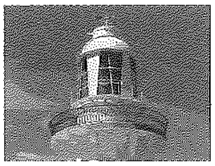
Grade 3 – Activity 35
Grade 4 – Activity 36
Grade 5 – Activity 37
Grade 6 – Activity 38
Grade 7 – Activity 39

a) For each activity you should answer the following questions:
b) In which grade(s) is the mathematical content that you identified as covered by this test item to be found in the mathematics curriculum?
c) Write out the specific curriculum wording for the identified content.
d) Draw up a list or a mind map in which you write about as many concepts as you can think of that are related to the concept of measurement as it is presented in this test item.
e) Explain your reasons for mapping the item content to your chosen curriculum reference(s).
f) Do you notice any differences between the concepts you first identified and the curricular references that you chose?
g) Do you think you might change the concepts you initially identified as necessary to solve the item correctly? If so, how?
h) Do you think you might change the curriculum references you initially identified as those which specify the concepts required to successfully work out the answer to the question? If so, how?
i) Do you teach this/these concepts in your mathematics class?
j) In what grade and during which term do you directly teach this/these concepts?
k) Do you link this/these concepts with others? If so, which ones?
l) If you don't teach these concepts in your grade, explain why not.

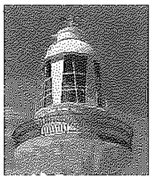
Activity 35

Grade 3 Measurement

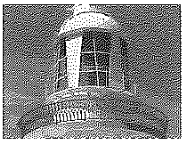
16. Which picture covers the biggest area?



(A)



(B)



(C)

(D)

Commentary

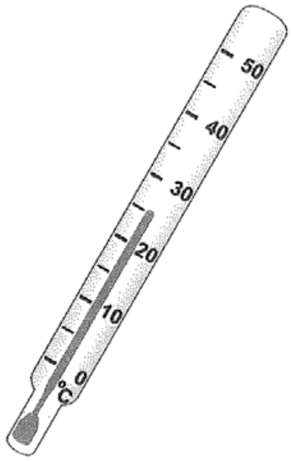
In order to answer this question correctly a grade 3 learner needs to know about the concept of area. This question does not call for measurement or calculation of area but rather the identification (using some sort of arbitrary unit) of which picture "covers the biggest area".

Explain how a learner could use an arbitrary unit to decide which of the pictures of a lighthouse in Activity 35 covers the largest area.

Activity 36

Grade 4 Measurement

5. What is the temperature shown on this thermometer?



(A) 20 °C
(B) 21 °C
(C) 25 °C
(D) 29 °C

Commentary

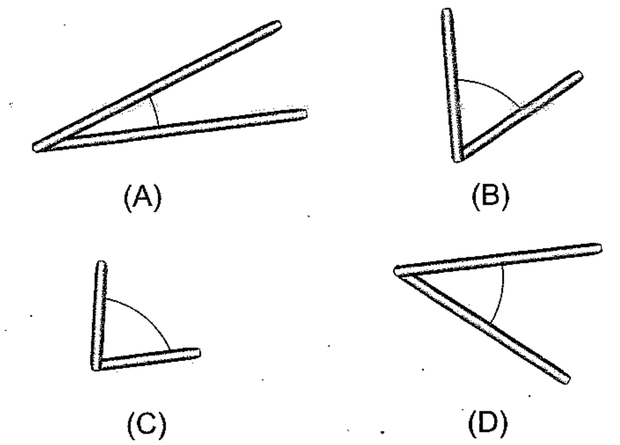
In order to answer this question correctly a grade 4 learner needs to be able to read a measurement (of temperature) from a scaled instrument.

Discuss the skills required by learners in order to read measurements from scaled instruments.

Activity 37

Grade 5 Measurement

19. Which of these shows the largest angle?



Commentary

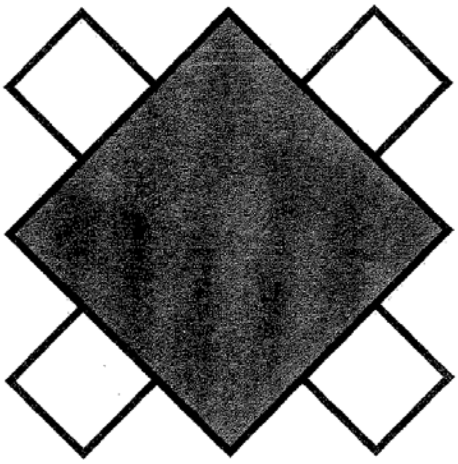
In order to answer this question correctly a grade 5 learner needs to be able to compare the sizes of angles represented diagrammatically.

Angles measure the size of an opening created by turning about a point. Discuss how you would explain the difference between a linear measure and the measurement of an angle.

Activity 38

Grade 6 Measurement

23. This shape has four small squares and one large square.



The area of each small square is 4 cm^2.

What is the area of the **whole** shape?

(A) 13 cm^2
(B) 36 cm^2
(C) 52 cm^2
(D) 160 cm^2

Commentary

In order to answer this question correctly a grade 6 learner needs to be able to calculate the area of a complex shape. The concept of area as "amount of surface covered" could be used by learners to decide how to work out what to do to calculate the total area of this shape.

Describe an activity that you could give to a grade 6 class to familiarise them with complex (shapes made up of more than one shape) shapes.

Activity 39

Grade 7 Measurement

27. Here are three bottles.

2.5 L 600 mL 1 L

What is the total capacity of the three bottles in **millilitres**?

(A) 950
(B) 1625
(C) 1850
(D) 4100

Commentary

In order to answer this question correctly a grade 7 learner needs to be able to work with (add) units of measurement (in this case litres and millilitres). Learners are also required by this question to be able to convert from litres to millilitres in order to find the total capacity of the three given bottles (in millilitres).

Explain the difference/similarity between the "total capacity" of the given bottles and the volume of liquid held by the three bottles.

Unit 3: Analysing learners' errors

Introduction
The first part of this unit introduces you to what teachers need to know in order to understand learners' misconceptions and errors. The second part introduces you to a process you can use to identify and reflect on learners' errors, with plenty of activities for practice and opportunities for reflection.

The activity of reflecting on and analysing learners' errors can help you to understand the thought processes of learners. Such understanding should enable you to develop more effective approaches to addressing learners' errors. An error can be the result of:

- a **mistake,** which is just a slip that the learner knows how to correct;
- a **misconception**, which causes the learner to make an error because he or she does not fully understand an underlying concept.

In this unit we focus on errors that arise from misconceptions.

Aims of the unit
This unit aims to introduce the reader to the kinds of knowledge that teachers need in order to understand and work with learners' errors and also to the value of using learners' errors as a resource for teaching and learning. It also aims to provide opportunities for teachers to learn how to analyse learners' errors. The unit focuses on the following:

- understanding error analysis;
- understanding different kinds of mathematical misconceptions;
- analysing learners' errors.

Understanding error analysis

Teachers come across errors in the mathematics classroom virtually every day. In order to work with learners' errors in the most effective way a teacher needs the relevant mathematical knowledge (subject matter knowledge); the ability to recognise the error and diagnose its cause; and understanding of how learners can relate previous to new knowledge. These kinds of knowledge are described in the reading, *Mathematical error analysis,* which follows.

Activity

Activity 1
Read the Reading for this Unit: *Mathematical error analysis*

h) After you have completed the reading write about why focusing on errors is likely to be a productive activity for a mathematics teacher.
i) According to Shulman and Ball, what different kinds of knowledge are involved in error analysis?

Reflection

Take some time to think about your own practices in relation to learners who make errors in your mathematics class. Reflect on your teaching practices with these questions in mind:

- How do you address learners' errors?
- Could you engage with learners' errors more effectively? Why or why not?

Commentary

Why is error analysis likely to be productive for teachers?

Proponents of error analysis explain that some of the errors learners make can be viewed as the attempts of these learners to make sense of new ideas. Therefore understanding learners' reasoning behind these errors can help teachers use errors productively to assist learners' in their acquisition of new mathematical knowledge. By investigating errors, teachers can gain a better understanding of the demands of the question, of the contextual elements (where these are important), and of alternative strategies or solutions. By engaging with error analysis teachers become more aware that the process of making sense may be different for each learner. Learners' errors must not be ignored but rather need to be understood by the teacher and used to inform subsequent teaching.

The central process of error analysis involves three stages:

- identify errors;
- diagnose their causes; and
- find a way to use the information from these errors in planning of future lessons.

Different kinds of knowledge involved

According to Ball there are different types of knowledge involved in doing error analysis. Subject knowledge (which Ball and Shulman call common content (or just content) knowledge) enables the teacher to identify an error ("is the answer right/wrong?"). Identification of an isolated error is just the first step. If a teacher can identify patterns in learners' work this shows a deeper understanding of the content from the perspective of the learner (this is referred to as "specialised content knowledge"). If the teacher can realise what learners were thinking when they made the error ("diagnose the cause of the error") then the teacher is using pedagogic content knowledge – a kind of knowledge which is special to teachers, since it enables them to understand their learners more deeply and be able to communicate with them more effectively.

Personal reflection

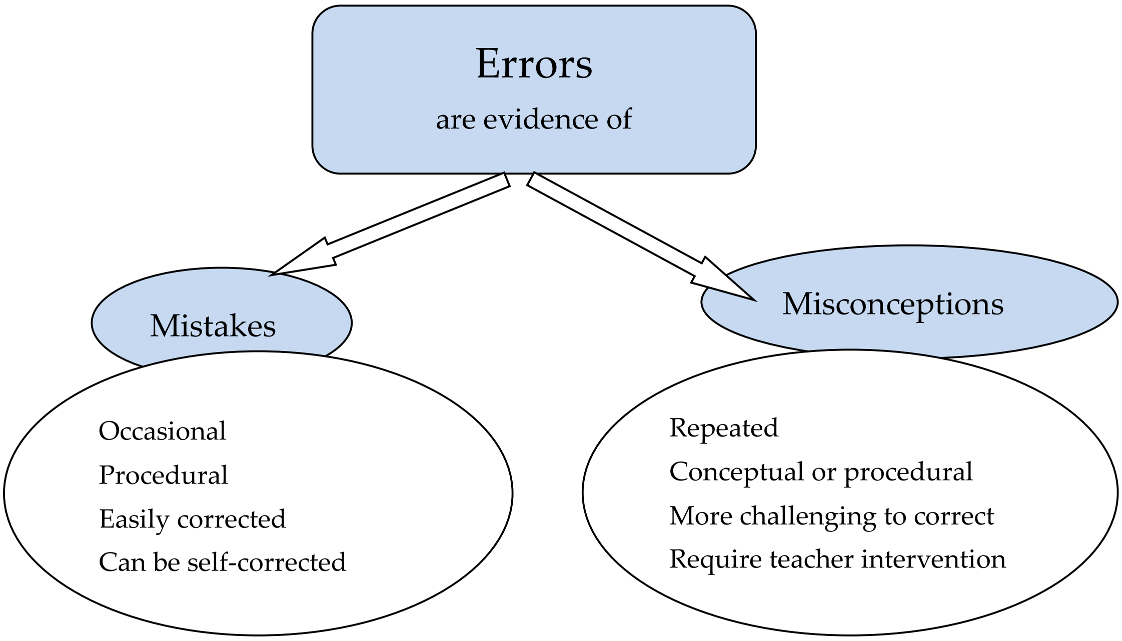
To become aware of the reasons why learners make errors in their daily mathematical tasks, teachers need to reflect on the ways they teach. If teachers only provide learners with examples which learners are expected to copy and follow without asking questions, they might remember a particular procedure for that particular example, but may not be able to handle a problem that is phrased differently. If learners keep on making the same errors in a test or exam, it may be time for the teacher to investigate the reasons behind these errors.

Understanding different kinds of mathematical misconceptions

The reading about the knowledge teachers need in order to do error analysis includes reasons for using learners' errors to inform teaching. It is important to understand the difference between errors that arise from mistakes and errors that arise from misconceptions.

For Olivier (1996) mistakes are wrong answers due to incorrect procedural working. Learners make such mistakes from time to time but these are easily corrected. Misconceptions arise from incorrect procedural or conceptual ideas and they occur repeatedly. They are not easy to correct because learners believe that what they are doing is right when in fact, they are making errors. Teachers need to deal with learners' misconceptions so that the learners will be able to progress to a more sophisticated mathematical understanding.

The diagram below illustrates the connection between "errors", "mistakes" (or slips) and "misconceptions".



Radatz (1979, p.167) writes about learners' mathematical errors in a slightly different way. He suggests that they can be due to any of the following:

- language difficulties;
- spatial/visualisation difficulties;
- learning gaps (inadequate prior learning);
- application of inappropriate rules or strategies.

In and extensive review of the research on mathematical misconceptions Smith, di Sessa & Roschelle (2007) identified a number of common threads in the academic study of mathematical misconceptions. Brief summaries of some of these threads are used to introduce the activities in this part of the unit.

Learners have different understandings of mathematical concepts

Learners do not always form the same conceptual understanding after listening to their teacher's explanations of a concept. It is part of the teacher's responsibility to check that all learners in his/her class come to full understanding of what he/she is teaching, by using various forms of assessment.

Example: A teacher may explain decimal number concept to her class. A learner who has not yet fully understood the explanations given about decimal numbers could use reasoning about whole numbers to deal with decimals as in the example in Activity 2.

Activity

Activity 2

A learner in your grade 5 class claims that 4,5 < 4,672. When asked why she has given this answer, the learner reasons that there are more numbers in 4,672 than in 4,5.

a) What is wrong with the learner's reasoning in the above explanation?
b) What is the concept embedded in the question?
c) What concept is interfering with the learner's reasoning in this instance?
d) In what way is this learners' understanding different to what the teacher expects?

Commentary

The reasoning the learner gives for her answer shows that she did not fully understand the teacher's explanation about decimal numbers. She drew on her understanding of whole number concept to compare the numbers 4,5 and 4,672, using the numbers 45 and 4 672. The learner needs to expand her number concept so that she correctly understands decimal numbers.

Misconceptions could originate in prior learning

Learners' minds are not empty slates when they enter a mathematics classroom. They have already learned some mathematics but their ideas about mathematical concepts may differ from each other, with some closer to and some further away from the normally accepted concepts. Clement (2008) calls these "pre-concepts". These pre-concepts are developed as part of a normal learning process but they need to be worked with so that learners will develop the correct conceptual understanding and be able to answer questions correctly.

Learners often generalise from their prior knowledge when they encounter new concepts. However learning does not necessarily involve the "stacking" of new knowledge on old knowledge. Instead old and new knowledge build on each other in complex ways as part of the learning process. Piaget (1969) refers to the process through which learners "make sense" of new knowledge as a process of both assimilation and accommodation.

Assimilation: If some new, but recognisably familiar, idea is encountered, this new idea can be incorporated directly into an existing set of ideas that relate to the new idea, i.e. the idea is interpreted or re-cognised in terms of an existing concept. In this process the new idea contributes to what the learner knows by expanding existing concepts.

Accommodation: Sometimes a new idea may be quite different from existing sets of ideas. Then it is necessary to re-construct and re-organise one's sets of ideas in order to accommodate the new. In this process the new idea contributes to extending what the learner already knows by introducing new concepts.

Example: A model that is often used in primary schools is that multiplication is repeated addition and always makes numbers bigger. In the repeated addition interpretation of multiplication, the operator must be a whole number and the product must be bigger than the numbers being multiplied. The process for adding whole numbers can be extended by assimilation to include adding decimal fractions. In the case of extending the process of multiplying from multiplying by a whole number to multiplying by a fraction, accommodation is required.

Activity

Activity 3

a) Give the problem below to your learners. Before doing the problem, ask them to explain what they understand about it
To trim 1 scarf you need 0,75 m of lace. How many scarves can you trim with 10 m of lace?

b) How did your learners respond to the problem?

c) How many ways could this question be explained?

d) What is the correct answer to the problem?

Commentary

This is a word problem which needs to be translated by learners into a mathematical equation before it can be solved. In this problem understanding of decimals and multiplication / division is required. If prior conceptions of whole number operations are incorrectly applied to decimals, this will result in incorrect reasoning. Learners could extend their knowledge of multiplication from whole numbers to multiplication of decimals numbers through assimilation. This means that they could transfer the idea that multiplication is repeated addition. However, the idea that multiplication will yield an answer that is "bigger" does not transfer, since multiplication by a decimal actually reduces the size of the number being multiplied. Understanding different effects of multiplication by whole numbers and decimals involves accommodation.

Misconceptions can be difficult to change

Some learners find it difficult to let go of oversimplified rules that they have learned in the early grades, especially if these seem to be simple and clear. Some misconceptions originate in teachers' efforts to make content and procedures simple for learners in the early years of schooling. Teachers need to find out what learners have been taught and where necessary to show them what is inadequate or incorrect about what they have previously learned.

Example: There are many general rules given out in maths which are not as general as they may sound. A common rule given out about fractions is "what you do to the top you do to the bottom". Learners remember this rule because is it short and rolls of the tongue easily but some of them forget the specific contexts in which the rule applies (simplification of fractions/generation of equivalent fractions) and they apply it to every context in which they operate on fractions. This leads them to incorrect answers, and yet they hold onto the rule because of its simplicity and clarity and seeming all-inclusiveness.

Activity 4

A learner in your class made the following mistake:

$\frac{1}{3} + \frac{1}{2} = \frac{1+1}{3+2} = \frac{2}{5}$

a) Explain the mathematical error that the learner has made.
b) What explanation would you give to help the learner to rectify the error?

Commentary

The learner has wrongly applied the rule "what you do to the top you do the bottom" and arrived at the incorrect answer. To assist learners to rectify this error the teacher could use concrete materials to demonstrate addition of fractions so that learners see that half a cake (or an apple) plus a third of a cake or apple cannot be two fifths of a cake or apple. The concrete demonstration will make two things clear to the learner. The first is that that unlike fraction parts cannot be added (e.g. $\frac{1}{3} + \frac{1}{2}$ cannot be added as they are). The second is that like fractions can be added by adding the number of fraction parts (e.g. $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$).

Misconceptions may interfere with learning

Misconceptions that are based on prior learning and that are resistant to change may interfere with new learning.

Example: In primary school, before negative numbers are encountered (in grade 7 in South Africa), learners are taught that "you cannot subtract a bigger number from a smaller number". A learner who has internalised this rule and is unable to move beyond it will say that 5 – 10 = 5. This follows the rule, of "subtract the smaller number from the bigger number". A learner who has started to modify this rule to incorporate negative numbers might come up with a rule that says "subtract the smaller number from the bigger number and make the answer negative". This rule would interfere with the progression of learning about subtraction of integers.

Activity

Activity 5

A grade 7 learner in your class uses the rule "subtract the smaller number from the bigger number and make the answer negative" to get the correct solution to: $-34 + 24 = -10$. The same learner says that $-34 - 23 = -11$.

a) Is the learner's answer of – 11 correct? Explain your answer.
b) What is the concept embedded in the question?
c) What concept is interfering with the learner's reasoning in this instance?

Commentary

This question involves subtraction of integers. The correct answer to the second question is – 57. The learner did not get this answer, but we can identify the assumption the learner may have made in finding the answer of –11. It seems that the learner has modified the general rule of "subtract the smaller number from the bigger number" to "subtract the smaller number from the bigger number and make the answer negative". This rule which the learner has invented (by over-generalising the basic "subtract the smaller number from the bigger number" rule) does not cover subtraction of a negative number from a negative number, as in the example $-34 - 23 = \_\_$. The concept of positive number operations is interfering with the concept of operations on integers.

Moving from misconceptions to correct conceptualizations is a process

Misconceptions cannot simply be uprooted and replaced with new, "correct" concepts. Learners have to become aware of their misconceptions through a process of repeated demonstrations and discussions.

Example: "Division makes things smaller" is a belief widely held by many learners. While this belief enables learners to divide by whole numbers, it does not work for division by fractions.

Activity

Activity 6

a) Give an example where division does "make things smaller".
b) Give an example where division does not "make things smaller".
c) How could you convince your learners that division does not always make things smaller?

Commentary

An example of division making things smaller is $4 \div 2 = 2$. An example of division not making things smaller is $1 \div \frac{1}{2} = 2$. Many examples should be given that contradict the belief that division makes things smaller in order to convince learners to give up their incorrect belief. Allow learners the opportunity to think of their own examples that uphold the correct rule and contradict the incorrect belief about the rule.

Language influences misconceptions

The language that teachers use to explain concepts often contributes to learners' misconceptions. Teachers sometimes use phrases to help learners to remember certain rules but in doing so they may actually create some misconceptions. For example, if teachers give the following statements as rules, they are using language that could promote the formation of misconceptions:

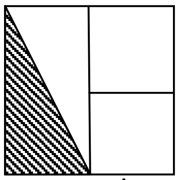
- "Flip and multiply" or "tip and turn" (when the multiplier is a fraction)
- "Carry over the bridge and change the sign" (when solving equations)
- "What you do to the numerator, you must do to the denominator" (when finding equivalent fractions)
- "Add a zero when you multiply by 10" (when multiplying by ten)
- "Move the comma by the number of zeros in the multiplier" (when multiplying by multiples of ten)

Example: The form of language used can also cause confusion to the learner, especially when a learner is studying maths in a language (such as English) that is not his or her home language.

Activity

Activity 7

A teacher showed the following example to her class, saying that she had shaded one quarter of the square. Chris argued with the teacher, saying that one quarter of the square had not been shaded.



a) Identify the reasoning of the learner.
b) Explain what you think might have caused the learner's error.

Commentary

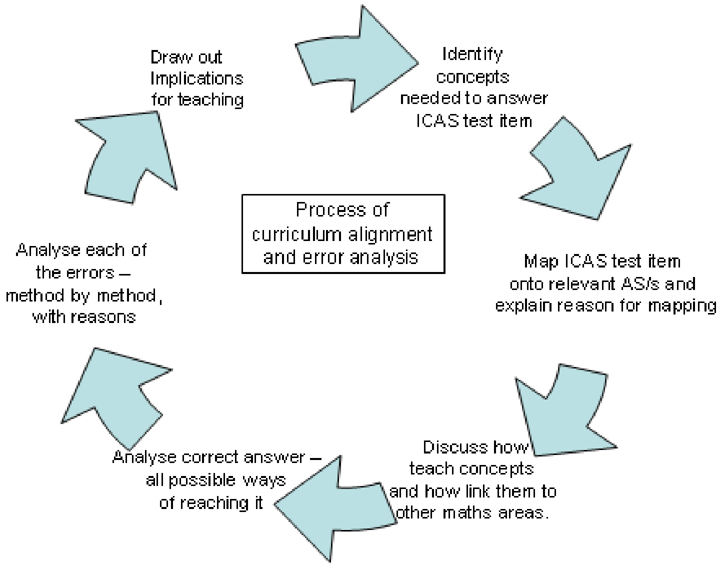
Much of learners' earlier work on fractions would have led them to understand that fraction parts of a whole must be equal. This learner does not think that one quarter of the shape has been shaded because the parts into which the square has been divided do not look the same, or "equal". Teachers commonly talk about "equal" parts, rather than "parts of equal size". In this teacher's example the square has been divided into quarters, but although the quarters do not look the same, they are of equal size.

Reflection

Reflect for a while on the different sources of learners' mathematical misconceptions.
How many have you encountered in your teaching?
What have you done when you encountered learners making mathematical errors?

Analysing learners' errors

Unit 2 focused on curriculum mapping. The diagram below shows the full cycle involved in aligning a test item with the curriculum and then analysing the errors that learners made when answering the test item in order to improve teaching. This unit focuses on error analysis. Unit 4 draws on the curriculum mapping and error analysis activities to help you give feedback to learners in relation to particular errors. Unit 5 is based on a series of transcripts of episodes from videotaped lessons of teachers and learners at work. The transcripts illustrate the misconceptions discussed in the previous section of this unit.



(Welch, 2010)

A process for error analysis

This activity offers you an opportunity to experience the DIPIP interpretive process of analysing mathematical test items by thinking about both the correct and incorrect answers to questions. Doing this should assist you to gain a better understanding of the mathematical concepts covered in the items and the misconceptions that learners may have in relation to these concepts. Knowing about this will enable you to teach these concepts more effectively.

The error analysis template that is used in the activities is given below. It has eight rows.

- The first two rows require the identification of the grade and particular test item under discussion.
- The third row requires you to think about the way(s) of working and thinking that a learner might use in order to get the answer correct (fill in more than one possible method and/or way of thinking, if necessary).
- The fourth row requires you to write about the expected level of the work. Is it as you would expect it for the grade, OR higher OR lower than you would expect?
- The fifth row requires you to write out the list of 'distractors' (the incorrect options provided in the multiple choice options) in order from the one

selected most often to the one selected least often. *You may trial these with your learners and see if they choose answers in a similar way.*

- The sixth row requires that for each distractor, you think and write about what learners might have done to obtain the answer in each incorrect choice. Fill in more than one possible method and/or way of thinking, if necessary.
- The seventh row begins the reflection, requiring that you write about any other issues with the task that you think are important but that have not been raised in the error analysis you have done.
- The eighth row requires you to write about issues that should be considered when teaching these concepts (for example, how to overcome some of the problems that have been raised).

		Teacher input
1	Grade level of item	
2	Test item (describe or write out)	
3	What are the way(s) of working and thinking that a learner might use in order to get the answer correct (fill in more than one possible method and/or way of thinking, if necessary).	
4	Indicate for *each method* whether you think it is at the expected level for the grade OR higher OR lower than you would expect.	
5	Write out the list of distractors in order from the one selected most often to the one selected least often. *You may trial these with your learners – and see if they choose answers in a similar way.*	
6	For each distractor, think and write about what learners might have done to obtain the answer in each incorrect choice. Fill in more than one possible method and/or way of thinking, if necessary.	
7	Write about any other issues with the task that you think are important but that have not been raised in the error analysis you have done.	
8	Write about issues to consider when teaching these concepts (for example, how to overcome some of the problems that have been raised).	

In the activities that follow, you will use this template row by row, slowly building up your understanding of all of the aspects involved in error analysis. The selected activities are linked to the same item (ICAS[9] 2006 Grade 5 Item 9) used in Unit 2. If you have not yet done the curriculum mapping of these items, you may wish to do this before you proceed with the error analysis of the activities, since curriculum mapping will help you to locate the concepts being tested in the test item within the curriculum that you teach.

[9] International Competitions and Assessments for Schools (ICAS) is conducted by Educational Assessment Australia (EAA), University of New South Wales (UNSW) Global Pty Limited. Students from over 20 countries in Asia, Africa, Europe , the Pacific and the USA participate in ICAS each year. EAA produces ICAS papers that test students in a range of subjects including mathematics.

When you work through error analysis activities, you develop your diagnostic reasoning – your ability to engage with learners’ thinking about the mathematics they are doing and with the mistakes or errors they make when they do this work.

Diagnostic reasoning: understanding learners’ thinking

In the DIPIP project teachers started to talk about thinking diagnostically about learners’ errors. The following three sections of the unit will take you through the DIPIP error analysis process to enable you to think diagnostically about learners’ errors in your mathematics classes.

Learners’ thinking about the correct answers

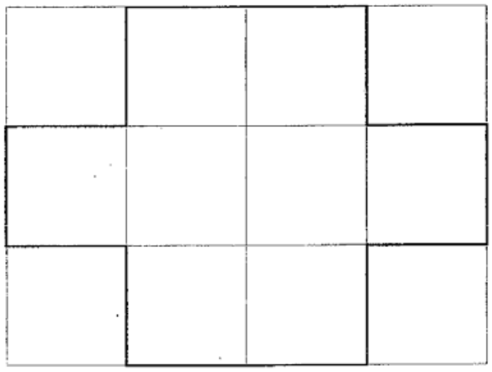
Before you focus on learners’ errors about mathematical content you should be sure that you know this content well yourself. As a teacher this will mean that you may be able to think of several ways (not just one) to find the correct solution to a question. You should not expect your learners to be able to solve every question using multiple strategies – one correct strategy is enough for them. Although more would be enriching, your first aim is for learners to be able to use just one.

Activity

Activity 8

Study the test item below:

9. Holly drew this shape on 2 cm grid paper.



What is the area of Holly's shape?

(A) 32 cm^2
(B) 28 cm^2
(C) 16 cm^2
(D) 8 cm^2

(ICAS 2006 Grade 5 item 9)

d) Solve the item yourself.
e) The grade level of the item is given in the test. Do you think that the item has been set for the appropriate grade? If not, which grade level would you say it is?
f) What are the way(s) of working and thinking that a learner might use in order to get the answer correct (fill in more than one possible method and/or way of thinking, if necessary).
g) Indicate for *each method* whether you think it is at the expected level for the grade OR higher OR lower than the group would expect.

Commentary

This is a challenging question for grade 5 (and even grade 6) learners and may be more easily done by grade 7 learners. By using their knowledge of what area is, to determine that each little square's area is $4cm^2$, and then multiplying 4 by the number of squares in the shape, which is 8, this will equal 32.

Some could have calculated the area of the whole shape (an 8 cm by 6 cm rectangle) and then subtracted the area of the four corner squares from the area of the whole shape. This would give the area of Holly's shape. It is also possible to view the top two squares as a rectangle (two by four), the next four squares as another rectangle (two by eight) the bottom two squares as another rectangle (two by four). Learners would then calculate the area of the three rectangles and add them to find the full final answer.

Reflection

Explain why it is important for a teacher to think about the correct solution to a problem before thinking about errors that learners could make when solving the problem.

Learners' thinking about the errors

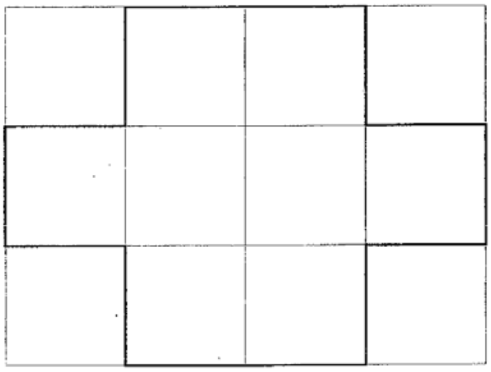
Learners may have different reasons for giving different answers. In the previous section of this unit you read about several different ways in which learners' misconceptions may arise. As a teacher you need to try to "get into the learner's head" to find out why he/she thinks the way he/she does in connection with a mathematical problem. If you are able to do this, you will be able to diagnose the learners' errors. This will help you to help your learners understand where they have gone wrong and start to realise what they should have done to answer the question correctly.

Activity

Activity 9

Study the test item below:

9. Holly drew this shape on 2 cm grid paper.



What is the area of Holly's shape?

(A) 32 cm^2
(B) 28 cm^2
(C) 16 cm^2
(D) 8 cm^2

(ICAS 2006 Grade 5 item 9)

DIPIP learner data for selection of answers:

A	14%
B	22%
C	23%
D	34%

a) Write out the list of distractors in order, from the one selected most often to the one selected least often. *You may trial these with your learners – and see if they choose answers in a similar way.*

b) For each distractor, think about and write what learners might have done to obtain the answer in each incorrect choice. Try to think of more than one possible method and/or way of thinking.

Commentary

D - Learners just counted 8 blocks because they did not realise that a square in the grid has an area of 2 cm by 2 cm.

C - Learners saw the 2 cm and multiplied it by the 8 squares which they counted. They must have reasoned that each square is 2 cm and there are 8 squares in Holly's shape.

B - Instead of calculating the area they calculated the perimeter by adding the length of the sides.

Reflection

Trial these distractor explanations given in the commentary above.
Reflect on the similarities or differences between your learners' thinking and these given explanations.

Reflecting on your teaching with errors in mind

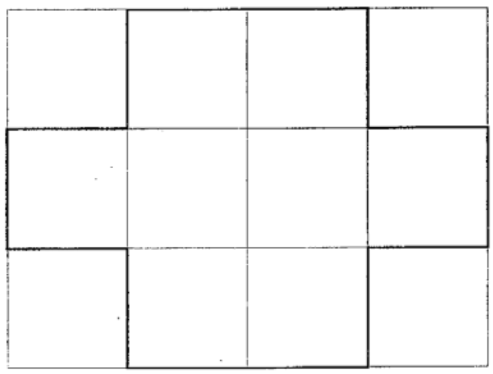
As a responsible teacher of mathematics you need to deal with learners' errors when you encounter them. You might not always be able to address an error on the spot (it may be too time consuming in the context of a lesson where time is limited) but you should make a point of speaking to any learner who you realise holds a misconception that will interfere with his/her mathematical learning.

Activity

Activity 10

Study the test item below:

9. Holly drew this shape on 2 cm grid paper.



What is the area of Holly's shape?

(A) 32 cm^2
(B) 28 cm^2
(C) 16 cm^2
(D) 8 cm^2

(ICAS 2006 Grade 5 item 9)

a) Write about any other issues with the task that you think are important but that have not been raised in the error analysis you have done.
b) Write about issues to consider when teaching these concepts (for example, how to overcome some of the problems that have been raised).

Commentary

In grade 5, areas of simple squares and rectangles is not often taught using a combination of shape(s) in a drawing as in this question from the ICAS test. Where such tasks are introduced in grade 5 (or even 6) teachers may only have used 1 cm by 1 cm squares. Teachers need to establish the basics but then move on to more challenging questions, as called for by the curriculum so that if learners write an international test (such as ICAS) with difficult questions such as this one, they will be prepared for them.

Reflection

Do you regularly refer to the curriculum for your grade to check that you are teaching content at the correct level?
Think about how you could disadvantage your learners if you do not teach them content at the appropriate level for their grade every year.

Further Error analysis activities

In **Unit 3: Further error analysis activities**, there are sets of activities, grouped according to mathematical content areas. These will give you the opportunity to do error analysis in relation to test items across a range of mathematical topics. You should select those items for which you have already done the curriculum mapping and error analysis activities in Unit 2.

The items have been selected in order to raise discussion of the mathematical content they present. There is one activity per grade, from grade 3 to grade 7, so you can work through the set of activities at the level or levels of your choice. The activities do not represent an entire mathematics curriculum since this is not possible within the scope of the unit, but they do present material which is often misunderstood by learners and about which teachers would benefit from deep understanding. The commentary given does not relate to the full range of questions you need to work through when you do these activities, but highlights key misconceptions for the given tasks. You should discuss your activity responses with a colleague if possible in order to gain the full benefit from these activities.

Conclusion

This unit has:

- introduced the idea that learners errors are productive for teaching;
- explained the differences between mistakes and misconceptions;
- explained different kinds of misconceptions that learners develop and the implications for learning and teaching; and
- provided you with processes for and practice in analysing mathematics test items in order to enable you to think more deeply about the errors learners make when they do mathematics.

The process of error analysis is based on the idea that learners have different understandings of mathematical concepts at times, some of which may need to be worked with in order for all learners to achieve full, correct understanding of the concept. Teachers can give learners a voice by encouraging them to explain their thinking in relation to the errors they have made and then use the information that they receive when they listen to learners' explanations in order to diagnose the source of learners' errors. Careful analysis of learners' errors can be productive for teaching and learning and can bring about deeper understanding of mathematical concepts for teachers and learners.

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Unit 3 Reading: Mathematical error analysis

This Reading was prepared by Yael Shalem especially for this module.

In Mathematics teaching, error analysis involves identifying and thinking about the errors in learners' work in order to understand the mathematical explanation of each error. Not all learners' errors are the result of mathematical misconceptions: some are simply careless 'slips' (Olivier, 1996) or mistakes which can easily be corrected if the slip or mistake is pointed out to the learner. Error analysis focuses on the errors made by learners which are based on misunderstanding of a concept. A learner who makes such an error believes that what he/she is doing is correct. Such errors are systematic and persistent and are performed by learners in a range of contexts (Nesher, 1987), both in South Africa and in other countries.

In the Data Informed Practice Improvement Project (DIPIP) error analysis was one of the main activities. In this project identification of mathematical errors, and thinking about the way in which these errors influence the mathematical thinking of learners, were informed by Ball's domains of teacher knowledge (Shalem, Sapire & Sorto, 2014). Key aspects of error analysis were found to span three of the knowledge domains described by Ball: (i) common content knowledge; (ii) specialised content knowledge and (iii) pedagogical content knowledge (Ball, Thames & Phelps, 2008).

Research in mathematics education has shown that a focus on errors, as evidence of reasonable and interesting mathematical thinking on the part of learners (Radatz, 1979; Kramarski & Zoldan, 2008), helps teachers to understand learners' thinking (Prediger 2010; Peng, 2010; Heritage, Kim, Vendlinski, & Herman, 2009), to adjust the ways they engage with learners in the classroom situation and to revise their approaches to teaching (Nesher, 1987; Brodie, 2013, Adler, 2005).

Teachers come across errors in the mathematics classroom virtually every day. How teachers handle these errors plays a major role in the learning process. The questions to be asked are what do teachers need to know in order to work with learners' errors productively and what does it take to work effectively with learners' errors? It can be argued that at a minimum level, in order to respond to learners' errors in the most effective way, a teacher needs to know the correct solution to the question and also some common errors that learners might make in relation to the question.

Working with learners' errors is not a routine activity, but rather a specialised capacity which relies on the teacher's complex understanding of the content being taught. As Peng and Luo (2009) argue, if teachers identify learner errors but interpret them with wrong mathematical knowledge, their evaluation of learner performance or their plan for a teaching intervention are both meaningless. Error analysis requires, therefore, professional judgement and teachers need to learn how to respond to learners' errors in a productive way.

If working with learners' errors relies on teachers' clear understanding of the content under investigation, it is important to understand what guides the judgements and critical decisions that teachers need to make in the classroom.

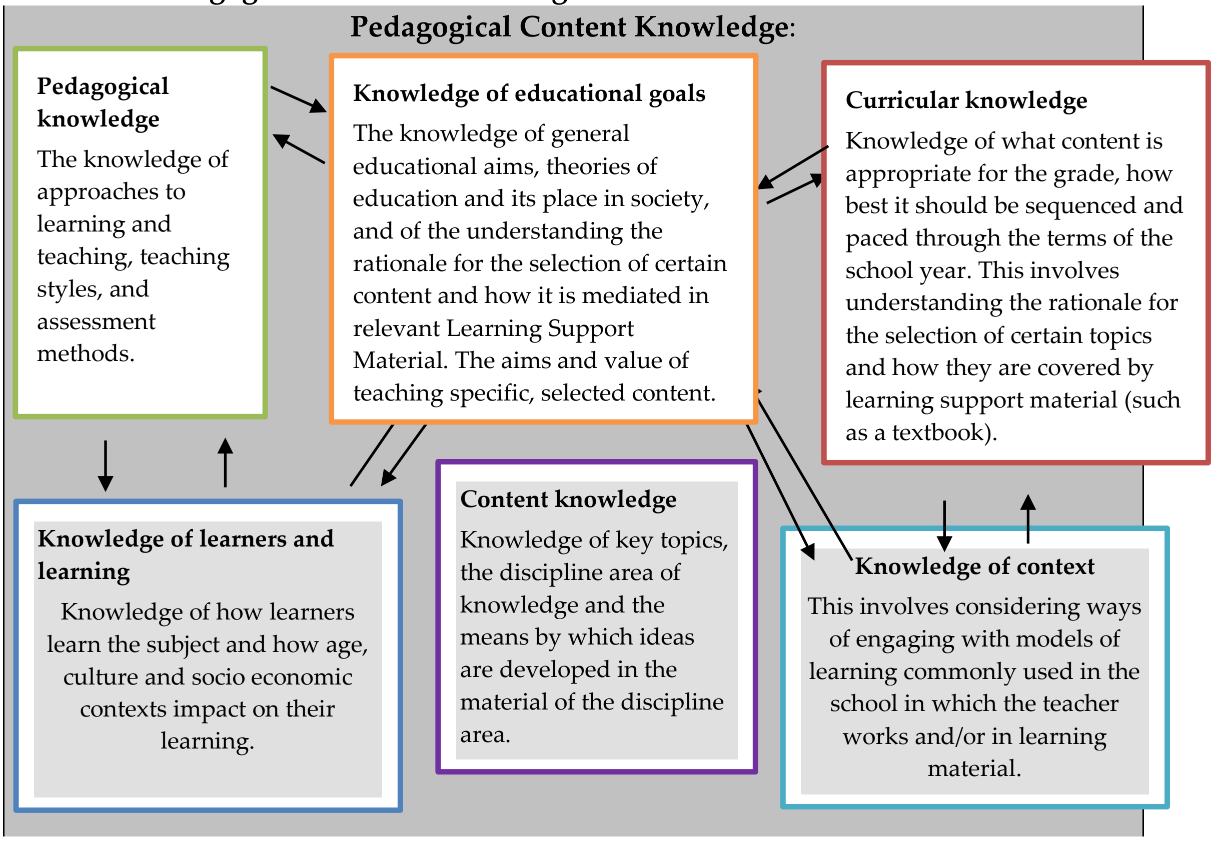
The central question that frames studies on teacher knowledge, in the field of mathematics education, is the following: "is there a professional knowledge of mathematics **for teaching** which is tailored to the work teachers do with curriculum materials, instruction and students?" (Ball, Hill & Bass, 2005, p16). This question points to an important idea- that professional knowledge for teaching consists of something very specific, some kind of knowledge that a pure content specialist (for example a mathematician rather than a mathematics teacher) does not need to develop. Lee Shulman, a famous theorist in the field of teacher education, introduced the term "pedagogical content knowledge" (PCK) to describe the unique specialisation involved in teaching. PCK, he says, is "that special amalgam [blend] of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (1987, p.8).

Many theorists have followed Shulman's innovative idea and developed different categorisations of teachers' knowledge of mathematics for teaching (Ball, Thames & Phelps, 2008; Adler, 2005; Hill, Rowan & Ball, 2005). Working productively with learners' errors requires several kinds of teacher knowledge. It is a complex practice that involves more than knowing the subject matter of mathematics. This does not mean that teachers' knowledge of the subject matter of mathematics is not important but it does mean that teachers also require other knowledge resources.

Teacher knowledge of mathematics – Shulman's view

The figure below draws on Shulman's development of his three-way classification of teachers' knowledge of mathematics. In his 1987 paper he identified a further four knowledge bases shown in Figure 1. The figure also shows that different components combine to make up pedagogical content knowledge (PCK).

Figure 1: Shulman's classification of the knowledge resources that make up teachers' Pedagogical Content Knowledge



Ball, Thames and Phelps (2008) have extended Shulman's notion of pedagogical content knowledge and its relation to subject matter. They use a new term to define PCK calling it "mathematical knowledge **for** teaching". They define this as "the mathematical knowledge needed to carry out the work of teaching mathematics" (p395). Following the distinctions made by Shulman between content and pedagogical content knowledge, Ball et al. have identified six sub-domains in relation to content knowledge and pedagogic content knowledge. These are shown in Table 1 below. The figure aims to show that working with learner errors relies on a variety of dimensions of teacher knowledge.

Table 1: Ball et al.'s (2008) classification of teachers' knowledge resources

Subject Matter Knowledge		Pedagogical Content Knowledge	
Common content knowledge (CCK)	Horizon content knowledge (HCK)	Knowledge of content and students (KCS)	Knowledge of content and curriculum (KCC)
Specialized content knowledge (SCK)		Knowledge of content and teaching (KCT)	

Briefly, the four domains of knowledge that are particularly relevant to error analysis can be defined as follows:

- **Common Content Knowledge (CCK).** This is general subject matter knowledge. A mathematics teacher with good subject matter knowledge uses mathematical terms correctly, is able to follow a procedure fully, and can evaluate whether a textbook defines a mathematical term correctly or incorrectly.
- **Specialised Content Knowledge (SCK).** This is mathematical knowledge specific to teaching and which, according to Ball et al, general mathematicians do not need.
- **Knowledge of Content and Students (KCS).** This orients teachers to the kind of knowing typical of learners of different ages and social contexts in regard to specific mathematical topics. Teachers develop this orientation from their teaching experience and from specialized educational knowledge of typical misconceptions that learners develop when they learn specific topics.
- **Knowledge of Content and Teaching (KCT).** This makes the link between the knowledge resources of the first two domains (Common Content Knowledge and Specialised Content Knowledge) and the knowledge resource of the third domain (Knowledge of Content and Students). Based on their knowledge of these three sub-domains, teachers use their knowledge of teaching to decide on the sequence and pacing of lesson content or which learner's contribution to take up and which to ignore.

Error analysis and teachers' knowledge of mathematics

The activity of mathematical error analysis draws on the different types of knowledge identified by Shulman, Ball and others. In terms of working with learners' errors, error analysis activities, using Ball's domains, can be identified in the following ways:

1. **Recognizing whether a learner's answer is correct or not**. This activity draws on Common Content Knowledge (CCK). Recognition of errors is a necessary component of teachers' content knowledge. Teachers who are able to *explain* the mathematical solution to a problem are likely to be able to recognize learners' mathematical errors. Teaching mathematics involves a great deal of procedural explanation which should be done fully and accurately so that learners understand the procedures and become competent in working with them. Teaching mathematics also involves conceptual explanations which should be done in such a way that concepts can be generalised by learners and applied correctly in a variety of contexts. So the first aspect of mathematical knowledge that teachers need, in order to "carry out the work of teaching mathematics", and specifically of error analysis, is to know the crucial steps involved in arriving at the correct answer, the sequence of these steps and, where appropriate[10], their links to other concepts.

[10] Some mathematical problems lend themselves more to procedural explanations while in others the procedural and the conceptual are more closely linked. There is a progression in mathematical concepts – so that what may be conceptual for a grade 3 learner (for example, basic addition of single digit numbers) is procedural for a grade 9 learner who will have progressed to operations at a higher level.

Looking for patterns in learners' errors.
Here teachers draw on Specialized Content Knowledge (SCK). A different aspect of error analysis is located in this sub-domain: "Looking for patterns in student errors or…sizing up whether a nonstandard approach would work in general" (Ball, Thames & Phelps, 2008, p400). Whereas teacher knowledge of the full explanation of the correct answer enables a teacher to spot the error, teacher specialized content knowledge enables a teacher to interpret a learner's solution and evaluate its plausibility (how likely it is to be acceptable), by recognizing the missing steps and/or conceptual links in the learner's solution.

Error analysis is a common practice among mathematicians in the course of their own work; the task in teaching differs only in that it focuses on the errors produced by learners… Teachers confront all kinds of student solutions. They have to figure out what students have done, whether the thinking is mathematically correct for the problem, and whether the approach would work in general (ibid, p397). So, for example, when marking their learners' work (Ball, 2011, Presentation), teachers need to judge and be able to explain to the learners which definition of a concept ('rectangle', in the following example) is more accurate:

- a rectangle is a figure with four straight sides, two long and two shorter
- a rectangle is a shape with exactly four connected straight line segments meeting at right angles
- a rectangle is flat, and has four straight line segments, four square corners, and it is closed all the way around.

DIPIP used the idea of "awareness of error" to refer to this type of knowledge of error.

2. **Thinking diagnostically about learners' reasoning when they make mathematical errors**. The knowledge resource in this sub-domain, Knowledge of Content and Students (KCS), enables teachers to explain and provide a rationale (set of reasons) for the way the learners are reasoning when they produce an error. The activity of error analysis involves teachers going beyond identifying the actual mathematical error and understanding the way learners may have been reasoning when they made the error.

   The emphasis in diagnostic reasoning is on the quality of the teachers' attempts to provide an explanation for how learners were reasoning mathematically when, for example, they chose a distractor in a multiple choice question. Teachers need to be able to provide multiple explanations of the error since learners need to hear more than one explanation of it. Because contexts of learning (such as age and social background) affect learners' understanding and because, for some topics, initial misconceptions are the starting point for learning, teachers need to develop a repertoire (range) of explanations, with a view to addressing differences in learners' current knowledge and readiness to move to more complex understandings. Some explanations are more accessible (easily understood) to learners while others, though more complex, are more accurate. Once they have been diagnosed, some errors may need to be explained in different ways to different learners.

Different knowledge resources contribute to the development of Pedagogic Content Knowledge (PCK) in relation to errors and enable teachers to identify, diagnose and respond to learners' mathematical errors in an appropriate way. Ball, Thames and Bass (2008) emphasise that knowledge resources are connected and yet remain distinct:

> … Recognizing a wrong answer is common content knowledge (CCK), whereas sizing up the nature of an error, especially an unfamiliar error, typically requires nimbleness (ability to move quickly and skilfully)in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of specialized content knowledge (SCK). In contrast, familiarity with common errors and deciding which of several errors students are most likely to make are examples of knowledge of content and students (KCS). (p401; explanation of nimbleness added)

Teachers with a deep knowledge of learners' mathematical errors will be able to address the range of errors and misconceptions that they have identified through doing error analysis. Many of the errors made by learners are common not only among South African learners but also among learners in classrooms all over the world. These errors (often based on misconceptions) are seen by some as a natural part of the learning process (Nesher, 1987). By learning about mathematical errors teachers are able to assist their learners to move beyond these misconceptions to reach a full and correct understanding of the foundational mathematics they are taught at school.

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Unit 3 Further Error analysis activities

Introduction

The following sets of activities, grouped according to mathematical content areas, will give you the opportunity to do error analysis in relation to test items across a range of mathematical topics. You should select those items for which you have already done the curriculum mapping and error analysis activities in Unit 2.

The items have been selected in order to raise discussion of the mathematical content they present. There is one activity per grade, from grade 3 to grade 7, so you can work through the set of activities at the level or levels of your choice.

If you would like to improve your understanding of the relevant content areas in the curriculum, we suggest that you consult *Mathematics for Primary School Teachers*, an openly licensed module digitally published by *Saide* and the University of the Witwatersrand (Wits), downloadable from OER Africa: http://www.oerafrica.org/ResourceResults/tabid/1562/mctl/Details/id/39030/Default.aspx.

The activities do not represent an entire mathematics curriculum since this is not possible within the scope of the unit, but they do present material which is often misunderstood by learners and about which teachers would benefit from a deeper understanding. The commentary given does not relate to the full range of questions you need to work through when you do these activities, but they highlight key misconceptions for the given tasks. You should discuss your full activity responses with a colleague if possible to gain the full benefit from these activities.

Place value

Learners' understanding of place value as used in our base ten numeration system is developed from the moment they start counting and recording numbers. They extend this understanding of one-by-one counting to knowledge of numbers with bigger values until they are able to read, represent and work with numbers of any size.

Activities 10 – 14 Place value

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the place value activities in order to learn more about how learners work with place value.

For an understanding of the content that underpins place value consult Unit 2 pp. 53-92 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

Grade 3 – Activity 10
Grade 4 – Activity 11
Grade 5 – Activity 12
Grade 6 – Activity 13
Grade 7 – Activity 14

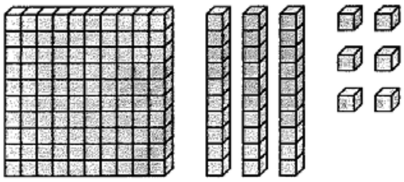
For each activity you should answer the following questions:

l) Solve the item yourself.
m) The grade level of the item is given in the test. Do you think that the item has been set for the appropriate grade? If not, which grade level would you say it is?
n) What are the way(s) of working and thinking that a learner might use in order to get the question correct (fill in more than one possible method and/or way of thinking, if necessary).
o) Indicate for *each method* whether you think it is at the expected level for the grade OR higher OR lower than the group would expect.
p) Write out the list of distractors in order from the one selected most often to the one selected least often. *You may trial these with your learners – and see if they choose answers in a similar way.*
q) For each distractor, think and write about what learners might have done to obtain the answer in each incorrect choice. If possible write out more than one method and/or way of thinking.
r) Write about any other issues with task that you think are important but that have not been raised in the error analysis you have done.
s) Write about issues to consider when teaching these concepts (for example, how to overcome some of the problems that have been raised).

Activity 10

Grade 3 Place value

12.



What number is shown by this model?

(A) 136
(B) 163
(C) 631
(D) 1036

DIPIP learner data for selection of answers:

A	**42%**
B	13%
C	17%
D	24%

Commentary

D – The learners do not have a good understanding of place value and may have been distracted by the size of the "big" block and written "10" for the ten rows they see in the block instead of "1" for the one hundred that the big block represents.

C – The learners may have read from right to left – their eye may have been attracted to the six blocks first. This could happen if they were unfamiliar with Dienes' blocks and if they do not have a well-developed understanding of [place value.

B – Here the learners have swopped the numbers 3 and 6. This may have happened because of poor focus or possibly dyslexia.

In Grade 3 learners are introduced to the 'hundreds' place value.

- What errors in the understanding of place value might emerge at this stage?
- What can teachers do to enable learners to form a firm basic understanding of the base ten numeration system that can be expanded in the years of schooling to come?

Activity 11

Grade 4 Place value

13. 186 – 47 = ?

(A) 149
(B) 141
(C) 139
(D) 39

DIPIP learner data for selection of answers:

A	19%
B	22%
C	**41%**
D	14%

Commentary

A – Learners who did this have partially understood how to subtract from a 3-digit number but they have not worked correctly with the tens and units, in the process of subtracting the 7 units. They did break down one of the tens to make 16 units from which they subtracted 7, but they did not compensate for this by reducing the number of tens (and so they subtracted 4 tens form 8 tens, rather than 4 tens from 7 tens).

B – Learners who selected this option show the common error of following the over-geneneralised rule "you cannot subtract a bigger number from a smaller number". Following this rule, they reverse the direction in which they are subtracting, in order to maintain subtraction of a smaller number from a bigger number. (This reversal usually takes place only in the places where learners encounter a "problem" (as in the units place in this example) for the rest of the operation, they follow the correct order.

D – This error is could be an indication of a slip (careless error – forgetting to record the hundreds). It could also indicated that learners might think they need to break down both the tens and the hundreds when they need to break down in order to subtract. A teacher would need to speak to a learner who chose this option to dig more deeply into the reasons for selection.

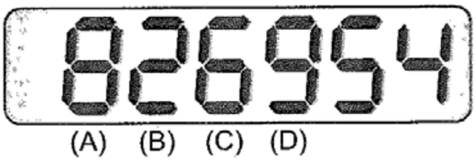
In Grade 4 learners expand their understanding of number concept to 4-digit numbers.

- What errors in the understanding of place value might emerge at this stage?
- What can teachers do to enable learners to build on their basic understanding of place value and work through misconceptions they may have formed in earlier years?

Activity 12

Grade 5 Place value

5. Which digit shows the number in the thousands column on this calculator display?



DIPIP learner data for selection of answers:

A	22%
B	12%
C	**35%**
D	24%

Commentary

A – Learners who selected this distractor may have looked only at the lettering giving indicators for the options and counted four places to the left from the units "place" (D). This is a test-related error since the layout may have tricked the learner into this incorrect reading, but it is based on poor place value concept, since a learner with a good understanding of place value would probably have looked at the whole digital display of the number and not used the multiple choice distractor letters as an indication of place value in a number.

B – This distractor is more of a filler – mathematically it does not stand in a "place" that may have been misunderstood (four from the left or four from the right). It does have its basis in a poor place value concept, since it is an invalid reading of the digits according to their positions in the numbers.

D – This distractor represents the error or reading place value from right to left inst4ead of from left to right. It also could be an indication of confusion between "hundreds" and "thousands".

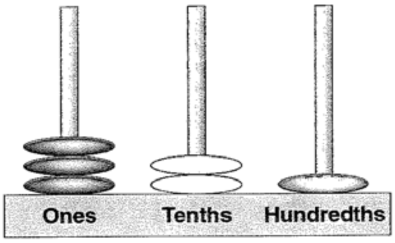
In Grade 5 learners expand their understanding of number concept to 6-digit numbers.

- What errors in the understanding of place value might emerge at this stage?
- What can teachers do to enable learners to build on their basic understanding of place value and work through misconceptions they may have formed in earlier years?

Activity 13

Grade 6 Place value

24. This is a bead counter.



Which number is shown on the bead counter?

(A) 1.23 (B) 3.21

(C) 123 (D) 321

DIPIP learner data for selection of answers:

A	19%
B	**22%**
C	32%
D	24%

Commentary

A – Learners who chose this option have reversed the order of the places represented by the counters on the abacus – they are reading hundredths as units, tenths as tenths and units as hundredths. This shows a fundamental lack of understanding of the way in which units, tenths and hundredths are related. The representation using an abacus may have lead to the confusion, since learners may not be familiar with using an abacus to represent decimal numbers. They did choose an option with a decimal comma in the answer, which indicates that they had taken note of the information giving place names on the abacus.

C – Learners who chose this distractor had not noticed that decimal fractions are being represented by the abacus. They are reading the number from the abacus as if the positions are from left to right, units, tens and hundreds.

D – Learners who chose this distractor had not noticed that decimal fractions are being represented by the abacus. They are reading the number from the abacus as if the positions are from right to left - units, tens and hundreds. This is not conventional for abacus representations, where the smallest place is usually on he left, similar to the way in which numbers are recorded using place value.

In Grade 6 learners expand their understanding of number concept to 9-digit numbers.

- What errors in the understanding of place value might emerge at this stage?
- What can teachers do to enable learners to build on their basic understanding of place value and work through misconceptions they may have formed in earlier years?

Activity 14

Grade 7 Place value

13. Which of these numbers is smallest?

(A) 0.1 (B) 0.09

(C) 0.109 (D) 0.0999

DIPIP learner data for selection of answers:

A	59%
B	**14%**
C	5%
D	20%

Commentary

A – The face value (the digit 1 – not noting the place in which this digit is standing) of this distractor is the root cause of the error made by learners who chose this distractor. 1 is the smallest number that can be identified, if one ignores place value.

C – The presence of the 1 (albeit in combination with a nine in the thousandths place) might have distracted learners who were again thinking more of "face value" (the digits you see, regardless of the places in which they are positioned). The 1 is in the tenths place and so this is actually the biggest number presented here.

D – The presence of 9s in "very small places" might have distracted learners who chose this as the smallest number. This number does have a 9 in the smallest place present in all of the numbers given in the options, but the actual number is bigger than 0,09 by 0,0099. A very small amount, but bigger none-the-less.

In Grade 7 learners expand their understanding of number concept to include decimal numbers.

- What errors in the understanding of place value might emerge at this stage?
- What can teachers do to enable learners to build on their basic understanding of place value and work through misconceptions they may have formed in earlier years?

Operations

Learners' understanding of operations (addition, subtraction, multiplication and division) begins when they have adequate number concept knowledge which allows them to operate on pairs of numbers. Early operation strategies should be closely linked to concrete examples but as learners develop this understanding they should start to use abstract procedures and not rely on counting to do their computations.

Activities 15 – 19 Operations

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the operations activities in order to learn more about how learners work with operations.

For an understanding of the content that underpins Operations consult Unit 3 pp. 93-124 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

Grade 3 – Activity 15
Grade 4 – Activity 16
Grade 5 – Activity 17
Grade 6 – Activity 18
Grade 7 – Activity 19

For each activity you should answer the following questions:

a) Solve the item yourself.
b) The grade level of the item is given in the test. Do you think that the item has been set for the appropriate grade? If not, which grade level would you say it is?
c) What are the way(s) of working and thinking that a learner might use in order to get the question correct (fill in more than one possible method and/or way of thinking, if necessary).
d) Indicate for *each method* whether you think it is at the expected level for the grade OR higher OR lower than the group would expect.
e) Write out the list of distractors in order from the one selected most often to the one selected least often. *You may trial these with your learners – and see if they choose answers in a similar way.*
f) For each distractor, think and write about what learners might have done to obtain the answer in each incorrect choice. If possible write out more than one method and/or way of thinking.
g) Write about any other issues with task that you think are important but that have not been raised in the error analysis you have done.
h) Write about issues to consider when teaching these concepts (for example, how to overcome some of the problems that have been raised).

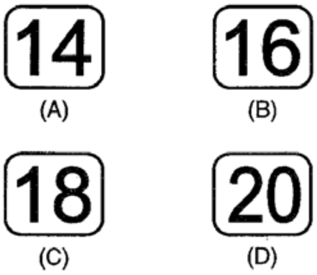
Activity 15

Grade 3 Operations

26. Here is a pattern.

Each row of three cards follows the same pattern.

What is the missing number card from the third row?



DIPIP learner data for selection of answers:

A	30%
B	**28%**
C	14%
D	18%

Commentary

This item requires the use of basic addition/subtraction to identify a pattern given in a block divided into three rows and three columns in which numbers are placed and one of the nine numbers is missing.

A – This choice shows that learners were distracted by the counting in 2s apparent in the first two rows of the given pattern block. They counted from row to row (6-8, 10-12, 18-20) and did not look at the pattern occurring simultaneously between the columns (6-10, 8-12).

C – This was the least popular distractor and is the most difficult to explain. It is an arbitrary number that may have seemed feasible to some learners. Teachers could ask particular learners who chose this answer to explain their reasoning to get to the root of the misunderstanding that resulted in choosing this distractor.

D – There is at least one visible 'pattern' which could have been followed to lead to the choice of this distractor. One is that in the second two rows, if one looks diagonally from right to left (going down from row 2 to row 3) the number 12 is repeated. This might have led to the conclusion that the number 20 should be repeated below and to the left of the number 20 in the row above it.

In the commentary above, it is assumed that learners do know how to add/subtract and have used this knowledge to look for the pattern. What other errors could have arisen if this basic knowledge was not in place?

Activity 16

Grade 4 Operations

22. 945 ÷ 9 = ?

(A) 15 (B) 100

(C) 105 (D) 150

DIPIP learner data for selection of answers:

A	16%
B	19%
C	**30%**
D	31%

Commentary

A – An answer of 15 to this this division question implies that knowledge of division is present (the learner has correctly divided 9 by 9 and 45 by 9) but knowledge of place value and division of the whole number 945 is shaky. The learner who answers this may think that she can break 945 up into two parts, 9 and 45, and simply divide them separately in order to find the quotient.

B – This answer shows evidence of a partial division (900 has been divided by 9) but the learner has not done the full division called for in the question. The answer does indicate an understanding of division but further questioning of learners who chose this option would reveal why they did not complete the division operation and how to help them to complete the division of a 3-digit by a 1-digit number fully.

D – An answer of 150 to this this division question implies that knowledge of division is present (the learner has correctly divided 9 by 9 and 45 by 9) but knowledge of place value and division of the whole number 945 is again shaky. The learner who answers this has also probably broken 945 up into two parts, 9 and 45, divided them separately in order to find the quotient. The problem that has arisen (with potential roots in place value misconception) is that the learner has recoded the answer to 45 divided by 9 in the tens place and not the units place.

Poor knowledge of the application of place value is at the root of many misconceptions that become evident when learners perform operations.

Does place value understanding play a role in all of the operations and if so, in what way?

Activity 17

Grade 5 Operations

31. $32.80 × 15 = ?

(A) $19 680 (B) $196.80

(C) $49 200 (D) $492

DIPIP learner data for selection of answers:

A	19%
B	31%
C	24%
D	**19%**

Commentary

The grade 5 curriculum in South Africa does not call on learners to perform operations on numbers with decimal places. Some errors that have arisen in this question would relate to understanding of place value including decimal places.

A – This answer is calculated by adding 5 x $3280 to $3280. Learners who did this calculation to find the answer to the question ignored the decimal point in the monetary value of $32,80. They also did not actually calculate 15 times the given value.

B – This answer is calculated by adding 5 x $32.80 to $32.80. Learners who did this may have thought that they were calculating 10 X $32.80 and adding 5 x $32.80 to make 15 times $32.80. In fact what they did was calculate 5 + 1 (6) times the value.

C – Numerically this answer contains the correct symbols but place value has not been properly used in the calculation and so the actual value of the answer is not correct – it is 100 times bigger than it should be. Learners who found this answer performed the correct multiplication but they did not take the decimal places into account.

Sometimes knowledge of operations is needed in context. The context in this question is financial, but the money in the context is dollars and not rands, the South African local currency. In what way do you think that this potentially unfamiliar context may have resulted in learner errors when they answered this question?

Activity 18

Grade 6 Operations

9. 7.35 – 5.85 = ?

(A) 1.20 (B) 1.50

(C) 2.00 (D) 2.50

DIPIP learner data for selection of answers:

A	10%
B	**38%**
C	14%
D	37%

Commentary

Decimal numbers is not in the CAPS for grade 6 in South Africa. Some errors that have arisen in this question would relate to understanding of place value including decimal places.

A – This distractor is difficult to explain (and it was the least chosen by learners in the sample). The problem lies in the tenths position – the learner appears to have subtracted the hundredths correctly, and also the units, but the answer of 2 in the tenths position does not relate to identifiable common solutions in this case. Teachers could interview learners to probe the reasoning behind this answer.

C – Learners who chose this distractor focussed of the subtraction of the whole numbers only and ignored the tenths and hundredths. Place value understanding and correct use of place value to subtract numbers which have more than one place could lie at the root of this problem.

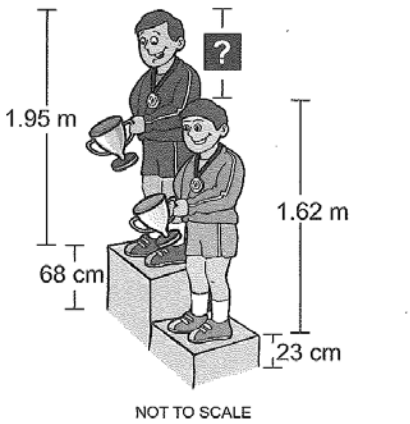
D – This answer indicates that learners did subtract fully, but when they did so they forgot to take away the unit they exchanged into tenths in order to subtract in the tenths place. This error in subtraction is common and occurs when learners do whole number subtraction as well as subtraction such as this one where decimals are involved. Explanations that probe the exchange could be used to assist learners to overcome this problem.

Sometimes in systemic tests learners are tested on mathematical knowledge that is beyond the scope of their current understanding. To what extent do you think this question disadvantaged South African learners and why?

Activity 19

Grade 7 Operations

24. Calvin and Steven are standing on steps of different heights.



What is the distance between the top of Calvin's head and the top of Steven's head?

(A) 33 cm (B) 45.33 cm

(C) 78 cm (D) 91.33 cm

DIPIP learner data for selection of answers:

A	42%
B	18%
C	**24%**
D	14%

Commentary

A – This most popular distractor represents the subtraction of two measurements shown in the illustration which appear to give the measurement to the top of each of the boy's heads. (1,95 m – 1,62 m = 33 cm). These measurements do not give the full measurement from the floor to the top of each boy's head and so this answer has not taken all of the given information into account.

B – This answer has been calculated by subtracting the two given pairs of measurements (1,95 m – 1,62 m = 0,33 m and 68 cm – 23 cm = 45 cm) and then adding the two answers together (45 + 0,33 = 45, 33), without proper regard for the units of measurement (one pair is given in metres and one pair in centimetres).

D – This answer has been calculated by subtracting the two given pairs of measurements for the heights of the boys (1,95 m – 1,62 m = 0,33 m) and adding the heights of the steps 68 cm + 23 cm = 91 cm) and then adding the two answers together (91 + 0,33 = 91, 33). This calculation does not take proper regard for the units of measurement (one pair is given in metres and one pair in centimetres) and it also shows confusion about which operations to use and how.

Solution of this word problem requires the application of operations in the context of the measurement of height. Which of the errors that arose in this question may have resulted from contextual issues rather than simply errors in relation to the understanding of operations? Think about this in relation to:

- The given illustration and information presented diagrammatically.
- Units of measurement in the context.

Fractions

Learners' understanding of fractions begins at an early age, almost at the same time as their concept of whole numbers is being developed. Initially it is based very much on concrete examples of fractional parts which can help them to visualise and then generalise the idea of a part relative to a whole. The fraction concept needs to be sufficiently well generalised until it is an abstract number concept which is not reliant on concrete images or objects.

Activities 20 – 24 Fractions

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the fractions activities in order to learn more about how learners work with fractions.

For an understanding of the content that underpins Fractions consult Unit 4 pp. 125-174 of the *Saide*/Wits module, *Mathematics for Primary School Teachers.*

Grade 3 – Activity 20
Grade 4 – Activity 21
Grade 5 – Activity 22
Grade 6 – Activity 23
Grade 7 – Activity 24

For each activity you should answer the following questions:

a) Solve the item yourself.
b) The grade level of the item is given in the test. Do you think that the item has been set for the appropriate grade? If not, which grade level would you say it is?
c) What are the way(s) of working and thinking that a learner might use in order to get the question correct (fill in more than one possible method and/or way of thinking, if necessary).
d) Indicate for *each method* whether you think it is at the expected level for the grade OR higher OR lower than the group would expect.
e) Write out the list of distractors in order from the one selected most often to the one selected least often. *You may trial these with your learners – and see if they choose answers in a similar way.*
f) For each distractor, think and write about what learners might have done to obtain the answer in each incorrect choice. If possible write out more than one method and/or way of thinking.
g) Write about any other issues with task that you think are important but that have not been raised in the error analysis you have done.
h) Write about issues to consider when teaching these concepts (for example, how to overcome some of the problems that have been raised).

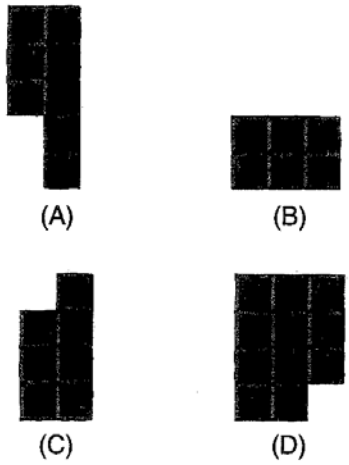
Activity 20

Grade 3 Fractions

22. Jill had a block of chocolate with 32 pieces.

She broke off a quarter of the block.

Which of these shows the quarter she broke off?



DIPIP learner data for selection of answers:

A	**25%**
B	31%
C	13%
D	23%

Commentary

B – This distractor, which was the most highly chosen one, was probably chosen because it "looks" most like the fraction parts into which learners have most often seen shapes being divided. The other shapes, given as potential quarters have unusual, irregular shapes. The misconception evident in this choice is called an over-generalisation. Learners chose it because it is the most familiar type of "quarter" of the four options, based on the generalisation that when a whole is divided into quarters they will look like this. They did not evaluate the size of the given part (six pieces) in relation to the whole (which was said to have 32 pieces).

C – The number count of the pieces in this distractor is 7 pieces – one short of the 8 pieces which would have been a quarter of a slab of chocolate with 32 pieces. Teachers could probe learners with questions on this choice since it might have been a miscount or it might have been some other reasoning.

D – Here, as with distractor C, learners would need to be probed to find out what their reasoning was behind the choice of this distractor. One thing that might have been part of a learner's reasoning here is that "1 quarter" (one block) was removed from the one column of 4 blocks in this shape. One out of four blocks is a quarter – but not a quarter of 32 blocks.

Learners in grade 3 still need to work with multiple variations of concrete representations of fractions. How do you think this exposure to lots of different concrete representations of fractions would help learners to establish a better fraction concept?

Activity 21

Grade 4 Fractions

12. Jay had a melon.

He ate $\frac{1}{4}$ of the melon.

Then Jay ate **another** $\frac{1}{4}$ of the melon.

What fraction of the melon did he eat in total?

(A) $\frac{1}{8}$ (B) $\frac{2}{8}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

DIPIP learner data for selection of answers:

A	15%
B	26%
C	**39%**
D	16%

Commentary

A – Learners may have chosen this answer if they incorrectly added two quarters to get one eighth.

B – This, the most popular distractor, may have resulted from a misreading of the question, linking the quarter in the last part of the question to the equivalent fraction of two eighths to give the answer.

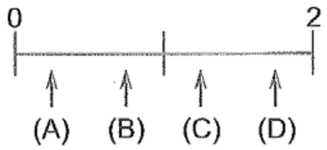
D – This answer may have been chosen if learners thought that they were meant to indicate how much melon was left – according to what was shown in the second illustration of a melon with one quarter eaten.

The visual element on this question (the whole melon and the melon with one quarter removed) may have created a distraction that misled learners when they answered this question.
In what way can visuals be distracting and potentially misleading in mathematical questions?

Activity 22

Grade 5 Fractions

25. Which arrow points to $1\frac{3}{4}$?



DIPIP learner data for selection of answers:

A	18%
B	24%
C	28%
D	**27%**

Commentary

A – Learners who chose this distractor did not look carefully at the given scale of the number line as indicated by the labels 0 and 2 and the markers on the number line. They have under-estimated the size of $1\frac{3}{4}$ and simply paced it just bigger than zero. This might indicate an over generalisation that fractions are bigger than zero but smaller than 1, without regard to their actual relative size.

B – The position of B on the number line is on the number $\frac{3}{4}$. Learners who chose this might have realised that this marker was on $\frac{3}{4}$ and forgotten about the whole number part of the given number that they had to place on the number line. ($1\frac{3}{4}$). They might also have been sticking to the belief that numbers with fractions are "less than 1".

C – The position of C on the number line is on the number $1\frac{1}{4}$. Learners who chose this might have realised that that number was greater than 1 but did not correctly work out how much more than 1.

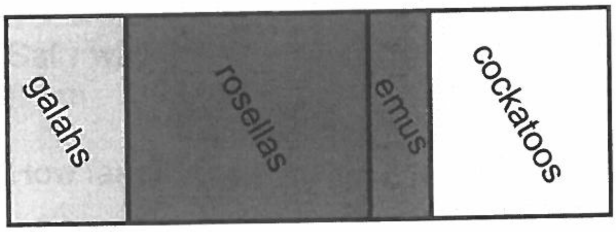
Learners seemed uncertain about this item as can be seen by the fairly even spread of percentages across the choices of answer. In what way would more regular exposure to number line representations of fractions would assist learners to realise the correct relative sizes of these numbers?

Activity 23

Grade 6 Fractions

14. Bill went on a walk in an Australian national park.

    He drew a graph to show the proportion of different birds he saw on his walk.



Approximately what fraction of the bords were rosellas?

(A) $\frac{1}{4}$ (B) $\frac{2}{5}$

(C) $\frac{2}{4}$ (D) $\frac{3}{5}$

DIPIP learner data for selection of answers:

A	27%
B	**16%**
C	26%
D	27%

Commentary

This was a difficult item in which the spread of choices indicates that very few learners knew how to answer it correctly and they may have been guessing on the distractors. It is a contextualised question (in a potentially unfamiliar context) and requires understanding of mathematical language of fractions as well as proportion.

A – This answer may have been selected because there were 4 types of birds. One of four equal sized parts is a quarter. But these four parts are not equal in size, and so the answer is not correct. The misconception here could thus relate to the understanding of fractions.

C – This answer may again have been selected because there were 4 types of birds – hence quarters were the fraction of choice, however since the piece that represents rosellas is bigger, and so it was assumed that two quarters of the birds were rosellas. This choice again represents a misconception of fractions as parts of a whole.

D – It is difficult to explain the choice of $\frac{3}{5}$ as an answer to this question. Teachers could ask learners probing questions to follow up on this choice and come to grips with the confusion here.

Learners in grade 6 are starting to work more often with numeric representations of fractions rather than visual ones. This question gave a visual representation which required measurement and calculation to work out the relative sizes of the fraction parts in the diagram. In what way could you support your grade 6 learners to deal with such a question?

Activity 24

Grade 7 Fractions

13. Which of these expressions is equivalent to $\frac{5}{7}$?

(A) $\frac{5}{7}+\frac{7}{5}$ (B) $\frac{5\times5}{7\times7}$

(C) $\frac{5+2}{7+2}$ (D) $\frac{5\times7}{7\times7}$

DIPIP learner data for selection of answers:

A	30%
B	33%
C	17%
D	**17%**

Commentary

A – This distractor presents a fraction being added to its reciprocal. This does not yield an equivalent fraction. Learners may have realised that if they multiplied a fraction by its reciprocal they get 1 (and they know this was not what the question was asking for) and so they decided that maybe adding the reciprocal could generate an equivalent fraction since it was presented as an option in the answer selection. Teachers could probe learners who chose this option with further questions to find out their explanations for choosing this option.
B – In this distractor the error involves multiplying the numerator by itself and the denominator by itself. This does not result in an equivalent fraction, although learners could think it is an application of the rule "what you do to the top you do to the bottom" when finding equivalent fractions.
C – Here the error involves adding the same number to the numerator and the denominator. This does not result in an equivalent fraction, although learners could think it is also an application of the rule "what you do to the top you do to the bottom" when finding equivalent fractions.

Overgeneralisation of the rule "what you do to the top you do to the bottom" lies at the root of the errors embedded in the distractors for this question.

- Do you use this "rule" when you teach your learners?
- How do you think this simplified rule confuses learners?

Ratio

Learners' understanding of ratio should also be developed based on concrete representations and activities. This concept needs to be sufficiently well generalised until it is an abstract number concept which is not reliant on concrete images or objects.

Activities 25 – 29 Ratio

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the ratio activities in order to learn more about how learners work with ratio.

For an understanding of the content that underpins Geometry consult Unit 1pp. 5-52 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

Grade 3 – Activity 25
Grade 4 – Activity 26
Grade 5 – Activity 27
Grade 6 – Activity 28
Grade 7 – Activity 29

For each activity you should answer the following questions:

a) Solve the item yourself.
b) The grade level of the item is given in the test. Do you think that the item has been set for the appropriate grade? If not, which grade level would you say it is?
c) What are the way(s) of working and thinking that a learner might use in order to get the question correct (fill in more than one possible method and/or way of thinking, if necessary).
d) Indicate for *each method* whether you think it is at the expected level for the grade OR higher OR lower than the group would expect.
e) Write out the list of distractors in order from the one selected most often to the one selected least often. *You may trial these with your learners – and see if they choose answers in a similar way.*
f) For each distractor, think and write about what learners might have done to obtain the answer in each incorrect choice. If possible write out more than one method and/or way of thinking.
g) Write about any other issues with task that you think are important but that have not been raised in the error analysis you have done.
h) Write about issues to consider when teaching these concepts (for example, how to overcome some of the problems that have been raised).

Activity 25

Grade 3 Ratio

33. Students at Happyville School can earn Bronze, Silver and Gold awards.

Students with 6 Bronze awards receive a Silver award.

Students with 4 Silver awards receive a Gold award.

How many Bronze awards are needed to receive a Gold award?

(A) 6×4
(B) $6 + 4$
(C) $6 \times 4 + 1$
(D) $6 + 4 + 1$

DIPIP learner data for selection of answers:

A	**20%**
B	24%
C	28%
D	21%

Commentary

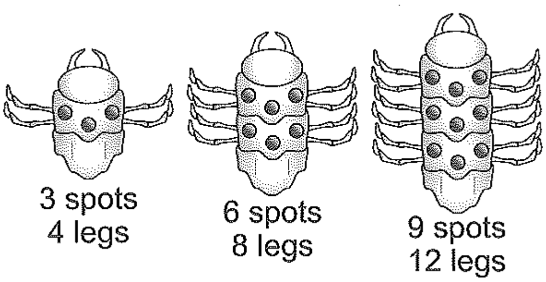
B – This distractor shows that learners think there is an additive relationship between the number of Bronze and Silver awards – they have added the number of each type of award and not taken into account that they need 6 Bronze awards to earn each Silver award.
C – This distractor shows that learners realise there is a multiplicative relationship between the number of Bronze and Silver awards but in this case they have also added 1 to the sum 6 + 4, possibly for the final Gold award earned.
D – As in B, this distractor shows that learners think there is an additive relationship between the number of Bronze and Silver awards – they have added the number each type of award and not taken into account that they need 6 Bronze awards to earn each Silver award. In this case they have again added 1 to the sum 6 + 4, possibly for the final Gold award earned.

What is the difference between a multiplicative and additive relationship between pairs of numbers? How does this relate to ratio?

Activity 26

Grade 4 Ratio

33. Sarah made these model bugs.



She then made a model bug that had 36 legs.

How many spots did that model bug have?

(A) 9
(B) 12
(C) 27
(D) 48

DIPIP learner data for selection of answers:

A	16%
B	25%
C	**33%**
D	21%

Commentary

A – Learners who chose this distractor may have worked out that the model bug with 36 legs means that the bug has 9 x 4 = 36 legs and they realise the factor 9 is a significant number in the solution of the question. The learners did not go on to calculate the correct number of spots (3 x 9 = 27) using the multiple of 9 that they identified.

B and D – It is difficult to say what learners may have been thinking in order to select these distractors. Teachers could interview the learners to establish their line of thought.

Pupils will often confuse the terms 'ratio' and 'proportion', and need a clear understanding of when each is appropriate. This item includes a pattern where the spots are given as pairs of proportional numbers.

Do learners have to know the meaning of the word proportion in order to solve this question?

Activity 27

Grade 5 Ratio/Rate

37. Maria was training for a race.

She ran 3 km each day except on Sundays when she ran 5 km.

She ran every day for 31 days, and started her first run on a Friday.

How many kilometres did she run in total?

(A) 90
(B) 101
(C) 103
(D) 155

DIPIP learner data for selection of answers:

A	28%
B	21%
C	**20%**
D	28%

Commentary

A – This distractor is difficult to understand, yet a high percentage of learners chose it. Teachers could interview their learners to find out how they calculated this answer in order to understand their confusion about rate calculations.
B – Learners who chose this distractor may have calculated it using 4 Sundays (for the 4 full weeks in 31 days) and all of the other days as weekdays. They did not take into account that the running started on Friday, in which case there was a Sunday in the first 3 days.
D – This answer was found by calculating 5 x 31 = 155. Learners who did this assumed that Maria ran 5 km every day.

Ratio questions are about proportions between units of the same kind. Questions about rate involve different kinds of units (such as in this question a relationship between the number of kilometres run and the number of days).
How could you assist your learners to work out the relationship between the parts of the question?

Activity 28

Grade 6 Ratio/Rate

30. Sam and Kevin are bricklayers.

Sam lays 150 bricks in 60 minutes.
Kevin lays 20 bricks in 10 minutes.

Working together, how many minutes will it take Sam and Kevin to lay 180 bricks?

(A) 25 (B) 40

(C) 70 (D) 100

DIPIP learner data for selection of answers:

A	10%
B	**11%**
C	47%
D	28%

Commentary

A – The learners that chose this option started (rightly) working out Sam's rate to one per 10 minutes, by dividing 60 by 6. They then divided 150 also by 6 to get 25. Since they see 25 as one of the options, they went for it as the correct answer.
C – The learners that chose this distractor may have said that since the bricklayers are working together, the time that it will take them would be the sum of the times given. Hence they added 60 and 10 to get 70 minutes.
D – The learners that chose this option may have added the number of bricks to arrive at 170 and then subtracted the sum of the minutes given i.e. 170 – (60 + 10) to arrive at 100. They were simply juggling the given figures. Teachers could interview learners to find out other explanations for this answer.

Ratio questions are about proportions between units of the same kind. Questions about rate involve different kinds of units (such as in this question a relationship between the number of bricks laid and the time taken to lay the bricks).
How could you assist your learners to work out the relationship between the parts of the question?

Activity 29

Grade 7 Ratio/Rate

3. This machine prints 119 copies of a book every 7 minutes.

How many copies does it print in 1 minute?

(A) 17
(B) 19
(C) 112
(D) 833

DIPIP learner data for selection of answers:

A	**50%**
B	24%
C	14%
D	10%

Commentary

B – This answer might represent a careless error in the division of 119 by 7. The learners who did this does understand that to reduce the rate to a "per minute" rate they need to divide, but then made an error in the calculation.

C – To get this answer, learners subtracted the number of minutes from the number of books given in the question. This shows a poor understanding of the rate relationship and how to simplify it.

D – To get this answer, learners multiplied the number of minutes by the number of books given in the question. This shows a poor understanding of the rate relationship and how to simplify it.

Ratio questions are about proportions between units of the same kind. Questions about rate involve different kinds of units (such as in this question a relationship between the number of minutes and the number of copies made).

How could you assist your learners to work out the relationship between the parts of the question?

Geometry

Geometry often involves visualisation of shapes when learners work under test conditions since they are not given concrete shapes in tests. Leaners should be given ample opportunities to work with concrete shapes and do visualisation activities so that they can develop the necessary skills to work with abstract representations of shapes.

Activities 30 – 34 Geometry

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the geometry activities in order to learn more about how learners work with geometry.

For an understanding of the content that underpins Geometry consult Unit 1pp. 5-52 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

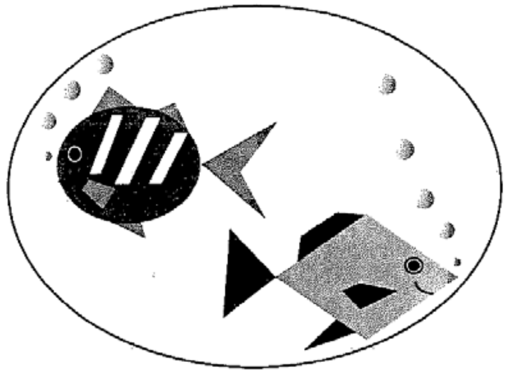
Grade 3 – Activity 30
Grade 4 – Activity 31
Grade 5 – Activity 32
Grade 6 – Activity 33
Grade 7 – Activity 34

For each activity you should answer the following questions:

a) Solve the item yourself.
b) The grade level of the item is given in the test. Do you think that the item has been set for the appropriate grade? If not, which grade level would you say it is?
c) What are the way(s) of working and thinking that a learner might use in order to get the question correct (fill in more than one possible method and/or way of thinking, if necessary).
d) Indicate for *each method* whether you think it is at the expected level for the grade OR higher OR lower than the group would expect.
e) Write out the list of distractors in order from the one selected most often to the one selected least often. *You may trial these with your learners – and see if they choose answers in a similar way.*
f) For each distractor, think and write about what learners might have done to obtain the answer in each incorrect choice. If possible write out more than one method and/or way of thinking.
g) Write about any other issues with task that you think are important but that have not been raised in the error analysis you have done.
h) Write about issues to consider when teaching these concepts (for example, how to overcome some of the problems that have been raised).

Activity 30
Grade 3 Geometry

18.



How many four-sided shapes are in this picture?

(A) 5
(B) 6
(C) 7
(D) 8

DIPIP learner data for selection of answers:

A	26%
B	17%
C	21%
D	**29%**

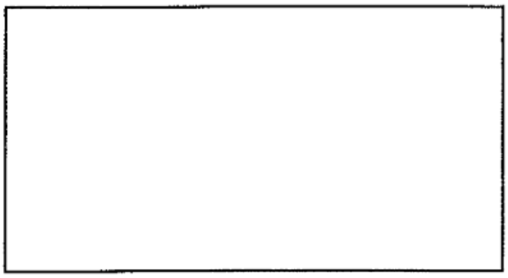
Commentary
A, B and C – The distractors in this item all represent different miscounts of the number of four sided shapes. There are some ways these miscounts can be explained but they may also represent carelessness on the part of the learner since there are so many shapes in the drawing and learners may have missed some of them. Teachers may wish to use probing questions to follow up on learner errors here.

Distinguishing between different types of geometric shapes is difficult for some learners. How can you help them to overcome these difficulties?

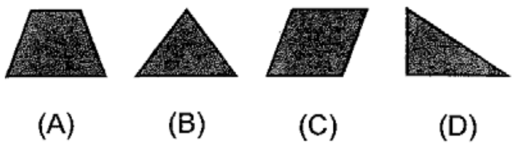
Activity 31

Grade 4 Geometry

20. Julia wants to tile this area of her bathroom wall.



Which of these tiles can she use to cover this area with no cutting, overlapping or gaps?



DIPIP learner data for selection of answers:

A	12%
B	13%
C	50%
D	**18%**

Commentary

A and B – These two distractors did not attract many learners, possibly because they do not represent familiar tiles that learners have seen tiling surfaces in real spaces around them.

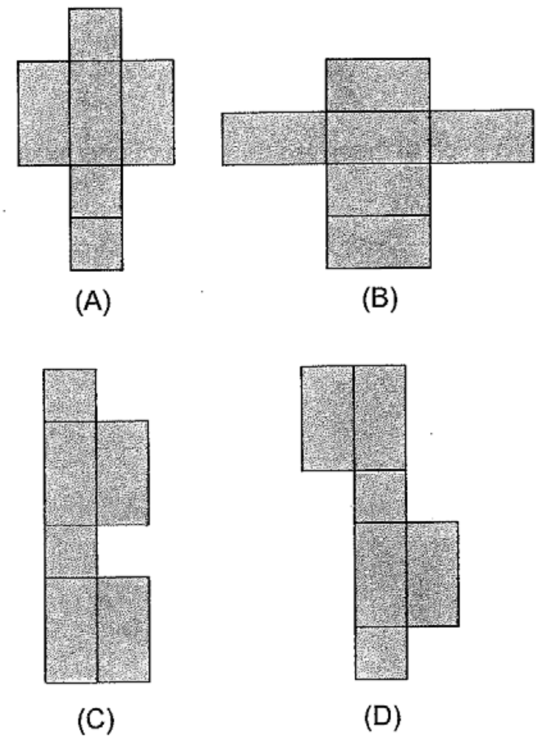
C – Learners may have chosen this distractor (the most highly chosen one in a big way) since it looks most like familiar tiles or tiling patterns that they have seen around them. This is an example of learners applying a generalisation from the familiar everyday world to a mathematical question.

The generalisation of familiar everyday examples to mathematical rules can create learner misconceptions. How can you use familiar examples in an effective way to teach correct mathematical concepts?

Activity 32

Grade 5 Geometry

33. Which of the following is a net of a rectangular prism?



DIPIP learner data for selection of answers:

A	20%
B	40%
C	16%
D	**15%**

Commentary

A and B – These two distractors were the most highly selected (particularly B) which is possibly evidence that learners chose a net that was laid out in a way in which they have usually seen nets for rectangular prisms laid out – with four central faces and two side faces, one on either side. These nets will not close into a rectangular prism as learners would have found if they were able to try them out concretely for themselves.

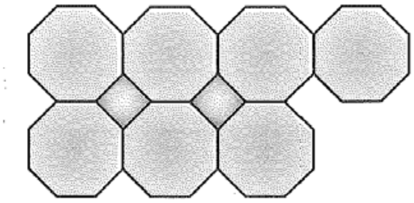
C – This distractor was about as popular as the correct answer itself, most likely because it is a new presented in an unfamiliar way.

Familiarity again influenced learners when they selected answers to this question. How could you help your learners avoid the over-generalisation about nets that they made when they answered this question?

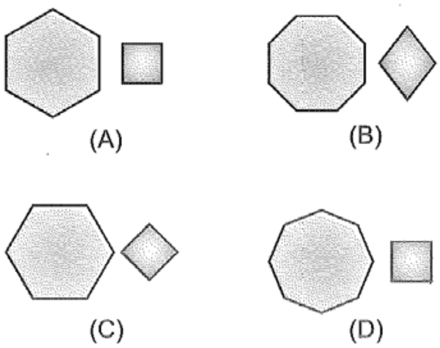
Activity 33

Grade 6 Geometry

15. This pattern is made out of tiles.



Which two tiles are needed to complete the pattern?



DIPIP learner data for selection of answers:

A	6%
B	44%
C	38%
D	**9%**

Commentary

A – The square in this distractor is one of the shapes but the other shape is an octagon and not a hexagon. Not many learners chose this distractor but those who did may have focussed on the square.

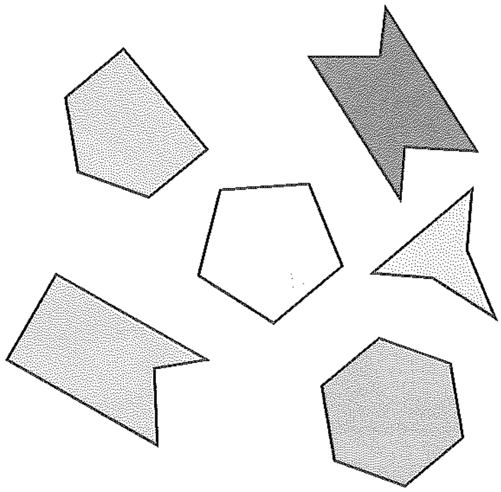
B and C – These were the two most popular distractors here. It is likely that the diamond shape drew the eye of learners who chose these distractors and dominated their choice which was then not made on a full and proper matching of the two shapes used to make the tessellation.

Visuals in questions can be distracting. How could you assist your learners to focus on the correct elements of given visuals in questions that they work through?

Activity 34

Grade 7 Geometry

1. How many of these shapes are pentagons?



(A) 4 (B) 3

(C) 2 (D) 1

DIPIP learner data for selection of answers:

A	**33%**
B	23%
C	26%
D	17%

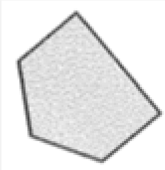
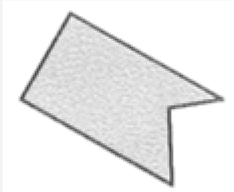
Commentary

The distractors in this question all represent counts of shapes. Errors in answers here may have related to misconceptualisation of a pentagon (a five-sided shape) or they may have resulted from careless counting. Teachers could follow up by discussing learners' choices with them to see which is the case.

B and C – It is difficult to tell how learners miscounted the number of pentagons but both of these choices represent a count of a smaller number of pentagons than there actually are given. Teachers could talk to learners to find out which ones they counted and why, and make sure they are able to identify all of the pentagons after the discussion.

D – Learners who said there was only one pentagon might have been focussing on the central shape only – it is a regular pentagon, which is often the example shown to learners.

Some 2-D shapes are convex and some are concave. Examples to clarify this difference, taken from the given set of shapes for this question are as follows:

Convex pentagon 	Concave pentagon 	

Do you think this idea might have caused some learners confusion when they were counting the number of pentagons in the given set of shapes, and if so, how?

Measurement

Measurement is all about quantifying characteristics of shapes. Only certain characteristics can be measured – those to which a "number" can be assigned. Knowledge of "what" can be measured incorporates an understanding of how this measurement is made. Too often teachers expect learners to measure and find quantities before they know what these quantities represent. It is important to use appropriate arbitrary units of measurement (such as steps to measure the length of the classroom) to develop the concept of each quantifiable characteristic before moving on to formal measurement and the use of standard units of measurement (such as metres, for the length of the classroom).

Activities 35 – 39 Measurement

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the measurement activities in order to learn more about how learners work with measurement.

For an understanding of the content that underpins Measurement consult Unit 6 pp. 197-210 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

Grade 3 – Activity 35
Grade 4 – Activity 36
Grade 5 – Activity 37
Grade 6 – Activity 38
Grade 7 – Activity 39

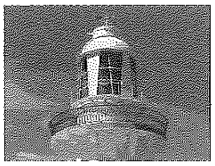
For each activity you should answer the following questions:

a) Solve the item yourself.
b) The grade level of the item is given in the test. Do you think that the item has been set for the appropriate grade? If not, which grade level would you say it is?
c) What are the way(s) of working and thinking that a learner might use in order to get the question correct (fill in more than one possible method and/or way of thinking, if necessary).
d) Indicate for *each method* whether you think it is at the expected level for the grade OR higher OR lower than the group would expect.
e) Write out the list of distractors in order from the one selected most often to the one selected least often. *You may trial these with your learners – and see if they choose answers in a similar way.*
f) For each distractor, think and write about what learners might have done to obtain the answer in each incorrect choice. If possible write out more than one method and/or way of thinking.
g) Write about any other issues with task that you think are important but that have not been raised in the error analysis you have done.
h) Write about issues to consider when teaching these concepts (for example, how to overcome some of the problems that have been raised).

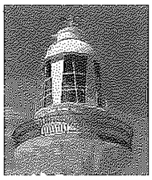
Activity 35

Grade 3 Measurement

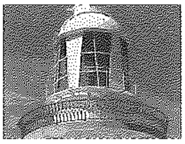
16. Which picture covers the biggest area?



(A)



(B)



(C)

(D)

DIPIP learner data for selection of answers:

A	**27%**
B	18%
C	44%
D	8%

Commentary

C – This was the most popular distractor. It is difficult to explain how learners made their selection on this item and teachers would probably do well to question their learners to find out the real reasons for their choices. But one reason we could think of to explain this error is that this image is more od a close-up of the tower and appears to have "more picture of the tower" in the given picture.
B – This was a fairly highly chosen distractor which could be because at a glance it seems to be as large as the other images (in size) and its height is greater than that of the other images. As with distractor C, it would be best to ask learners to explain the reasons for their choices in this question and to respond to their particular explanations in an appropriate manner.
D – This was not a very popular distractor which may be because it does appear to be smaller than the other images.

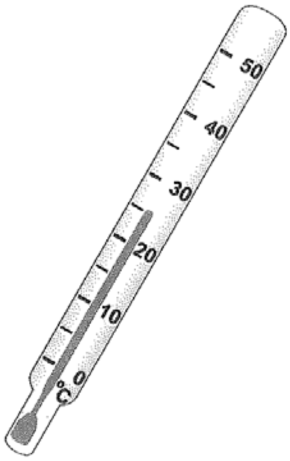
The language used in this question may have caused confusion.

- In what way is the wording of this question confusing?
- How could it be better worded to avoid this confusion?

Activity 36

Grade 4 Measurement

5. What is the temperature shown on this thermometer?



(A) 20 °C
(B) 21 °C
(C) 25 °C
(D) 29 °C

DIPIP learner data for selection of answers:

A	23%
B	16%
C	**46%**
D	11%

Commentary

A – This measurement (the most popular incorrect answer) represents a reading in which learners have rounded off (down) to the nearest marked unit (numerically) on the scale markings of the thermometer. This shows a possible confusion in the skill of readings a calibrated measuring instrument.

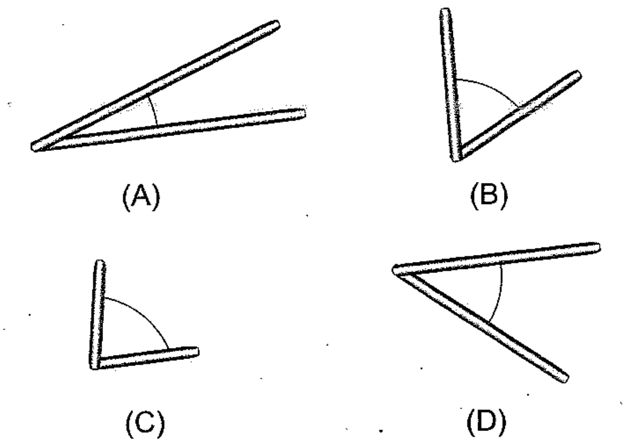
B and D – Not as many learners chose these distractors. Both of them show readings which are one unit away from marked units on the thermometer scale. These readings are more close to the correct reading (4 degrees away, on either side) but there is not a good reason for choosing them since if one is using the single degree markings on the thermometer scale, one should then give the accurate reading itself. It would be interesting to question learners who chose either of these options to find out what reasoning informed their choices.

More learners chose distractor A than distractor B although B is actually closer to the correct measurement than A. Why do you think this happened?

Activity 37

Grade 5 Measurement

19. Which of these shows the largest angle?



DIPIP learner data for selection of answers:

A	36%
B	10%
C	**34%**
D	13%

Commentary

A – The angle in this choice is the smallest, but it has the longest arms. One of the common errors that learners make is to confuse the size of the angle with the length of the arms.

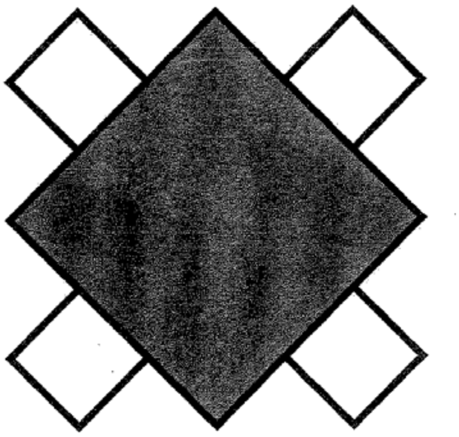
B and D – These two angles are very close in size and both have shortish arms, so although they are bigger than the angle in distractor A (and smaller than the correct answer given in option C), they were not chosen by as many learners.

How could you go about teaching the concept of angles to avoid the misconception that longer arms mean a larger angle?

Activity 38

Grade 6 Measurement

23. This shape has four small squares and one large square.



The area of each small square is 4 cm^2.

What is the area of the **whole** shape?

(A) 13 cm^2
(B) 36 cm^2
(C) 52 cm^2
(D) 160 cm^2

DIPIP learner data for selection of answers:

A	22%
B	30%
C	**23%**
D	21%

Commentary

A – Learners who chose this answer had counted the number of small squares in the whole shape. They calculated the total number of the square units in the tessellation and ignored the measurement of the given small square

B – This distractor represents the solution worked out based on only the area of the inner large square (9 x 4 = 36 cm^2)

D – Learners who chose this worked out the area as if the length of the side of each little square was 4 cm. The area concept seems to have been ignored and they used the given area as a length (4 x 4 = 16) and they added a zero (it seems) because it made the number bigger. Teacher would do well to interview learners who chose this distractor to find out the way in which the reasoned when they worked out this answer.

Misconceptions of area are often related to confusion between 1-dimensional and 2-dimensional measurements.

- What are 1-dimensional and 2-dimensional measurements?
- Why do you think this confusion arises?

Activity 39

Grade 7 Measurement

27. Here are three bottles.

2.5 L 600 mL 1 L

What is the total capacity of the three bottles in **millilitres**?

(A) 950
(B) 1625
(C) 1850
(D) 4100

DIPIP learner data for selection of answers:

A	36%
B	25%
C	13%
D	**25%**

Commentary

A – Learners who chose this distractor converted litres to millilitres by multiplying by 100 instead of by 1 000.
B – Learners who chose this as the solution may have converted the 1 litre to 1 000 ml, added this to 600 ml but then (incorrectly) added 2,5 l as 25 ml to get a total amount of 1 625 ml.
C – Learners who chose this as the solution may have converted the 1 litre to 1 000 ml, added this to 600 ml but then (incorrectly) added 2,5 l as 250 ml to get a total amount of 1 850 ml.

There is a relationship between an understanding of place value and an understanding of conversion between units of measurement?

- What is this relationship?
- How would this understanding have helped learners avoid errors when answering this question?

Unit 4: Giving feedback to learners

Introduction

- 'The most powerful single moderator that enhances achievement is feedback' (Hattie, 1992, in Clarke, 2005).
- 'When giving feedback to learners it is important to praise, to ask questions and to encourage' (Lipp & Davis-Ockey, 1997).
- 'Teachers should be aware of the impact that comments, marks and grades can have on learners' confidence and enthusiasm and should be as constructive as possible in the feedback that they give' (Assessment Reform Group, 2002 in Clarke, 2005).
- 'Being prepared to change feedback practices is one thing, but implementing change quite another' (Lee, 2011).
- Feedback says to a student, "somebody cared enough about my work to read it and think about it!" Most teachers want to be that "somebody". Feedback is just-in-time, just-for-me information delivered when and where it can do the most good (Brookhart, 2008).

Unit 3 of this module focused on analysing learners' misconceptions and errors and on the valuable information that such analysis can provide to teachers. Understanding the different kinds of misconceptions that learners have, the different kinds of errors that they make and the reasons why they make them is likely to assist teachers to adapt their teaching to meet learners' needs. Understanding of learners' misconceptions and errors should also enable teachers to give feedback to learners on what they have understood and / or been able to do well and what they still need to learn. However, findings from research into how teachers give feedback and how learners respond, indicate that feedback can be either productive or unproductive for learning, depending on the nature of the feedback, when it is given and how it is given (Black, 2004).

Aims of the unit

The content and activities in this unit have been designed to enable readers to develop:

- greater understanding of the important role of feedback in learning and teaching;
- greater understanding of different kinds of feedback;
- ability to give feedback that meets learners' needs and contributes to learning.

What is feedback?

Most dictionaries give at least two different meanings for the verb 'feed'. Firstly, 'to feed' means to provide food and to offer nourishment. Secondly, 'to feed' means to supply something or to put something into something else as in 'The equipment enables you to feed sound directly into the headphones'. One meaning of the adverb 'back' is 'the place where someone or something was before'.

All of these meanings are helpful for understanding what is involved when teachers give feedback to learners. Feedback is based on something that learners have already done so that when it is given, it should encourage learners to think 'back' or to think 'again' about a task they completed for their teacher (e.g. a homework exercise). Most importantly, feedback should supply nourishment for learners' minds so that, with new understandings, they are able to move forward in their learning.

Activity 1

j) Think back to your days as a learner – at school, teachers' college or university. Write a few notes to describe what you remember as being a very positive experience of receiving feedback on your work from a teacher or lecturer. Then do the same for an experience that you remember as being very negative.

k) After you have done this, make a few more notes about what made the one experience positive and the other negative for you.

l) To conclude this activity, complete this sentence: It is important for teachers to understand how to give 'nourishing feedback' because …

Commentary

Researchers have found that feedback that focuses on the qualities of the work and the strategies used by learners is experienced as positive by most learners. Feedback that focuses on the learner is not very helpful whether it consists of praise (e.g. "Good girl, well done!") or criticism (e.g. "How could you be so stupid?"). Such personal comments give no guidance to learners about what they did well or why and how they need to improve. In addition negative personal comments may be so discouraging to learners that they stop trying.

Now that you have thought about your own positive and negative experiences of receiving feedback, think about how the learners in your class are likely to feel about the feedback that you give them and what they are likely to do in response to it.

What matters about feedback?

This is the title of a chapter written by Shirley Clarke in which she discusses a number of problems associated with what she terms "traditional feedback".

One of these problems is that poor feedback can lead to learners becoming demoralised (lacking in confidence) so that they stop trying to learn. Another problem is that learners who receive quite high marks may become complacent (self-satisfied) and as a result see no reason to work hard. She explains how feedback that consists only or mainly of marks contributes to these problems and then describes some characteristics of effective feedback.

Activity 2

Study **Unit 4: Reading 1** - Clarke (2005), *What matters about feedback.*

a) List the forms of non-verbal feedback described on page 1 of the reading.
b) Tick the forms of non-verbal feedback that you use.
c) Describe any other forms of non-verbal feedback that you also use or have seen other teachers use.
d) Verbal feedback can be oral or written. On page 2 of the reading Clarke summarises two sets of findings about students' responses to verbal feedback. Write a few lines in response to one finding in each set that interests you most.
e) According to Butler's study (as described by Clarke on page 3 of the reading) marks are not the most effective way to give feedback on learning. Explain why you agree or disagree with her.
f) On page 3 and 4 of the reading, Clarke writes about five characteristics of effective feedback. For each one give an example of how you could apply this characteristic in your classroom.

Commentary

As stated by Brookhart (2008: 1), feedback says to a learner, 'Somebody cared enough about my work to read it and think about it'. However, whether the feedback is effective in motivating learners to continue on their learning journeys depends on a number of factors.

Think about the kinds of feedback that you use regularly in your classroom. In what ways could these kinds of feedback support or prevent learning in your class?

Giving effective feedback

The Clarke reading ends with seven key principles to guide teachers in giving effective feedback. In the list below, these have been slightly reworded as follows:

- Feedback needs to be focused on the learning objective of the task and not on comparisons with other learners.

- Verbal and non-verbal language from the teacher gives powerful messages to the learner about his/her ability.
- Giving a mark to every piece of work is likely to discourage learners who are struggling and to make those who receive good marks become self-satisfied and less likely to work hard.
- Teachers need to give **specific** feedback which indicates what has been done well and where improvements are needed.
- Feedback needs to show learners what steps to take in order to demonstrate full understanding of a topic at a particular grade level.
- Learners need opportunities (both in class and for homework) to improve on the work on which they have received feedback.
- Teachers need to train learners in self and peer assessment so that they can reflect on their own work and give constructive feedback to their peers.

The following is an important principle to add to Clarke's list:

- Teachers need to decide when and how (e.g. orally or in writing) to give feedback.

Brookhart (2008) developed two tables which teachers may find useful for thinking about feedback. The first focuses on strategies for giving feedback and the second on the content of feedback.

Table 1: Feedback Strategies (Brookhart, 2008: 5)

Feedback strategies can vary in …	**In these ways…**	**Recommendations for good feedback**
Timing	When given How often	Provide immediate feedback for knowledge of facts (right / wrong). Delay feedback slightly for more comprehensive reviews of student thinking and processing. Never delay feedback beyond where it would make a difference to students. Provide feedback as often as practical for all major assignments.
Amount	How many points made How much about each point	Prioritize – pick out the most important points. Choose points that relate to major learning goals. Consider the student's developmental level.
Mode	Oral Written Visual	Select the best mode for the message. Would a comment in passing the student's desk suffice? Is a conference (meeting) needed? Interactive feedback (talking with the student) is best when possible. Give written feedback on written work or on assignment cover sheets. Use demonstration if "how to do something" is an issue or if the student needs an example.
Audience	Individual Group / class	Individual feedback says, "The teacher values my learning". Group / class feedback works if most of the class missed the same concept on an assignment, which presents an opportunity for re-teaching.

Table 2: Feedback Content (Brookhart, 2008: 6)

Feedback content can vary in	In these ways	Recommendations for good feedback
Focus	On the work itself On the process the student used to do the work On the student's self-regulation On the student personally	When possible, describe both the work and the process – and their relationship. Comment on the student's self-regulation if the comment will foster self-efficacy (being able to move on independently). Avoid personal comments.
Comparison	To criteria for good work (criterion- referenced) To other students (norm-referenced) To student's own past performance (self-referenced)	Use criterion-referenced feedback for giving information about the work itself. Use norm-referenced feedback for giving information about student processes or effort. Use self-referenced feedback for unsuccessful learners who need to see the progress they are making, not how far they are from the goal.
Function	Description Evaluation / judgement	Describe. Don't judge.
Valence (orientation)	Positive Negative	Use positive comments that describe *what* is well done. Accompany negative descriptions of the work with positive suggestions for improvement.
Clarity	Clear to the student Unclear	Use vocabulary and concepts the student will understand. Tailor the amount and content of feedback to the student's developmental level.
Specificity	Nit-picky (too focused on small details) Just right Overly general	Suit the degree of specificity to the student and the task. Make feedback specific enough so that students know what to do but not so specific that the work is done for them. Identify errors or types of errors, but avoid correcting every one (e.g. copy editing or supplying right answers) which doesn't leave students anything to do.

In preparation for Activity 3 in which you will work with some of Brookhart's feedback strategies and content read the extracts from an interview based on a question taken from a worksheet.

A Grade 4 teacher set the following question for her class:

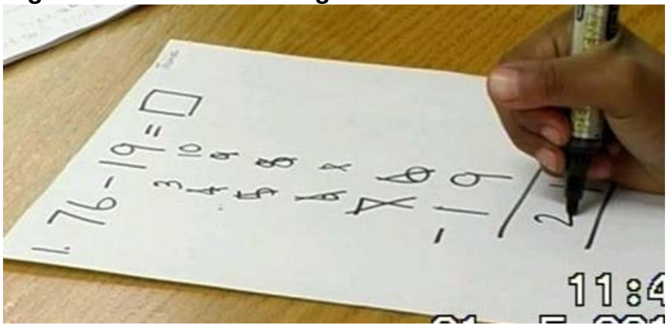
QUESTION 5

Amy's mass is 38kg. Kim weighs double Amy's mass.
After a year, Kim loses 19kg.

How much does Kim weigh now ? ________kg (2)

One of her learners wrote the following working in her solution to this question:

Figure 1: Learner working



The teacher spoke to the learner about her work (in an interview which lasted 18 minutes), to try to find out what her reasoning had been when she did this working. The extracts below are taken from this interview.

> She said, "What you have done, in this question … you have added all the, um, kilograms because it's double the weight and then, after that, you decided to subtract. Now something that I'm very interested in is the way you were subtracting. Can you please, while subtracting, explain to us step by step what you were doing?"
> She then allowed the learner time to explain herself, and this is what the learner said: "Because you can't subtract 6 (*points to the 6 of 76*) from 9 (*points to the 9 of 19*). So I took 1 away from the 7 (*points to the 7 of 76, crosses out the 7 and writes 6 above it*) so it becomes 6 and this one becomes 7 (*crosses out the 6 of 76 and writes 7 above it*). Now you still can't subtract it, so I took away 1 again (*crosses out the 6 written above the 7 of 76 and writes 5 above it*) and it became 5. And I added this one (*crosses out the 7 written above the 6 of 76 and writes 8 above it*), so it became 8."
> The learner paused, and the teacher said, "Alright. Ok." And so the learner carried on with her detailed explanation, saying, "And I still couldn't get it, so I took there (*crosses out 5 on the left side and writes 4 above it*) made it 4 and then I added (*crosses out 8 on right hand side and writes 9 above it*) and made it 9. So 9 minus 9 obviously is going to become zero so then I took it again and 4 took away 1 (*crosses out 4 on left hand side and writes 3 above it*), made it 3. And I added 10 (*crosses out 9 on right hand side and writes 10 above it*). 10 take away 1, um 9, I got 1. And 3 minus 1 was 2. (*writes 21 in answer line*) So that's how I got my answer."
> When the learner had finished this explanation of the incorrect working, the teacher said to her, "Well you explained to me very well. Right, now Jamie, I'm going to give you another sum so you can do the subtraction again, ok?"

The teacher then provided the learner with another question that tested the same skill (subtraction of a 2-digit number from a 2-digit number) but this time she provided counting blocks to help the learner as she worked through the subtraction. She helped the learner to work through a simpler question and then to re-do the original question using the counting blocks. She then allowed the learner to re-do the original question (using blocks) as if it were a "new" question. This working was followed by the following discussion in which the teacher asked the learner to compare her original answer to 76 – 19 to her new answer to the same question:

Minute 11

Speaker	Utterance
Teacher	Are they different sums or the same?
Learner	They are the same sums.

Teacher	They are the same. Right now what was your answer here (points to original white paper), there (points to red paper)?
Learner	There (points to red paper) it was 57.
Teacher	And here (points to white paper)?
Learner	21.
Teacher	21. So what did you do different to get your different answers?
Learner	Uh I used tens and units (pointing to blocks lying on the red paper).
Teacher	Ok and here (points to white paper)?
Learner	Here (points to white paper) I didn't use tens and units, I just used the numbers.
Teacher	Where did you get the numbers from?
Learner	The numbers from the sum.
Teacher	But, but, what did you do differently from here (points to white paper) than there (points to red paper)?
Learner	Here (points to red paper) I used blocks. Here (points to white paper) I just used the numbers.

Activity 3

a) Could you identify the learner's error before you read the explanation that she gave when she spoke to the teacher?
b) How did the teacher encourage the learner to explain her working?
c) Why was it valuable that the teacher allowed the learner the time to explain the WHOLE piece of working she had done and not just a part of it?
d) Did the teacher criticize the learner? What was the result of the way in which she treated the learner?
e) How did the teacher plan to help the learner overcome her problem?

Commentary

Most mathematics teachers would have identified the error that the learner made in her original working very quickly and may not have allowed the learner to go through all of the steps in her working so painstakingly without offering any criticism. This interview presents an alternative to offering immediate feedback (in line with Brookhart's principle relating to the *timing* of the feedback: 'Delay feedback slightly for more comprehensive reviews of student thinking and processing'). This strategy is ultimately successful since the teacher then allows the learner the opportunity to work through another question of a similar nature but this time using counting blocks (in line with Brookhart's principle of *mode* of feedback, 'Use demonstration if "how to do something" is an issue or if the student needs an example'). The value in allowing the learner the chance to explain her whole procedure is that the teacher does not interrupt or devalue the learner's self-expression. She indicates that the learner has "explained to me very well" (the *valence* principle – positive about the way in which the working has been explained) and then indicates that she is going to move on to another example. In this way she does not judge the learner (the *function* principle).

The teacher does not criticise the learner but rather gives her an opportunity to find out for herself what was wrong with her own working and how to correct it, through using alternative examples and manipulatives. After working through other examples she allows the learner to compare her original solution to her new solution (applies the comparison principle through a comparison with 'student's own past performance (self-referenced)' (Minute 11). The teacher brings the learner to a point of self-realisation of what went wrong the first time she attempted the subtraction – as the learner puts it, "Here I didn't use tens and units, I just used the numbers". This is a deep realisation. When she first attempted the question she used only the face values of the digits and did not think about their place value.

She says this in her own words, which is a powerful demonstration that the feedback given to this learner (through use of an alternative demonstration where the learner is actively involved) has brought the learner to an understanding of her error and enabled her to perform the operation correctly.

It is important to focus on the value and the opportunity in what learners bring to a conversation rather than directing them to your way of thinking and reasoning. You need to make links to what your learners think, rather than simply direct them to your way of thinking.
Can you think of an example from your own classroom in which you made such links successfully, or an opportunity that you missed?

Giving feedback on different mathematical topics

So far most of the information and activities in this unit could be used by all teachers. The rest of the unit will focus specifically on when and how to give feedback on different mathematical topics. The topics will be presented in relation to different mathematical test items. These activities will give you an opportunity to apply what has been introduced in general terms to specific ways of giving feedback in the mathematics classroom.

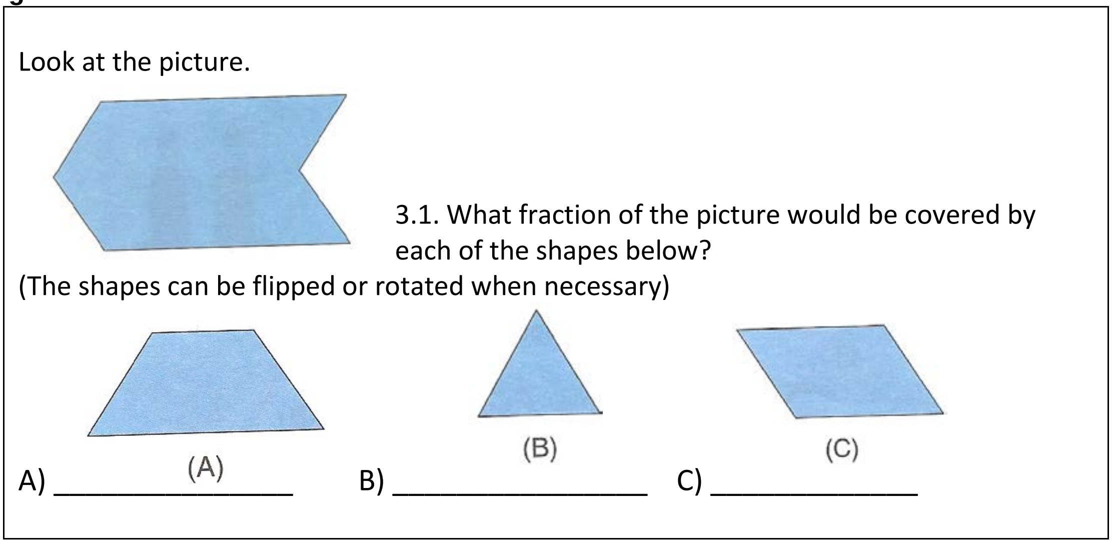
Giving oral feedback

An extract from a teacher-learner discussion has been selected to illustrate some of the ways in which a mathematics teacher can give oral feedback to assist learners to understand their errors and to develop new understanding.

The discussion is focused on a test question designed by a group of Grade 7 teachers. The learner featured in the extract had mistakenly used different sized shapes to determine fractional parts of a whole.

The question (Figure 2 below) presents a shape (the top shape in the diagram below) which is to be regarded as a "whole" into which three other shapes (the three smaller shapes below, labelled A, B and C) need to be fitted. The wording of 3.1 says "What fraction of the picture would be covered by each of the shapes below?" which implies that the learner needs to find out what fraction of the bigger shape each of the given smaller shapes represents. The question is testing whether the learner can work out the fractional value of parts of a whole. The activity requires that the learner checks how many of each of the smaller shapes (taken one at a time and repeated as many times as necessary) are needed to cover the whole shape above. A further instruction indicates that learners may flip or rotate the shapes if necessary in order to make them fit into the given bigger shape.

Figure 2: Extract from Grade 7 learner test

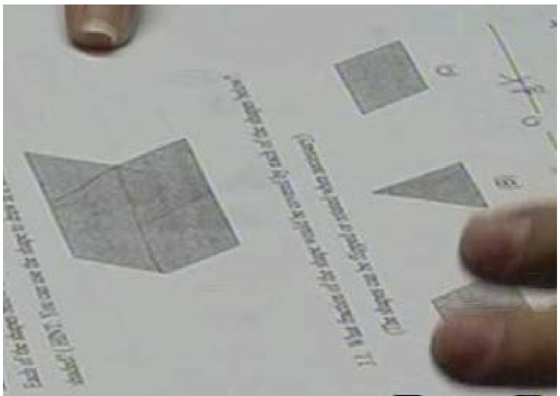

Look at the picture.

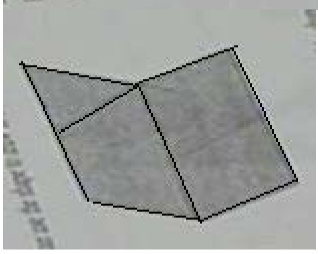
3.1. What fraction of the picture would be covered by each of the shapes below?
(The shapes can be flipped or rotated when necessary)

(A) (B) (C)

A) ______________ B) ______________ C) ______________

Figure 3: Learner working





The darker lines highlighted in this snap shot of the learner's work show the way in which the learner thought she would fit the three different smaller shapes into the bigger shape.

Extract from the discussion

The discussion was 7 minutes long. The extract below presents minutes 5-7 of the discussion.

Minute 5

Speaker	Utterance
Teacher	How many of these…like you did the trapezium (*points to trapezium*) to fit in there (*points to top shape*), how could we draw it to try and get these (*points to parallelogram*) to go in there (*points to top shape*)? Or what would you do to try and get them into there?
Learner	Um… (*rubs out previously drawn lines*) I think…I would draw in half again (*draws a horizontal line through top shape*) and then there would also be (*draws lines to make 4 rhombi*) four that could fit.
Teacher	Ok, perfect. So there's also four that fits. So now if we go back to the question: what fraction would be covered by each of the shapes? So if I took one of these (*points to parallelogram*) and put it in any one of them (*points to learner's 4 drawn parallelograms*), what part of the whole is coloured in?
Learner	A quarter.
Teacher	Again it's a quarter. Ok, now the tricky one. The triangle. How are we going to work out, if I take one of these (*points to triangle in 2nd row*), and put it into the whole (*points to top shape*), what fraction that is?
Learner	(*rubs out lines again*) I'd draw a half (*draws horizontal line through top shape again*),

Commentary – Minute 5

The teacher encourages the learner to think conceptually while she works through the next step of the activity (which involves fitting parallelograms into the bigger shape).

The procedural explanation includes all of the required key steps ("How many of these…like you did the trapezium (*points to trapezium*) to fit in there (*points to top shape*), how could we draw it to try and get these (*points to parallelogram*) to go in there (*points to top shape*)? Or what would you do to try and get them into there?")

After the learner responds, the teacher gives **two kinds of feedback**. Firstly, she affirms that the learner has come up with a correct solution ("Ok, perfect. So there's also four that fits"). Secondly, she asks a series of questions that help the learner to think about what fraction of the whole the parallelogram represents. ("So if I took one of these (*points to parallelogram*) and put it in any one of them (*points to learner's 4 drawn parallelograms*), what part of the whole is coloured in?") Again the learner responds correctly and the feedback affirms the correct response, challenges the learner to move forward and provides guidance for how to do so. The learner proceeds with the task. With reference to Brookhart's recommendation that feedback should foster learners' ability to work independently, the teacher has been successful.

Minute 6

Speaker	Utterance
Learner	…and then triangles (*draws lines to make 8 triangles*)…like how I did in the beginning…and then, yes…
Teacher	Ok, so now if we take one of them and covered part of that whole, what fraction of the whole would have been coloured in?
Learner	An eighth.
Teacher	An eighth. Ok, so now we know what those answers are: a quarter (*points to*

	trapezium), an eighth (*points to triangle*) and a quarter (*points to parallelogram*). Ok, so now it says here: if you only have two of these (*points to trapezium*), ok…what fraction of this whole picture (*points to top shape*) would be coloured in?
Learner	A half.
Teacher	It's a half.
Learner	Oh, I see now.
Teacher	Can you see?
Learner	Yes
Teacher	Alright, so now you got confused because you thought you had to take each of those (*points to the 3 shapes in 2nd row*) and stick them in separately (*points to top shape*), is that correct?
Learner	Yes
Teacher	Originally.
Learner	Yes
Teacher	But now how did you get a third (*points to learner's answer*)…if you took, like we drew, if you've got that parallelogram and that one's coloured in (*shades parallelogram on top shape*), and if we took the one triangle, and coloured it in (*shades triangle in top shape*),

Commentary – Minute 6

For most of minute 6 the teacher and learner exchanges are very similar to those in minute 5. Towards the end the teacher gives a slightly different kind of feedback. She tells the learner what she thought was the source of her error and checks whether the learner agrees. (Alright, so now you got confused because you thought you had to take each of those (*points to the 3 shapes in 2nd row*) and stick them in separately (*points to top shape*), is that correct?)

After the learner agrees, the teacher's feedback signals to her that she has now moved forward in her understanding ("Originally.") and when the learner agrees the teacher takes her back to her original solution. "But now how did you get a third (*points to learner's answer*)" and links this question to the discussion.

Minute 7

Speaker	Utterance
Teacher	…and then the trapezium that you had originally drawn, and coloured that in (*shades in trapezium on top shape*). Now how did you get originally the answer of a third (*points to learner's answer*), because that's not a third of the picture (*points to top shape*), is it?
Learner	Because I thought that it could be any size something to do with three, and so I just put…because all three of them fit in there even though it's a different size, I just put a third for each of them.
Teacher	For each of them. Oh, ok. Perfect. Right. That's it.

Commentary – Minute 7

The teacher completes her feedback, given in the form of a question which gets a little closer to the learner's original error (calling parts "thirds" when they are not actually thirds of the given whole because they are not all equal in size). The learner is able to explain, with an awareness of what she has learnt from the discussion, that she had mistakenly called shapes of different sizes 'thirds' ("Because I thought that it could be any size something to do with three"). The teacher's final feedback to the learner ("For each of them. Oh, ok. Perfect. Right. That's it.") tells the learner that she (the teacher) now fully understands the nature of the error. Both teacher and learner have new understandings as a result of the discussion which kept the learner's work in focus throughout (Brookhart, 2008).

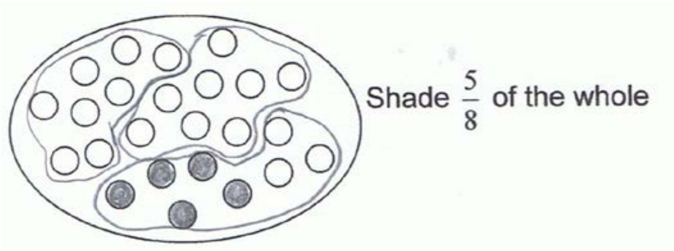
Giving written feedback

When you go through the work that learners have written either in a test or in their classwork books you have the chance to give them feedback in relation to the mistakes, errors or misconceptions in their work. If you just use big (or small) red crosses to mark work wrong, you do not give learners any means of finding out why they made an error. Giving feedback on written work is a powerful teaching opportunity which you should use productively whenever it arises.

The activity that follows gives some insight into feedback that a teacher could give, based on written work produced by a learner.

Activity 4

Study the way in which the learner has responded to the task instruction "Shade 5/8 of the whole". The learner has drawn in the groupings around the small inner circles contained within the outer ring.



h) Analyse the learner's work in order to identify what the learner has done.
i) Write feedback to the learner on the working in which you explain what is incorrect.
j) Write feedback in which you suggest how the learner should proceed from where he first made an error so that he arrives at the correct answer.

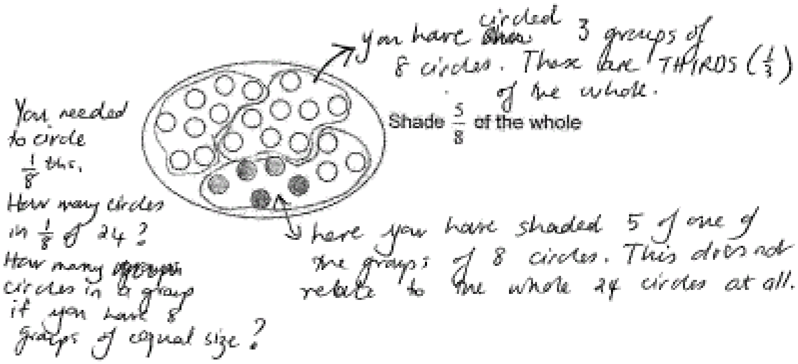
Commentary

This learner's work is not correct at all. The task is simple (it does not include many steps) and so the learner did not have the opportunity to show partially correct working here. Thus there is no opportunity to explain what has been correctly done in this working.

The learner has made two errors in his working here. Firstly, he confused the value of the denominator with the group size into which the whole should be divided. The denominator gives the number of parts into which the whole must be divided. Instead, the learner has divided the whole into parts which are the size of (or as big as) the denominator. So what he did was divide the whole into groups which have 8 in each group and he ended up with 3 such groups. Secondly, he shaded 5 of the 8 small circles in one of the groups that he circled, because he read from the numerator that he wanted 5/8. He has ignored the other two groups (which were part of the original whole of 24 small circles). He has reduced the whole from 24 small circles to 8 small circles, and shaded 5 of them.

In order to answer this question correctly, the learner had to divide the 24 circles into 8 groups of equal size. He would have found that there were 8 groups of 3 small circles making up the whole 24 small circles. In doing this, he would have divided the whole into eighths. One eighth of 24 is 3. Then to shade 5/8 of the whole, as requested, he would have shaded 5 of his small groups of three – shading a total of 15 small circles.

The ways in which a teacher chooses to write on such working will differ, but you may have written something along the following lines:



There is quite a lot of feedback here! You may want to write less and suggest that the learner comes to speak to you. Then you could work through the example and explain all of these ideas slowly to him.

Reflection

Do you think that giving feedback in the way that you have done in the above activity would be beneficial to your learners? If so, describe how it would benefit them.

Further feedback activities

In **Unit 4: Further feedback activities**, there are sets of activities, grouped according to mathematical content areas. These give you opportunities to practise giving feedback across a range of topics. You should select those items for which you have already done the curriculum mapping and error analysis activities in Units 2 and 3. The items have been selected in order to raise discussion of the mathematical content they present. There is one activity per grade, from grade 3 to grade 7, so you can work through the set of activities at the level of your choice. All of the activities are accessible, so should you wish to study a topic more fully, you could access more of the activities related to that topic than just the one set for the grade level at which you teach. The activities do not represent an entire mathematics curriculum since this is not possible within the scope of the unit, but they do present material which is often misunderstood by learners and about which teachers would benefit from a deeper understanding.

Summary

This unit has addressed several questions: Why is feedback important? When and how do you give feedback to learners? Can you plan for feedback? What helps you to give better feedback to learners who are struggling with mathematics?
The content and activities in the unit have been designed to enable readers to develop:

- greater understanding of the important role of feedback in learning and teaching;
- greater understanding of different kinds of feedback;
- ability to give feedback that meets learners' needs and contributes to learning.

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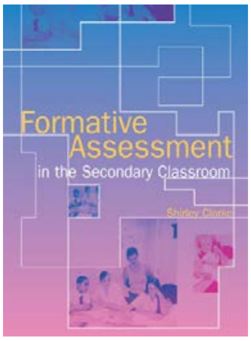
Unit 4 Reading: What matters about feedback

From: Clarke, S. (2005). *Formative Assessment in the Secondary Classroom*. Hodder and Stoughton, pp 67-73.



This highly practical guide focuses on learning objectives, effective questioning, self- and peer assessment, and feedback as the key elements of formative assessment. Down to earth and direct, with examples from across the secondary curriculum – and many accounts from practising teachers – it shows how formative assessment can bring a dramatic culture shift to teaching and learning in your own classroom. The numerous "ways in" described here – developed by teachers, and underpinned by research principles – will encourage you to become an action researcher in your own exploration of formative assessment.

"The most powerful single moderator that enhances achievement is feedback". *(Hattie, 1992)*

Whether feedback is oral or written, there are some key features which can be drawn from a great deal of classroom research.

The impact of traditional feedback on student motivation and achievement

When the Black and Wiliam (1998) review of formative assessment was published, the aspect which received most media attention was their findings about teachers' feedback to students. The traditional forms of feedback have, in many cases, led to regression in students' progress. Key negative elements are the giving of grades for every piece of work and external rewards, such as merit marks. Also loaded with potential to reinforce for students a sense of failure and lack of ability are: the teacher's tone of voice; body language; how difficulty with learning is talked about; the over-use of teaching assistants with certain students; and the words used by teachers when interacting with students.

Hargreaves, McCallum and Gipps (2001), in their research on feedback strategies used by teachers, found a range of 'approval' and 'disapproval' strategies. Non-verbal strategies for expressing approval included the teacher

- nodding,
- making eye contact,
- smiling,
- laughing,
- putting an arm around or patting the student, and
- taking on a mild manner in order to be approachable.

Non-verbal means of expressing disapproval included

- pulling faces,
- staring hard,
- clicking fingers, or
- making disapproving noises.

All of these strategies give clear messages to students about how the teacher feels about their ability.

The LEARN Project (University of Bristol, 2000 see www.qca.org.uk) consisted of interviews with over 200 students between Years 3 and 13 about their perceptions of assessment. The key findings were:

- Students were often confused by effort and attainment grades.
- Students sometimes felt that their effort was not recognized by teachers.
- Students preferred feedback that was prompt and delivered orally.
- Students were often unable to use feedback effectively.
- Students felt that feedback that was constructively critical helped improve their performance.

My own findings have been:

- Students believe that the purpose of marking is for the teacher to find out what they have got right or wrong, rather than for their own benefit.
- Students are rarely given time to read marking comments.
- Students often cannot read or understand the teachers' handwriting or comments.
- Students are rarely given time to make any improvement on their work because of the teacher's feeling of pressure to get onwith coverage.
- Many teachers worry that giving pupils 'time' to make any improvements on their work at the start of the lesson means a 'bitty' and informal or chaotic start.

A closer look at grading

'Teachers should be aware of the impact that comments, marks and grades can have on learners' confidence and enthusiasm and should be as constructive as possible in the feedback that they give.' (Assessment Reform Group, 2002)

Ruth Butler (1988) carried out a controlled experimental study in which she set up three ways of giving feedback to three different groups of same age/ability students:

- Marks or grades
- Comments
- Marks or grades and comments (the most common approach by UK teachers).

The study showed that learning gains (measured by exam results) were greater for the comment-only group, with the other two groups showing no gains. Where even positive comments accompanied grades, interviews with students revealed that they believed the teacher was 'being kind' and that the grade was the real indicator of the quality of their work.

Giving grades or marks for every piece of work leads to inevitable complacency or demoralisation. Those students who continually receive grades of, say, B or above become complacent. Those who continually receive grades of B or below become demoralised. Interestingly, girls and boys find different reasons for any apparent failure. Dweck (1986) found that girls attributed failure to lack of ability. This was because teachers' feedback to boys and girls was such that it would lead to girls feeling less able while enabling boys to explain their failure through lack of effort or poor behaviour.

In marking students' work with grades (competitive task orientation), teachers can be said to have focused students continually on the level of their ability compared to their peers. With a focus on feedback against the learning objectives of the task, however, students are enabled to improve realistically against past performance. It is important, of course, to know how one's performance compares with one's peers or against set criteria, but when this is the feedback for every piece of work, complacency or demoralisation sets in, thus impeding progress.

Many studies have shown that work marked by 'comment-only', with grades given only at end of units, increases motivation and achievement — findings which cannot be ignored.

On the whole, feedback has been a mainly negative experience for most students. Token comments at the bottom of work praising effort do not fool students, because the grades or rewards, and spelling and grammar corrections, tell students the 'truth' about their work.

What we now know about effective feedback

1. Focus feedback on the learning objective/success criteria of the task
Teachers have, in the past, apparently focused their written feedback on four main elements: presentation, quantity, surface features of any writing (especially spelling) and effort. Most school assessment policies also draw significant attention to these four main elements, which reinforces this practice. While these aspects are important, we have overemphasised them, so that the main focus of the lesson has been marginalised. What you put in is what you get out, so the message to students has been clear: get these things right and you will do better.

Effective feedback involves being explicit about the marking criteria. The other four features should be attended to every now and again rather than at every stage, and this needs to be reflected in any whole-school assessment policy.

2. Aim to close the gap
Sadler (1989) established three conditions for effective feedback to take place:

> The learner has to (a).possess a concept of the standard (or goal, or reference level) being aimed for, (b) compare the actual (or current) level of performance with the standard, and (c) engage in appropriate action which leads to some closure of the gap.

Improvement suggestions, therefore, need to be focused on how best to close the gap between current performance and desired performance, specific to the learning objectives in hand.

Give specific improvement suggestions
Kluger and DeNisis's (1996) research review showed that feedback only leads to learning gains when it includes guidance about how to improve. Terry Crooks (2001), as a result of his review of literature about feedback and the link with student motivation, concluded that:

> the greatest motivational benefits will come from focusing feedback on:
> - the qualities of the student's work, and not on comparison with other students;
> - specific ways in which the student's work could be improved;
> - improvements that the student has made compared to his or her earlier work' .

"Specific" and "improved" are two key words in Crook's recommendations. We have tended to be too general in the past to be helpful to students (e.g. 'some good words here' or broad targets such as 'remember to include more detail in your prediction). We have also tended to focus feedback on correction rather than improvement.

It is often the case that, instead of giving specific, concrete strategies to help students move from what they have achieved to what we want them to achieve, teachers instead simply reiterate the desired goal — a reminder prompt. For example, *'You need to improve these two long sentences*,' merely reiterates the learning goal of *'To be able to write effective long sentences*'. Better advice would be, for instance, *'Improve these two long sentences, using some short noun phrases, such as* old features, thin lips blue, grating voice *or similar*.' Giving 'for instances' and specific advice is key to the quality of an improvement suggestion.

4. Students make the improvement suggested
The traditional model of feedback is to make suggestions for improvement which one hopes will be taken account of when the same learning objective is revisited at a later date. The main reason that the same comments, appear over and over again on students' work is because students have not had an opportunity either to (a) carry out the improvement on that piece of work, according to the specificities of the improvement suggestion, or (b) revisit the skill in another context quickly enough. Only when this takes place will the improvement become embedded and able to be applied in further contexts. As Black and Wiliam (1998) found: 'For assessment to be formative, the feedback information has to be used.'

5. Relinquishing control
In most classrooms, the teacher defines the goal, judges the achievement and tries to close the gap between achieved and desired performance. Formative assessment research emphasises the importance of the involvement of the student, so we need to be careful not to oversimplify the process of giving effective feedback, leaving the student with no stake in the process. We need to model effective marking, aiming to gradually relinquish control so that students are trained to be effective self- and peer markers and assessors.

Teachers' findings

The teachers in the Black et al study stopped giving grades and focused on giving students improvement suggestions which were specific to the learning objective of the work (e.g. 'Well explained so far but add reasons why the Haber process uses these conditions) rather than general (e.g. 'Well explained but you could have given more detail'). One teacher said:

> At no time during the first 15 months of comment-only marking did any of the students ask me why they no longer received grades. It was as if they were not bothered by this omission. I found this amazing, particularly considering just how much emphasis students place on the grades and how little heed is taken of the comments generally. ...When asked by our visitor how she knew how well she was doing in science, one student dearly stated that the comments in her exercise book and those given verbally provide her with the information she needs. She was not prompted to say this!! (Black et al., 2003)

In summary

The Assessment Reform Group (2002), in *Assessment for Learning: Ten Principles*, said, as a result of collating the research about feedback:

> Assessment that encourages learning fosters motivation by emphasising progress and achievement rather than failure. Comparison with others who have been more successful is unlikely to motivate learners. It can also lead to their withdrawing from the learning process in areas where they have been made to feel they are "no good". Motivation can be preserved and enhanced by assessment methods which protect the learner's autonomy, provide some choice and constructive feedback, and create opportunity for self-direction.

Key principles

- Feedback needs to be focused on the learning objective of the task and not on comparisons with other students.
- Verbal and non-verbal language from the teacher give powerful messages to the student about his or her ability.
- Grading every piece of work leads to demoralisation for lower achievers and complacency for higher achievers.
- We need to give specific feedback focusing on success and improvement, rather than correction.
- We need to focus improvement suggestions on closing the gap between current and desired performance.
- Students need opportunities to make improvements on their work.
- We need to train students to effectively self and peer assess their work.

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Unit 4 Further feedback activities

Introduction

The following sets of activities, grouped according to mathematical content areas, will give you the opportunity to give feedback across a range of topics. You should select those items for which you have already done the curriculum mapping and error analysis activities in Units 2 and 3.

The items have been selected in order to raise discussion of the mathematical content they present. There is one activity per grade, from grade 3 to grade 7, so you can work through the set of activities at the level or levels of your choice.

If you would like to improve your understanding of the relevant content areas in the curriculum, we suggest that you consult *Mathematics for Primary School Teachers*, an openly licensed module digitally published by *Saide* and the University of the Witwatersrand (Wits), downloadable from OER Africa: http://www.oerafrica.org/ResourceResults/tabid/1562/mctl/Details/id/39030/Default.aspx.

The activities do not represent an entire mathematics curriculum since this is not possible within the scope of the unit, but they do present material which is often misunderstood by learners and about which teachers would benefit from a deeper understanding. The commentary given does not relate to the full range of questions you need to work through when you do these activities, but it suggests ideas for feedback on each of the given tasks. You should discuss your full activity responses with a colleague if possible to gain the full benefit from these activities.

Place value

Learners' understanding of place value as used in our base ten numeration system is developed from the moment they start counting and recording numbers. They extend this understanding of one-by-one counting to knowledge of numbers with bigger value until they are able to read, represent and work with numbers of any size.

Activities 10 – 14 Place value

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the place value activities in order to learn more about how learners work with place value.

For an understanding of the content that underpins place value consult Unit 2 pp. 53-92 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

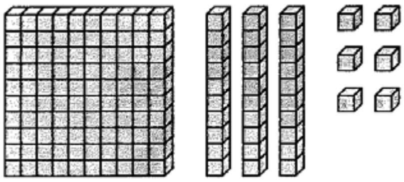
Grade 3 – Activity 10
Grade 4 – Activity 11
Grade 5 – Activity 12
Grade 6 – Activity 13
Grade 7 – Activity 14

For each activity you should answer the following questions:

t) What mathematical concept/skill is the focus of this test item?
u) What is the nature of the error and how would you give feedback to learners in relation to the error?
v) Should the feedback be oral or written?
w) Would objects / images ("visual" elements) be useful in the feedback? Explain how you would use such elements if you think this would be appropriate.

Activity 10
Grade 3 Place value

12.



What number is shown by this model?

(A) 136
(B) 163
(C) 631
(D) 1036

DIPIP learner data for selection of answers:

A	**42%**
B	13%
C	17%
D	24%

For the purpose of this activity focus on distractor D.

Commentary
The focus of this test item is the learner's ability to identify a number modelled using place value blocks. The error in distractor D relates to a poor understanding of place value. Feedback to learners could involve both written and oral discussion (linked to the use of place value blocks). The error could link to a lack of familiarity with the blocks, which resulted in the learner writing "10"36 (after counting the ten columns of ten blocks in the biggest block) instead of "136" (correctly representing the "one hundred" as "1" in the hundreds place. Learners need to be given the opportunity to write more 3-digit numbers based on visual representations such as this one, and feedback about the correct way to write a 3-digit number using place value.

Why should the feedback to a learner who cannot identify a number that has been modelled using place value blocks involve a discussion of writing numbers in base ten?

Activity 11

Grade 4 Place value

13. 186 – 47 = ?

(A) 149
(B) 141
(C) 139
(D) 39

DIPIP learner data for selection of answers:

A	19%
B	22%
C	**41%**
D	14%

For the purpose of this activity, focus on distractor B.

Commentary

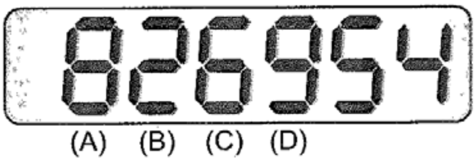
The focus of this test item is the learner's ability to subtract a 2-digit number from a 3-digit number. The error made in distractor B indicates that the learner has not come to grips with the use of place value and breaking down of numbers in order to subtract correctly when the units in the number being subtracted (subtrahend) are "higher" than the units in the number from which the subtraction is being made (minuend). This relates to basic number sense, without which operations on numbers cannot be done. Learners need to know that you cannot change the direction of subtraction to suit yourself. Feedback here could involve the use of blocks to represent the minuend – and then on-going use of the block to "take away" the subtrahend (breaking down the blocks if needed to facilitate the subtraction. The learner should write down the correct working while working with the blocks. This feedback would thus involve both concrete and written work.

Do you think that learners who know how to subtract 2-digit numbers from 3-digit but who have never had to break down numbers as required by the question in this activity should be able to do this activity anyway?

Activity 12

Grade 5 Place value

5. Which digit shows the number in the thousands column on this calculator display?



DIPIP learner data for selection of answers:

A	22%
B	12%
C	**35%**
D	24%

For the purpose of this activity focus on distractor D.

Commentary

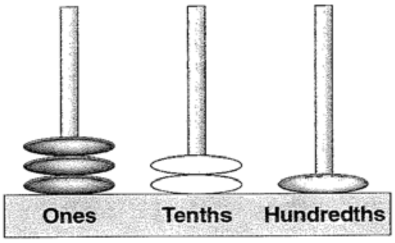
The focus of this test item is the learner's ability to identify the thousands digit in a large number. Feedback to learners who chose distractor D should highlight the importance of reading a number from left to right, where the number with the highest place appears on the left (according to the way in which we record numbers using base ten place value).

The digital display in this question may have confused learners. How could you avoid this confusion in your own learners?

Activity 13

Grade 6 Place value

24. This is a bead counter.



Which number is shown on the bead counter?

(A) 1.23 (B) 3.21

(C) 123 (D) 321

DIPIP learner data for selection of answers:

A	19%
B	**22%**
C	32%
D	24%

For the purpose of this activity focus on distractor C.

Commentary

The focus of this test item is the learner's ability to select a decimal number represented by an abacus diagram. In giving feedback to learners about this question the teacher needs to make sure that learners look closely at the diagram and read the labels – the abacus rods in this diagram are for fractional parts and not whole numbers. The learners who chose distractor C thought that the abacus rods represented whole numbers. Feedback here could involve leading questions given to learners to help them to realise how the rods are labelled and hence how to read the number represented correctly.

The abacus display might have confused learners.
How could you avoid this confusion on the part of your learners?

Activity 14

Grade 7 Place value

13. Which of these numbers is smallest?

(A) 0.1 (B) 0.09

(C) 0.109 (D) 0.0999

DIPIP learner data for selection of answers:

A	59%
B	**14%**
C	5%
D	20%

For the purpose of this activity focus on distractor A.

Commentary

The focus of this test item is the learner's ability to order numbers in decimal notation up to ten-thousandths. Learners who chose distractor A chose the "smallest number" by looking at the digits (as whole numbers) and not their positions according to place in which they occur. Feedback given to learners needs to highlight the necessity to read and determine the total value numbers using place value. The relationship between the sizes of decimal numbers needs to be explained – this can be done orally and using drawings/concrete representations of the relative sizes of digits in different decimal places.

"The total value of a digit in a number is determined by its size and position."

What does this mean?

Operations

Learners' understanding of operations (addition, subtraction, multiplication and division) begins when they have adequate number concept knowledge which allows them to operate on pairs of numbers. Early operation strategies should be closely linked to concrete examples but as learners develop this understanding they should start to use abstract procedures and not rely on counting to do their computations.

Activities 15 – 19 Operations

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the operations activities in order to learn more about how learners work with operations.

For an understanding of the content that underpins Operations consult Unit 3 pp. 93-124 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

Grade 3 – Activity 15
Grade 4 – Activity 16
Grade 5 – Activity 17
Grade 6 – Activity 18
Grade 7 – Activity 19

For each activity you should answer the following questions:

a) What mathematical concept/skill is the focus of this test item?
b) What is the nature of the error and how would you give feedback to learners in relation to the error?
c) Should the feedback be oral or written?
d) Would objects / images ("visual" elements) be useful in the feedback? Explain how you would use such elements if you think this would be appropriate.

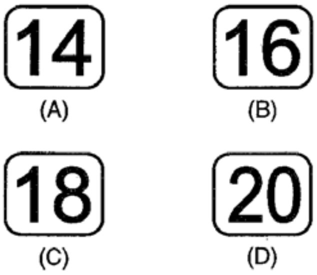
Activity 15

Grade 3 Operations

26. Here is a pattern.

Each row of three cards follows the same pattern.

What is the missing number card from the third row?



DIPIP learner data for selection of answers:

A	30%
B	**28%**
C	14%
D	18%

For the purpose of this activity focus on distractor A.

Commentary

The focus of this test item is the learner's ability to determine the number in a pattern using simple addition. Distractor A is might have been chosen because learners did not look closely at the patterns in the columns and rows of the diagram but who decided to use a familiar number pattern – the even numbers, counting up from 10. Feedback to learners who made this error should highlight the necessity to look closely at the diagram and work out the pattern using the given diagram. Feedback could be oral and should involve questioning about the operations involved in developing the patterns in the columns and rows of the diagram for this question.

Familiarity with certain number patterns may be a disadvantage to learners if they rely on identifying these patterns rather than working out the rule for how a number pattern develops.
How can you avoid your learners basing choices on familiar rules rather than logical working and conclusions drawn based on the evidence in a question?

Activity 16

Grade 4 Operations

22. $945 \div 9 =$?

(A) 15 (B) 100

(C) 105 (D) 150

DIPIP learner data for selection of answers:

A	16%
B	19%
C	**30%**
D	31%

For the purpose of this activity focus on distractor D.

Commentary

The focus of this test item is learner's ability to divide a 3-digit number by a 1-digit number. Learners who chose distractor D need feedback to guide them in the correct recording of the digits in the quotient. They need to be systematic and record each digit as they calculate the quotient. Feedback could be oral, based on this example as well as other similar examples, to enable the learner to realise why the correct answer to this question is 105 and not 150.

What is the educational value of repeated examples which are similar and yet different?

Think of a few "similar and yet different" examples to the question $945 \div 9 =$?

Activity 17

Grade 5 Operations

31. $32.80 × 15 = ?

(A) $19 680 (B) $196.80

(C) $49 200 (D) $492

DIPIP learner data for selection of answers:

A	19%
B	31%
C	24%
D	**19%**

For the purpose of this activity focus on distractor B.

Commentary

The focus of this test item is the learner's ability to multiply a given amount of money by a 2-digit number. The answer in distractor B is completely incorrect and seems to represent a guess rather than a calculation. The answer has dollars and cents in it, like the one factor in the multiplication. Feedback given to learners who chose this distractor should convey to them the importance of reading a question and then performing the operation required by the question. This is important since distractors in multiple choice questions can be designed to "mislead" learners.

The context of this question may have confused learners – they may have been led to think that the answer should have "dollars and cents" like in the question.
How can you enable your learners not to be misled by contextual factors in a question?

Activity 18

Grade 6 Operations

9. 7.35 – 5.85 = ?

(A) 1.20 (B) 1.50

(C) 2.00 (D) 2.50

DIPIP learner data for selection of answers:

A	10%
B	**38%**
C	14%
D	37%

For the purpose of this activity focus on distractor D.

Commentary

The focus of this test item is the learner's ability to subtract two numbers with two decimal places. Feedback given to learners who chose distractor D should enable them to realise that when they subtracted they forgot to rename the units when they broke down 7 units into 6 units and 10 tenths in order to facilitate the subtraction of the tenths. Oral feedback accompanied by written algorithms would be appropriate, based on several examples which are "similar but different" to the one in the question.

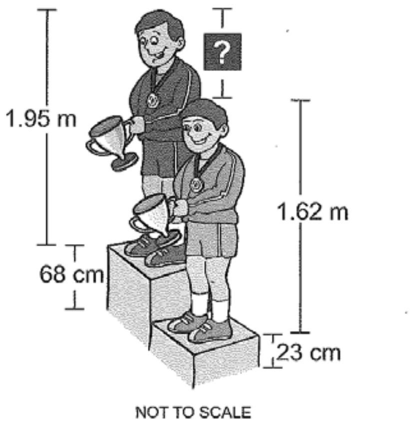
What is the educational value of repeated examples which are similar and yet different?

Think of a few "similar and yet different" examples to the question 7,35 – 5,85 = ?

Activity 19

Grade 7 Operations

24. Calvin and Steven are standing on steps of different heights.



What is the distance between the top of Calvin's head and the top of Steven's head?

(A) 33 cm (B) 45.33 cm

(C) 78 cm (D) 91.33 cm

DIPIP learner data for selection of answers:

A	42%
B	18%
C	**24%**
D	14%

For the purpose of this activity focus on distractor A.

Commentary

The focus of this test item is the learner's ability to use addition and subtraction with decimals. The question is linked to a context and so some feedback might be needed to enable learners to properly interpret the context. Learners who chose distractor A only referred to the two measurements given in the "top" part of the drawing. They did not calculate the total heights before subtracting to find the missing measurement.

How can you help your learners to interpret contextualise questions correctly?

Fractions

Learners' understanding of fractions begins at an early age, almost at the same time as their concept of whole numbers is being developed. Initially it is based very much on concrete examples of fractional parts which can help them to visualise and then generalise the idea of a part relative to a whole. The fraction concept needs to be sufficiently well generalised until it is an abstract number concept which is not reliant on concrete images or objects.

Activities 20 – 24 Fractions

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the fractions activities in order to learn more about how learners work with fractions.

For an understanding of the content that underpins Fractions consult Unit 4 pp. 125-174 of the *Saide*/Wits module, *Mathematics for Primary School Teachers.*

Grade 3 – Activity 20
Grade 4 – Activity 21
Grade 5 – Activity 22
Grade 6 – Activity 23
Grade 7 – Activity 24

For each activity you should answer the following questions:

l) What mathematical concept/skill is the focus of this test item?
m) What is the nature of the error and how would you give feedback to learners in relation to the error?
n) Should the feedback be oral or written?
o) Would objects / images ("visual" elements) be useful in the feedback? Explain how you would use such elements if you think this would be appropriate.

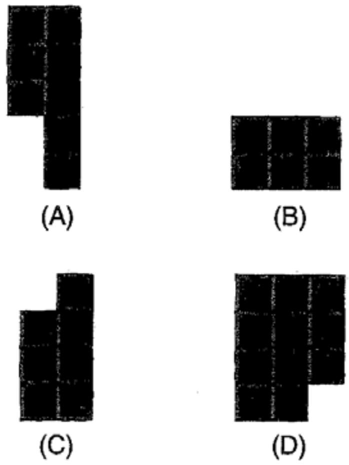
Activity 20

Grade 3 Fractions

22. Jill had a block of chocolate with 32 pieces.

She broke off a quarter of the block.

Which of these shows the quarter she broke off?



DIPIP learner data for selection of answers:

A	**25%**
B	31%
C	13%
D	23%

For the purpose of this activity focus on distractor B.

Commentary

The focus of this test item is the learner's ability to find one quarter of 32. Many learners selected this distractor which could have appealed to them because of its familiarity. Feedback to learners should work from the wording of the question and be based on an independent drawing, from which the solution is found. The solution will then be known to the learners (they should have worked out that one quarter of 32 is 8) so that they can count the blocks in the different multiple choice answer options rather than judge them based on their "looks".

Why is it important to calculate the answer before selecting an option in a multiple choice question?

Activity 21

Grade 4 Fractions

12. Jay had a melon.

He ate $\frac{1}{4}$ of the melon.

Then Jay ate **another** $\frac{1}{4}$ of the melon.

What fraction of the melon did he eat in total?

(A) $\frac{1}{8}$ (B) $\frac{2}{8}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

DIPIP learner data for selection of answers:

A	15%
B	26%
C	**39%**
D	16%

For the purpose of this activity focus on distractor B.

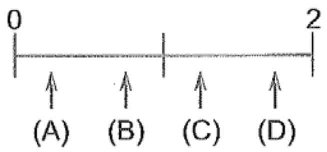
Commentary

The focus of this test item is the learner's ability to add two quarters to make a half. Learners who chose this distractor may think that when adding fractions you add the numerator and the denominator. Oral feedback (explanations) on this should be based on concrete illustrations of fractional parts which are added. Several examples of addition of various fractions (with the same denominator) can then be used to allow the learners to generalise the correct rule for addition of fractions.

Why is it better to allow learners to generalise a rule based on several examples? Do you allow your learners to "discover" rules for themselves based on examples that you give to them?

Activity 22
Grade 5 Fractions

25. Which arrow points to $1\frac{3}{4}$?



DIPIP learner data for selection of answers:

A	18%
B	24%
C	28%
D	**27%**

For the purpose of this activity focus on distractor C.

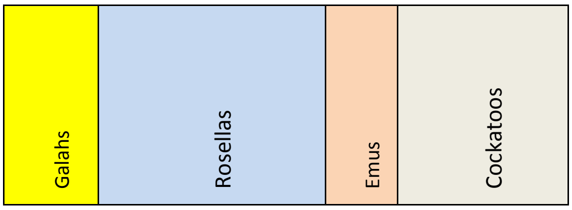
Commentary

The focus of this test item is the learner's ability to identify a fractional amount on a simple number line. Learners who chose distractor C did not look closely at the scale of the number line. They chose a number bigger than 1, and it could be that they worked out the "three quarters" by counting up to the third arrow (of the four arrows). Your feedback needs to clarify the use of the scale on the given number line in order to work out the correct position of the arrow to indicate $1\frac{3}{4}$.

Why is it important to teach learners that the scale on a number line should always be correct?

Activity 23
Grade 6 Fractions

14. Bill went on a walk in an Australian national park.
He drew a graph to show the proportion of different birds he saw on his walk.



(A) $\frac{1}{4}$ (B) $\frac{2}{5}$

(C) $\frac{2}{4}$ (D) $\frac{3}{5}$

DIPIP learner data for selection of answers:

A	27%
B	**16%**
C	26%
D	27%

For the purpose of this activity focus on distractors A and D.

Commentary

The focus of this test item is the learner's ability to interpret a bar graph in terms of fractions. To make this interpretation, learners had to work out the relative sizes of the shaded parts in the diagram. The two most popular distractors here represent different errors of thought – distractor A involves a guess that the rosellas take up $\frac{1}{4}$ of the space while distractor D involves a mis-reading of the graph – from the starting point to the end of the rosellas, which would be $\frac{3}{5}$. This is a difficult question since working from a small printed drawing could easily lead to misinterpretations. Feedback should point to the potential of misinterpreting the drawing and the need for careful and correct reading of the given diagram.

Learners need to be given many opportunities to work on contextualised questions.
Why is this becoming ever more important in the context of mathematics questions learners do?

Activity 24

Grade 7 Fractions

13. Which of these expressions is equivalent to $\frac{5}{7}$?

(A) $\frac{5}{7}+\frac{7}{5}$ (B) $\frac{5\times5}{7\times7}$

(C) $\frac{5+2}{7+2}$ (D) $\frac{5\times7}{7\times7}$

DIPIP learner data for selection of answers:

A	30%
B	33%
C	17%
D	**17%**

For the purpose of this activity focus on distractor B.

Commentary

The focus of this test item is the learner's ability to find an expression that is equivalent to a simple fraction. Learners who chose distractor B "did the same to the top and the bottom" in a different way to the one that we expect. Feedback to learners on this question could involve working through each of the distractors using correct mathematical procedures – as well as a discussion of what is being done. This should help them to find the correct answer and to demonstrate to themselves why distractor B is based on a distortion of the "rule" of equivalence.

Why is it better to allow learners to generalise a rule based on several examples?

Do you allow your learners to "discover" rules for themselves based on examples that you give to them?

Ratio

Learners' understanding of ratio should also be developed based on concrete representations and activities. This concept needs to be sufficiently well generalised until it is an abstract number concept which is not reliant on concrete images or objects.

Activities 25 – 29 Ratio

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the ratio activities in order to learn more about how learners work with ratio.

Grade 3 – Activity 25
Grade 4 – Activity 26
Grade 5 – Activity 27
Grade 6 – Activity 28
Grade 7 – Activity 29

For each activity you should answer the following questions:

l) What mathematical concept/skill is the focus of this test item?
m) What is the nature of the error and how would you give feedback to learners in relation to the error?
n) Should the feedback be oral or written?
o) Would objects / images ("visual" elements) be useful in the feedback? Explain how you would use such elements if you think this would be appropriate.

Activity 25

Grade 3 Ratio

33. Students at Happyville School can earn Bronze, Silver and Gold awards.

Students with 6 Bronze awards receive a Silver award.

Students with 4 Silver awards receive a Gold award.

How many Bronze awards are needed to receive a Gold award?

(A) 6×4
(B) $6 + 4$
(C) $6 \times 4 + 1$
(D) $6 + 4 + 1$

DIPIP learner data for selection of answers:

A	**20%**
B	24%
C	28%
D	21%

For the purpose of this activity focus on distractor C.

Commentary

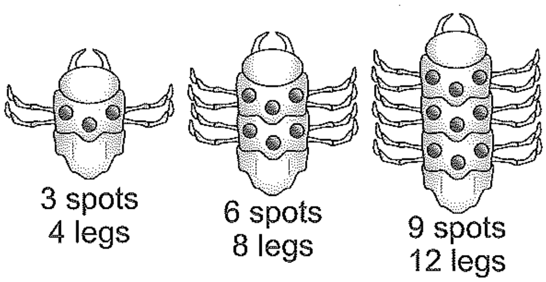
The focus of this test item is the learner's ability to select operations needed to solve a word problem. The problem wording should lead learners to set up two ratios (Bronze to Silver and Silver to Gold) which then need to be used together to find the ratio between Bronze and Gold. Feedback to learners should clarify this, the use of the ratios and the relationship between the ratios. You could draw sets of medals in the given ratios to illustrate the relationship.

Interpretation of contextualised questions is a skill learners need to have when they do standardised tests.
Are you giving your learners sufficient examples and opportunity to develop this skill?

Activity 26

Grade 4 Ratio

33. Sarah made these model bugs.



She then made a model bug that had 36 legs.

How many spots did that model bug have?

(A) 9
(B) 12
(C) 27
(D) 48

DIPIP learner data for selection of answers:

A	16%
B	25%
C	**33%**
D	21%

For the purpose of this activity focus on distractor B.

Commentary

The focus of this test item is the learner's ability to find a number using the relationship between two number patterns. Feedback to learners who chose distractor B should be aimed to allow them to realise that they realised they were looking for a number of spots, which are counted up in threes, but not the one that comes consecutively after 9. They need to be shown that they should have worked out how many steps in the pattern are needed to get up to a bug with 36 legs, and then work out how many spots that bug would have, based on the pattern for increasing numbers of spots.

Multistep problems are more difficult than single step problems.
Are you giving your learners sufficient examples and opportunity to develop their ability to do multistep problems?

Activity 27

Grade 5 Ratio/Rate

37. Maria was training for a race.

She ran 3 km each day except on Sundays when she ran 5 km.

She ran every day for 31 days, and started her first run on a Friday.

How many kilometres did she run in total?

(A) 90
(B) 101
(C) 103
(D) 155

DIPIP learner data for selection of answers:

A	28%
B	21%
C	**20%**
D	28%

For the purpose of this activity focus on distractor A and D.

Commentary

The focus of this test item is the learner's ability to solve a multi-step ratio problem. (To calculate the total distance covered). Feed back to learners could be given to show them the usefulness of a schedule to find out how many days she ran for each of the different distances. They need to find out that she ran 5 km/day for 5 days and 3knm/day for 7 days.

Interpretation of contextualised questions is a skill learners need to have when they do standardised tests.
Are you giving your learners sufficient examples and opportunity to develop this skill?

Activity 28
Grade 6 Ratio/Rate

30. Sam and Kevin are bricklayers.

Sam lays 150 bricks in 60 minutes.
Kevin lays 20 bricks in 10 minutes.

Working together, how many minutes will it take Sam and Kevin to lay 180 bricks?

(A) 25 (B) 40

(C) 70 (D) 100

DIPIP learner data for selection of answers:

A	10%
B	**11%**
C	47%
D	28%

For the purpose of this activity focus on distractor C.

Commentary

The focus of this test item is the learner's ability to use a rate calculation to find the time taken for a task. In selecting distractor C learners that may have thought that since Sam and Kevin are working together, from their conception of 'together', the time that it will take them would be the sum of the times given. Hence they added 60 and 10 to get 70 minutes. Feedback given should show to learners that in this question "together' will mean a shorter time, since they work at the same time and complete the job sooner. This is a difficult question that may be above the level of grade 6 learners.

Multistep problems are more difficult than single step problems.
Are you giving your learners sufficient examples and opportunity to develop their ability to do multistep problems?

Activity 29

Grade 7 Ratio/Rate

3. This machine prints 119 copies of a book every 7 minutes.

How many copies does it print in 1 minute?

(A) 17
(B) 19
(C) 112
(D) 833

DIPIP learner data for selection of answers:

A	**50%**
B	24%
C	14%
D	10%

For the purpose of this activity focus on distractor B.

Commentary

The focus of this test item is the learner's ability to simplify a ratio (divide a 3-digit number by a 1-digit number in context). Feedback given to learners here should involve an explanation of the division required since the answer of 19 to this question involves a careless error of computation.

This is a contextualised division question that learners did relatively well in on the test.
Why do you think they did well?

Geometry

Geometry often involves visualisation of shapes when learners work under test conditions since they are not given concrete shapes in tests. Leaners should be given ample opportunities to work with concrete shapes and do visualisation activities so that they can develop the necessary skills to work with abstract representations of shapes.

Activities 30 – 34 Geometry

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the geometry activities in order to learn more about how learners work with geometry.

For an understanding of the content that underpins Geometry consult Unit 1pp. 5-52 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

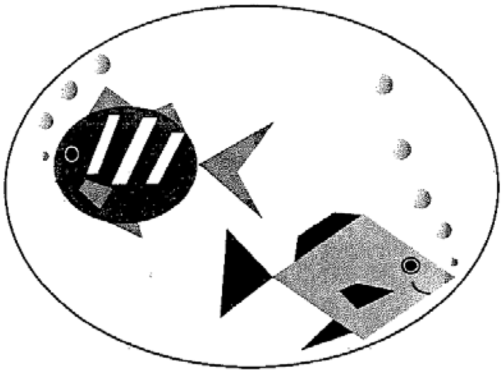
Grade 3 – Activity 30
Grade 4 – Activity 31
Grade 5 – Activity 32
Grade 6 – Activity 33
Grade 7 – Activity 34

For each activity you should answer the following questions:

l) What mathematical concept/skill is the focus of this test item?
m) What is the nature of the error and how would you give feedback to learners in relation to the error?
n) Should the feedback be oral or written?
o) Would objects / images ("visual" elements) be useful in the feedback? Explain how you would use such elements if you think this would be appropriate.

Activity 30
Grade 3 Geometry

18.



How many four-sided shapes are in this picture?

(A) 5
(B) 6
(C) 7
(D) 8

DIPIP learner data for selection of answers:

A	26%
B	17%
C	21%
D	**29%**

For the purpose of this activity focus on distractor A.

Commentary

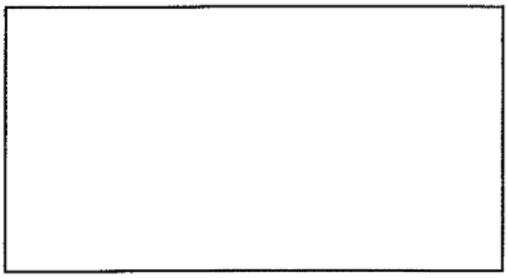
The focus of this test item is the learner's ability to identify a variety of four-sided shapes. Feedback here should involve working with learners and asking them to show you the different four-sided shapes that they can see. In this way you will be able to identify which of the shapes they have seen and which they still need to find. You can keep pushing them to look for more four-sided shapes until they find all of them.

Learners may be able to identify four-sided shapes in isolation but they also need to be able to find them in more complex drawings.
Do you give your learners opportunities to identify shapes out of the familiar setting and in unusual orientations?

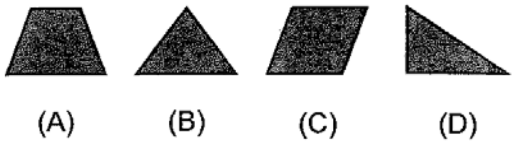
Activity 31

Grade 4 Geometry

20. Julia wants to tile this area of her bathroom wall.



Which of these tiles can she use to cover this area with no cutting, overlapping or gaps?



DIPIP learner data for selection of answers:

A	12%
B	13%
C	50%
D	**18%**

For the purpose of this activity focus on distractor C.

Commentary

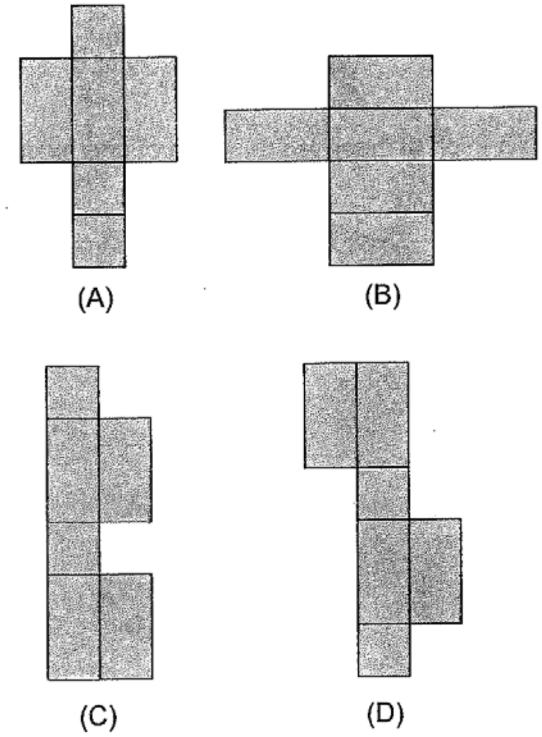
The focus of this test item is the learner's ability to select a tessellating shape. Feedback on this question should include a discussion and demonstration (using cut-up shapes) about tessellation in which you show the learners although all of the shapes in the distractors can tessellate, only D can do so to cover the larger rectangle without being cut or leaving gaps – as required by the question.

Why is it important to allow learners to cut out several shapes and experiment with tessellation with concrete material?

Activity 32

Grade 5 Geometry

33. Which of the following is a net of a rectangular prism?



DIPIP learner data for selection of answers:

A	20%
B	40%
C	16%
D	**15%**

For the purpose of this activity focus on distractor B.

Commentary

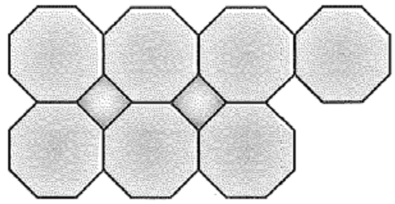
The focus of this test item is the learner's ability to recognise the net of a rectangular prism. Feedback to learners about this question should include explanations as well as demonstrations. You should allow your learners to make the nets shown in the question on a piece of paper and then allow them to experiment with the nets see which one works correctly.

What does the learners' selection of distractor B in the above question tell you about the need to vary the types of nets you show to your learners?

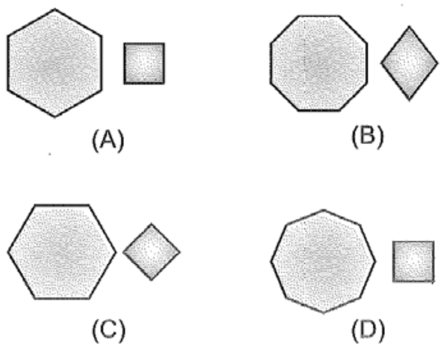
Activity 33

Grade 6 Geometry

15. This pattern is made out of tiles.



Which two tiles are needed to complete the pattern?



DIPIP learner data for selection of answers:

A	6%
B	44%
C	38%
D	**9%**

For the purpose of this activity focus on distractor B.

Commentary

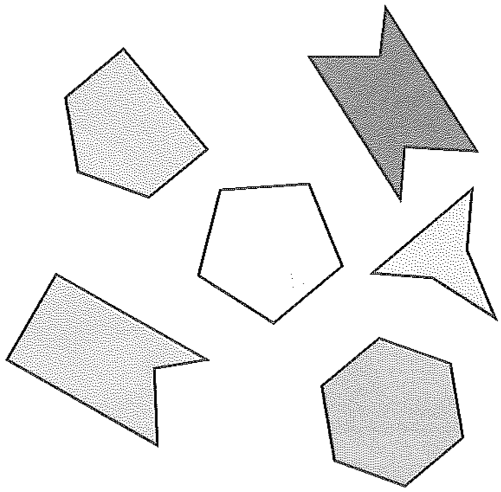
The focus of this test item is the learner's ability to complete a pattern of simple shapes. The feedback given to learners in relation to this question should involve both explanations and practical work. You should allow your learners to draw and cut out the shapes given in the question above in order to find out which ones work. The learners may have been confused by the orientation of the shapes – you should discuss this with the learners.

Learners should be able to visualise shapes in different orientations. How can you enable them to develop this spatial skill?

Activity 34

Grade 7 Geometry

1. How many of these shapes are pentagons?



(A) 4 (B) 3

(C) 2 (D) 1

DIPIP learner data for selection of answers:

A	**33%**
B	23%
C	26%
D	17%

For the purpose of this activity focus on distractor C.

Commentary

The focus of this test item is the learner's ability to count the number of pentagons in a picture. Learners who counted only two of the four pentagons in this drawing may not have counted the concave pentagons (those which have "indented" parts). Your feedback here should involve questioning the learners to find out which of the shapes they did count and then pushing them to identify the remaining pentagons, by suggesting that they count the numbers of sides in each of the shapes.

Two-dimensional shapes can be convex or concave.
Do you allow your learners sufficient examples of both of these categories of 2-D shapes?

Measurement

Measurement is all about quantifying characteristics of shapes. Only certain characteristics can be measured – those to which a "number" can be assigned. Knowledge of "what" can be measured incorporates an understanding of how this measurement is made. Too often teachers expect learners to measure and find quantities before they know what these quantities represent. It is important to use appropriate arbitrary units of measurement (such as steps to measure the length of the classroom) to develop the concept of each quantifiable characteristic before moving on to formal measurement and the use of standard units of measurement (such as metres, for the length of the classroom).

Activities 35 – 39 Measurement

The next five activities present questions based on learner work from Grade 3-7. You can select the activity in relation to the grade(s) that you teach. You may choose to do more than one of the measurement activities in order to learn more about how learners work with measurement.

For an understanding of the content that underpins Measurement consult Unit 6 pp. 197-210 of the *Saide*/Wits module, *Mathematics for Primary School Teachers*.

Grade 3 – Activity 35
Grade 4 – Activity 36
Grade 5 – Activity 37
Grade 6 – Activity 38
Grade 7 – Activity 39

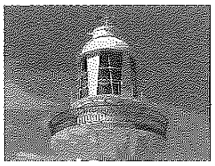
For each activity you should answer the following questions:

a) What mathematical concept/skill is the focus of this test item?
b) What is the nature of the error and how would you give feedback to learners in relation to the error?
c) Should the feedback be oral or written?
d) Would objects / images ("visual" elements) be useful in the feedback? Explain how you would use such elements if you think this would be appropriate.

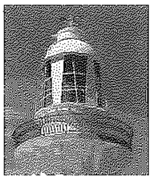
Activity 35

Grade 3 Measurement

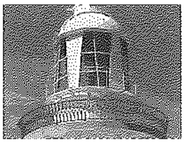
16. Which picture covers the biggest area?



(A)



(B)



(C)

(D)

DIPIP learner data for selection of answers:

A	**27%**
B	18%
C	44%
D	8%

For the purpose of this activity focus on distractor C.

Commentary

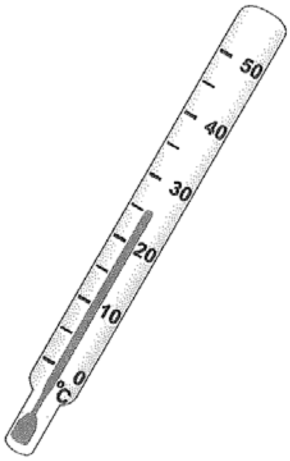
The focus of this test item is the learner's ability to identify which rectangle has the largest area. To give feedback to learners who selected C as the correct answer it could be useful to probe with questions to find out why they chose this particular answer. You could then help the learners to work out which shape has the largest area – they could use an arbitrary unit of area (such as a small square that they could cut out from a scrap of paper) or they simply need to compare the shapes carefully – looking at the sizes of the shapes in relation to each other, and not in relation to the size of the lighthouse picture in the image.

Why are arbitrary units useful in decision making about relative sizes?
Do you teach your learners about arbitrary units of area?

Activity 36

Grade 4 Measurement

5. What is the temperature shown on this thermometer?



(A) 20 °C
(B) 21 °C
(C) 25 °C
(D) 29 °C

DIPIP learner data for selection of answers:

A	23%
B	16%
C	**46%**
D	11%

For the purpose of this activity focus on distractor A.

Commentary

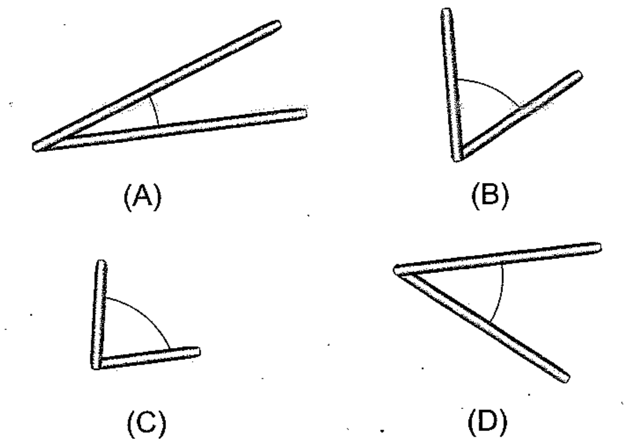
The focus of this test item is the learner's ability to read a thermometer. The demarcations on a thermometer are similar to those on a ruler in that they are accurately marked. Learners who chose distractor A probably focused only on the demarcations that had labels and did not work out the actual measurement using the scale to decide what the demarcation at the midpoint between 20 and 30 would be. When you give feedback to a learner who has made this error you could discuss the use of a scale and the reading of a measuring instrument that has been marked using the given scale.

Learners need to have practical experience of reading scaled instruments.
Do you allow your learners to use thermometers to read temperatures?

Activity 37

Grade 5 Measurement

19. Which of these shows the largest angle?



DIPIP learner data for selection of answers:

A	36%
B	10%
C	**34%**
D	13%

For the purpose of this activity focus on distractor A.

Commentary

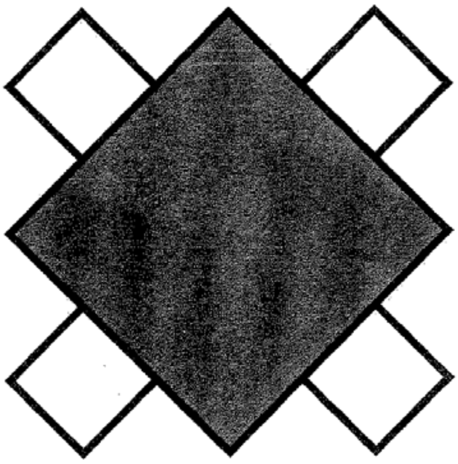
The focus of this test item is the learner's ability to differentiate angle size from orientation and side length. Learners who selected distractor A fell into the "trap" that an angle with longer arms is bigger. They were not able to differentiate angle size (the size of the opening) and side length. Your feedback to this question should include a discussion of the definition of an angle and then a practical demonstration of the openings made by arms which are rotated more or less about a point.

Discuss an activity in which you could allow learners to investigate that the lengths of the arms of an angle do not determine the side of the angle.

Activity 38

Grade 6 Measurement

23. This shape has four small squares and one large square.



The area of each small square is 4 cm^2.

What is the area of the **whole** shape?

(A) 13 cm^2
(B) 36 cm^2
(C) 52 cm^2
(D) 160 cm^2

DIPIP learner data for selection of answers:

A	22%
B	30%
C	**23%**
D	21%

For the purpose of this activity focus on distractor B.

Commentary

The focus of this test item is the learner's ability to calculate the total area of a figure in relation to a given portion of the area. Learners who selected distractor B did not count the number of smaller squares that make up the bigger shape correctly. Feedback to learners about this error should involve a discussion of the way in which the learners decided that there were 9 of the smaller squares in the larger shape. Then you should allow the learners to cut out one small square and use the cut-out to find out exactly how many of them fit into the bigger shape.

Why is it important for learners to work practically with cut-out shapes to find out relative areas of shapes?

Activity 39

Grade 7 Measurement

27. Here are three bottles.

2.5 L 600 mL 1 L

What is the total capacity of the three bottles in **millilitres**?

(A) 950
(B) 1625
(C) 1850
(D) 4100

DIPIP learner data for selection of answers:

A	36%
B	25%
C	13%
D	**25%**

For the purpose of this activity focus on distractor A.

Commentary

The focus of this test item is the learner's ability to convert between litres and millilitres. Learners who selected distractor A thought that they could just add the given numbers – they ignored the units. Feedback in relation to this error should involve an explanation of how to interpret the question (look at the different units given in the wording of the question) and also conversion from litres to millilitres.

Learners need to respect the difference between different units of measurement as well as be aware of the relationships between certain units.

How can you teach your learners so that they understand both of these ideas? (Give an example using units of capacity.)

Unit 5: Using learners' errors to inform lesson planning

Introduction

The planning and preparation of lessons is one of the major tasks of teachers. Researchers have found that while such planning and preparation take time, the more carefully these are done the more successful the lessons are likely to be and the better the results achieved by learners (Howie, 2003). When planning lessons it is important to think about the errors that learners made in their previous work and to decide how to respond to these. This unit focuses on using learners' errors to inform lesson planning.

The content and activities in this unit will support you in planning lessons that show awareness of errors and cater for a range of learners and learning needs. The unit presents transcripts of short episodes from mathematics lessons to situate the tasks within real mathematics lessons by way of examples of how learners' errors arise in mathematics lessons.

The unit is divided into two parts. The first part takes you through six examples of learner misconceptions that the teachers in the project encountered in the context of some actual lessons they taught during the first two phases of the DIPIP project. The focus of the second part of the Unit is on planning lessons so that teachers are prepared to address and deal with errors when they arise in their classroom teaching.

Aims of the unit

The main aim of this unit is to demonstrate the importance of keeping learners' errors in mind when planning lessons. One of the main causes of learners' errors is their misunderstanding of mathematical concepts (mathematical misconceptions). Not all lesson planning is based on identifying and understanding learners' errors (i.e. error analysis) but using learners' errors to inform the planning of good quality lessons is the topic of this unit. The unit focuses on the following:

- the value of using learner data in teaching;
- understanding that different kinds of mathematical misconceptions lead to learners' errors;
- the importance of keeping learners' misconceptions in mind when planning good quality lessons.

The value of using learner data in teaching

As explained in earlier units, the DIPIP project focused on identifying and working with learners' errors. While the first two phases of DIPIP were based on the university campus, in the third phase the project continued in a number of schools. This module (across the five units) has drawn on the activities that were carried out in DIPIP Phases 1&2 only. In a Learning Brief based on findings from the DIPIP

Phase 3 project in the years 2011 to 2013, Professor Karin Brodie writes about the benefits of using assessment data to improve teaching practice. She notes that teachers benefit from facilitation in this process of engaging with data. If you have worked through all of the units in this module, it is likely that you will have begun to realise these benefits. After reading *Using Assessment Data to Improve Practice,* complete Activity 1.

Activity 1

After reading *Using Assessment Data to Improve Practice* answer the following questions:

a) What are the six activities that teachers undertook in their classrooms in the third phase of the DIPIP project?
b) Why is it useful for teachers to analyse data on an on-going basis rather than only on "big" tests such as the ANA?
c) In what way does working collaboratively enable teachers to engage more meaningfully with assessment data?

Commentary:

Phase 3 DIPIP teachers in the years 2011 to 2013 worked on six activities similar to those in the first two phases of the project but this time they worked collaboratively in their schools and used more of their own classroom data in the process. They analysed test data, interviewed learners, identified concepts for discussion, read further research about the concepts they had identified, planned and taught lessons addressing the concepts and reflected on their lessons. Such activities, if carried out on an on-going basis enable teachers to meaningfully address errors that arise in the context of classroom teaching. In the learning brief, Brodie concludes that error analysis can enable teachers to "identify learners' needs and therefore their own learning needs" (p.5). Big tests that happen only once a year, can lead to analysis which does not benefit current learners since they will only be "in time for the next year's cohort of learners" (p.4). Collaboration helps teachers to benefit more from the activity of error analysis, since when teachers work on their own when they analyse data, the findings show that they tend to "resort to simple compliance rather than pushing deeply into the data" (p. 4).

Understanding that different kinds of mathematical misconceptions lead to learners' errors

In this part of Unit 5 you will read transcripts of episodes from several mathematics lessons recorded in DIPIP phases 1&2. Each one provides contextual evidence of a particular learner misconception (For a discussion of mathematical misconceptions, see Unit 3). Although not everyone is in agreement about how best to address misconceptions, we find them a useful organisational tool to think about learners' needs. The activities in Unit 5 give you another opportunity to think more deeply about learners' misconceptions but, also, more specifically on how teachers respond to learners' needs (for example how being aware of misconceptions help teachers to guide learners when they come across a concept that does not match their prior mathematical knowledge).

The sub-titles in this section relate to some of the aspects of the academic study of mathematical misconceptions which were identified by Smith, di Sessa & Roschelle (2007). While these aspects can be listed separately, in reality there is a complex interplay between them.

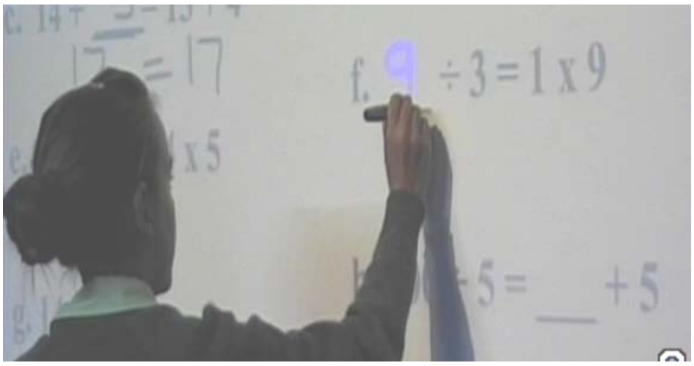
Each transcribed episode illustrates one or more of the following:

- Learners have different understandings of mathematical concepts
- Misconceptions could originate in prior learning.
- Misconceptions can be difficult to change.
- Misconceptions may interfere with learning.
- Moving from misconceptions to correct conceptualizations is a process.
- Some misconceptions are related to language.

Transcript 1: Learners have different understandings of mathematical concepts

A Grade 7 teacher conducted a lesson in which learners showed different understandings of the mathematical concept of equations. The lesson was about how to make true number sentences – sentences in which the values on both sides of an equal sign should be the same. The teacher had prepared many equations which the learners had to balance. She called up individual learners to do the working on the board. Read the transcript and then complete Activity 2 below it.

Figure 1: Grade 7: Solution of equations



Transcription of episode:

Teacher: OK the next one Nicki come try. (*Learner goes to sum which reads blank space divided by 3 = 1 x 9. She writes 9 in the space.*)
Teacher: Okay, right, let's see if she's correct. Is that right?
Learner's generally: No.
Teacher: Ok hands up who says no. If you say it's wrong there's got to be a reason why you say it's wrong. Dave why do you say it's wrong?
Learner: Because 9 divided by 3 is 3 and 1 times 9 is 9.
Teacher: Correct, okay so this one here is not balanced. So then, Dave, how can I get it to be balanced?
Learner: You have to say 27 divided by 3 which will equal to 1 times 9 which is 9 and the 27 divided by 3 will be 9.
Teacher: Ok so you've got to be careful. When you work out this sum, this side (*points to right of* =) 1 times 9 is 9, but it doesn't mean that this answer we get here goes into that hole on that side.

We've got to make sure that both sides are balanced and, as Dave says, If we go over it and went 9 divided by 3, that would give us 3. And 3 is not equal to 9. So just be careful of that. Right, we learn from our mistakes.

Activity 2

In the lesson, the learner had to complete the number sentence, so that the Left Hand Side (LHS) is equal to the Right Hand Side (RHS).

Answer the following questions:

a) Why do you think the learner wrote "9"?

b) How did the teacher respond to this? Did she explain the error in more than one way to the learner?

c) Design a worksheet to test learners' conceptions of the use of the equal sign. Be sure to include examples where you challenge learners to carefully consider their solutions.

Commentary:

A possible explanation for the learner's error is that she looked at the RHS of the equation, 1 x 9, and simply wrote the answer 9. She did not consider the division operation. Learners often think of the equal sign as meaning "it must give the answer". This thinking leads to misconceptions when they are asked to complete number sentences. The teacher explained that the learner had to check the values of the LHS and RHS of the number sentence to find out whether they are the same. She did not give another explanation. She could have expanded the explanation by writing the value of the LHS underneath the sentence, as she did with the RHS.

$\_\_\_ \div 3 = 1 \times 9$

Learner wrote:

$9 \div 3 = 1 \times 9$

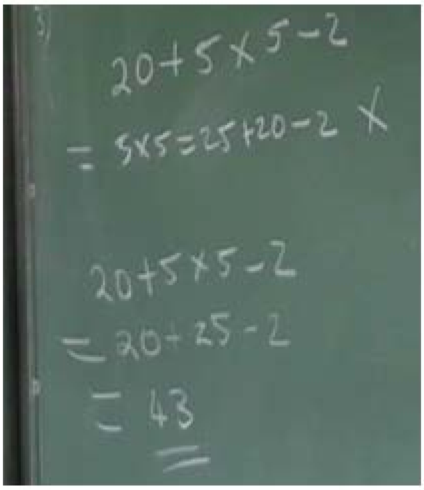
$3 \neq 9$

The opportunity could have been used to strengthen the concept of the equal sign, by explaining the ≠ sign. A teacher with a deep knowledge of learners' misconceptions and of the inaccurate constructions that learners might come up with, will be better prepared to guide these learners when they come across concepts that do not match the constructs they have developed from prior experiences.

Transcript 2: Misconceptions could originate in prior learning

Theo taught a lesson in which learners' prior knowledge of how the equal sign is used prevented them from solving equations at a higher level. The episode took place in a grade 9 class, when he was teaching his learners how to balance equations. The learners in the class worked individually on some equations which he had prepared for them and then they worked together as a whole class to go over the solutions. Throughout the discussion the teacher kept trying to reinforce the idea that when you want to balance equations, you need to make sure that the LHS and RHS of the equal sign should be equal. Read the transcript and then complete Activity 3 below it.

Figure 2: Grade 9: Using the equal sign



Transcription of episode (minutes 18-21):

Teacher: You got this question, hm. Let us look at something like: $20 + 5 \times 5 - 2$, right? (*Writes on board*). If you want to find out the solution… What am I using? What operation am I using by the way?

Learner: BODMAS

Teacher: We are using BODMAS, right? Order of operations, right? If I am going to find solutions there I am … step by step. But I can't say: five times five is equal to twenty five. (*Writes on board*: $= 5 \times 5 = 25$ *below the previous step).* Then I say 25 plus 20 then minus 2. (*Teacher proceeds to write:* $= 5 \times 5 = 25 + 20 - 2$ *on board*.)

Teacher: This is not correct, because five times five equals to twenty five plus twenty minus two – these two things are not equal to each other (*points to* $= 5 \times 5 = 25 + 20 - 2$). This is wrong. (*puts a* ✗ *next to the step*). Some people are …. You've got. They are writing the bits and pieces of the steps there. What are we supposed to do? You are supposed to work out what you have. If I've got 20 there (*pointing to the 20*), I am working these out first, right? So the 20 still remains there plus the solution to that (*pointing to the* 5×5). What is the solution to that?

Learner: Twenty five

Teacher: Twenty five minus 2 (*writes on board*). I never change this is equal to that (*pointing at the two steps*). That's what I am using the equal sign. If you are using the equal sign like this (*points to* $= 5 \times 5 = 25 + 20 - 2$) then you are writing wrong statements, because they are not equal to each other. So the next thing will then be to simplify: (*Writes on board* $= 20 + 25 -2 = 43$)

Teacher: So what do you see about this solution? Is not the same as the one you want, right? We then go on the work the next one. Teacher returns to writing on board. So this is incorrect (*puts a* ✗ *on the board next to the line* $20 + 5 \times 5 - 2$). It is unacceptable to write like that. You must lay out in such a way that the equal sign keeps its meaning. Cause equal sign mean what. What you work out on the left hand side, or once the proceedings that take place is equal to the statement above it, OK?

Activity 3

After reading the transcript "using the equal sign" answer the following questions:

a) Was the teacher clear in his explanation about the use of the equal sign? Give a reason for your answer.
b) Would you have done anything differently? If so what?
c) Think about an error made by your learners when they worked on strings of numbers. Suggest how you could address this error in planning for your next lesson.

Commentary:

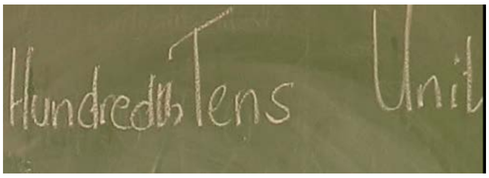
The teacher was clear in his explanation about the use of the equal sign. This is what he said, referring to the working on the board: "This is not correct, because five times five equals to twenty five plus twenty minus two – these two things are not equal to each other (*points to* $= 5 \times 5 = 25 + 20 - 2$). This is wrong. (*puts a* ✗ *next to the step*)." He then worked through the question again, showing the correct working, and reminded the class that their working should be correctly laid out. "You must lay out in such a way that the equal sign keeps its meaning."

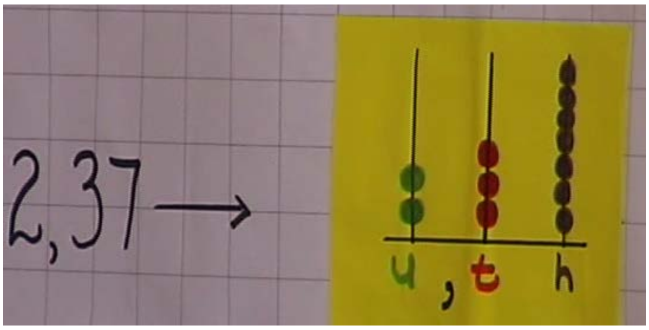
In this lesson the teacher had planned ahead of time and had thought about the errors that might crop up in his lesson which enabled him to address the error when it came up. This sort of planning allows a teacher to anticipate errors. He was on the look-out for learners who wrote sloppy working (disregarding the meaning of the equal sign) and he kept on pointing out to them what was wrong with it and how to do it in the right way. In this instance, he demonstrated the error and what should have been done differently.

Transcript 3: Misconceptions can be difficult to change

Fiona taught a lesson in which learners' misunderstanding of place value meant that they had difficulty equating different representations of the same number. The episode took place in a grade 6 class, when she was teaching her learners how to represent values on an abacus and to use numeric place value. The lesson was about decimal fractions, specifically tenths and hundredths. The teacher tried to make the connection between tens and units (which are whole numbers) and tenths and hundredths (which are fractions), and the way in which they are related, according to place value. This led to some confusion in the lesson, particularly because the teacher and the learners often talked about decimal fractions using whole number names (for example, they read 0,12 as "zero comma twelve" when they should have read it as "zero comma one two"). Read the transcript and then complete Activity 4 below it.

Figure 3: Grade 5: Place value





Transcription of episode (minutes 21-25):

Teacher: Now what do I have there? Yes, Themba?
Learner: Ma'am, it's zero comma twelve.
Teacher: Zero comma twelve. Now explain to me why zero comma twelve? Themba? (*writes 0,12 on board*) I'm waiting.
Learner: Ma'am, because there are no units and there are tens and hundreds.
Teacher: There's no units, good.
Learner: And there are tens and hundreds.
Teacher: And then the other part after the fraction is a decimal…I mean, after the comma is a decimal, right? And in the place values of the decimals what do I have? What words?
Learner: One.
Teacher: One what?
Learners: One tenth.
Teacher: One tenth and…?
Learners: Two hundredths.
Teacher: And two hundredths. I can break it up, isn't it? Now I can break it up in the expanded form. Remember when we did 32? What did I say 32 was equal to if I broke it up into a place value?
Learners: Three tenths.
Teacher: Three tenths, which is?
Learner: Thirty.
Teacher: Thirty…
Learner: And two hundreds.
Teacher: Sorry!?
Learner: Two units!
Teacher: (*writes 32 = on the board*) Why are you telling me two hundreds, Shadrack? I'm waiting for an answer please…you've done place values, am I right?
Learners: Yes.
Teacher: How many numbers are there? (*pointing to the 32 on board*)
Learner: Two.
Teacher: So what are the two values? (*pointing to the 32 on board*)
Learners: Tens and units.
Teacher: Units and?
Learners: Tens.
Teacher: So I've got three tens plus…? (*writes 3T + on board*)
Learners: Two units. (*writes 2U on board*)
Teacher: Which equals to (*writes = on board*)…thirty plus two. (*writes 30 + 2 on board*)
Learner: Thirty plus two.
Teacher: That is (your?) numbers, am I right?
Learner: Yes.
Teacher: Now, what am I saying? (*points to* $\frac{12}{100}$ *on board*) Zero comma twelve (*writes 0,12= on board*). What is zero comma twelve equal to?
Learner: Zero units.
Teacher: Zero units. (*writes 0U+ on board*)
Learner: Plus ten. One tenth.
Teacher: One?
Learners: Tenth.

Teacher: One with a small t (writes 1t), or I can write it as one out of ten (*writes* $\frac{1}{10}$ on board) and I can write it as…?
Learner: One.
Teacher: I'll come back to that later on…plus…(*writes +on board*)
Learner: Two hundreds.
Teacher: Two with the small h (*writes 2h* on board). The small h tells us it's hundreds. Right. Look over here. (*points to equation written on bottom of decimal numbers chart: 2,37→abacus drawing← 2 +* $\frac{3}{10}$ + $\frac{7}{100}$) Can you see? What number have I written here? Otto?
Learner: Two comma thirty-seven.
Teacher: Two…
Learners: Comma thirty-seven.
Teacher: Now explain to me what does two comma three seven mean? Now I'm putting it on an (*puts hand on drawing of abacus*) abacus; on the abacus I've got my rods (*traces finger down rod),* and my beads (*points to bead with thumb*). And now I got to put in my abacus counters.
Teacher: What is the two? (*points to the 2*)
Learners: Two units?
Teacher: Two wholes or two units. So on my first rod, I'm going to put two beads with the term capital U, for ones (*points to U on drawing of abacus*). Ok. Then what am I putting? I'm putting something.
Learners Comma.
Teacher: I'm putting the comma. Why am I putting the comma? Why am I putting the comma? Come the others, don't want to be hearing Sam only. Yes, Kamo?
Learner: Because then the numbers (are?) two hundred and thirty-seven.
Teacher: It's not two hundred and thirty-seven. It's not two hundred thirty seven…there's a difference between two hundred and thirty-seven and two comma thirty-seven. What is the difference, that's what I want to know from you?

Activity

Activity 4

This lesson deals with place value. The teacher discusses the values of decimal fractions by converting common fractions with denominator 10 or 100 to decimal fractions.

a) Give your views on the teacher's explanation of $\frac{12}{100}$ and 2,37.
b) Are you aware of an error made by the teacher and the learners? If so, what is this?
c) Why do you think the error was so prevalent?
d) How would you have dealt with the error?
e) Describe the interaction between teacher and learners in the episode.
f) What would you have done differently in the lesson that you have seen a part of? Give at least three suggestions.
g) Explain how you would teach the connection between a common fraction and its decimal counterpart to grade 6 learners.

Commentary:

The teacher did not make the connection between the common fraction $\frac{12}{100}$ and the decimal format 0,12 when she was trying to get the class to speak about the values of the digits in this number. She used incorrect language to speak about this number, calling it "Zero comma twelve". This is a pattern of speech which occurred throughout the lesson – used by both the teacher and the learner. On the other hand, when the teacher went through her explanation in relation to the number 2,37, she did link this to the fraction format of the decimal. The explanation that made this link was slightly more successful. The teacher used the correct language herself after completing this section of the explanation – she said "two comma three seven", correcting the learner who had just said "Two comma thirty-seven".

So we see that both the teacher and learners used incorrect language to describe a decimal fraction (for example referring to 2,37 as ***two comma thirty seven).*** This could be due to the interference of whole number concepts with decimal number concepts. This type of misconception is resistant to change and the correct concept has to be explicitly taught. One way to address this would be to constantly refer to the place value table. The place value table could have been used to illustrate the position of the tenths and hundredths. Learners have to be able to express the fraction in words. For example, they should be asked to <u>say</u> "***twelve hundredths***", emphasizing the "th" sound, and then write it down.

T	U	,	t	h
	0	,	1	2

This is 12 hundredths

The lesson is mostly teacher centred – the teacher does most of the talking, or starts sentences which she expects learners to finish. The learners only answer questions. They are generally not actively involved in the lesson in any other way.

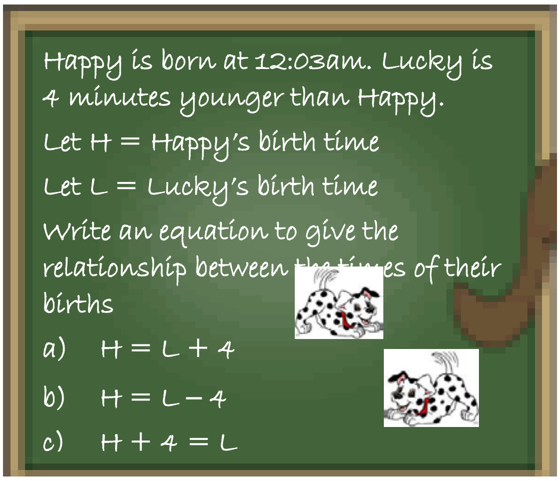
Reflection

- Are you thinking differently about how you would introduce decimal fractions and how a decimal fraction and a common fraction are linked?
- Have you ever thought about how difficult it is for young learners to understand that the same number can be written in completely different formats? (For example $\frac{1}{2}$, 0,5 and 50%)

Transcript 4: Misconceptions may interfere with learning

Hhlengi taught a lesson in which learners' prior misconceptions about the equal sign prevented them from understanding how to use an equal sign correctly. The episode took place in a grade 4 class, when she was teaching her learners the purpose of the equal sign (LHS and RHS of the equal sign should balance). In the lesson, she used a multiple choice question where the answers were different ways of expressing a solution using equations. There were four answer options a, b, c and d. Because the equations can be written in different ways, some options were actually equal (a = d and b = c). The question was difficult for Grade 4s and the discussion was complex. Read the transcript and then complete Activity 5 below it.

Figure 4: Grade 4: Time and solution of equations



Transcription of episode (minutes 9-15):

Teacher: Ok it's very much the same as we had yesterday, but it's slightly different. It still tells you that Happy was born at 12:03, Lucky is 4 minutes younger than Happy. Now we're going to do it in a different way. Have you heard what an equation is? Who knows? Who's heard about the word equation? Who's heard the word equation? (*No Learners raise hands.*)
Teacher: Nobody? Ok so what word is in there that you've heard before? Something that you do know – equation. What does it sound like?
Learner: Equivalent.
Teacher: Equivalent – what else?
Learner: Equal.
Teacher: Equal, good. So can you see I have an equal sign in all of these So what do you think an equation is? If we have an equals sign, what is an equation?
Learner: The same value.
Teacher: Right when we write up something we write a sum and it has the same value on this side and on that side. So in other words what is here (*gestures to her left side*) is equal to what is here (*gestures to her right side*). Like all the sums you do. 27 plus 3 is equal to 30. That is an equation. Because if I take 27 and I add 3 I get 30, so it is an equation. (*Points to board.*) So now I've written it slightly differently. Ok for each of these I've said write an equation to give the relationship between the times of their births. So one of these is going to give you a right answer. If you say that ***H*** is the time that Happy is born and you say that ***L*** is the time that Lucky is born. So if I have ***H*** – what would ***H*** mean?
Learner: Happy.
Teacher: And what time was Happy born?
Learner: At 12, 12, (*hesitant*). T: What time was Happy born?
Another Learner: 3 minutes past 12.
Teacher: 3 minutes past 12. 12:03 ok. So this actually would have been 12:03 (*points to board*). That's the time and then you can work out what time Lucky was born. So you can take out your white boards or you can just think in your heads, whichever you prefer. Look at the 4 equations and tell me which of those is right. So which of these shows you the time that Lucky is born and the time that Happy is born? No, first think about it. I'll give you all a few minutes to think about it. (*Learner calls Teacher and shows her board.*) Don't hold it up yet. So *a*, *b*, *c* or *d* – which one do you think. There may actually be more than one that's right by the way. You must have a look at each one and see if it's right or not. And write down the one that is right. (*Learner reacts*)
Teacher: Yes, think again. Look at it, think about it. Work it out in your head. (*Camera shows Learner writing on her white board*) and write which one you think is right. Just leave it let the others have a chance. Yours is right, turn it upside down. One of yours is right (*speaking to 2 Learners*). We'll see now and give everyone a chance.
Teacher: Ok put it down. Who says *a* is right – put up your hands. (*Some hands are raised.*) Who says *b* is right – put up your hands. (*No hands appear to be raised.*) Who says *c* is right? (*Many*

hands are raised enthusiastically.) And who says *d* is right. (*Camera doesn't show Learners.*) Ok so (*says name of learner*) you say *b* and *c*. Ok tell us why you say *b* and *c*. Listen to her explanation the rest of us.

Learner: Because it's like a sum but this sum can be mixed up because b says Happy it's kind of saying that um Lucky was. You just take away 4 minutes.

Teacher: Ok.

Learner: Ma'am is it fine if you chose *c*?

Teacher: Tell me why you chose *c*.

Learner: It's … ma'am I chose *c* because Happy was born at 3 minutes past 12 and if you added a 4 minutes extra it will equal to Lucky, that time he was born.

Teacher: That is the perfect explanation. So who said *a*? (*Some hands are raised.*) So let's test *a* and see if it could be or not. Ok if you say it's *a* you're telling me that Happy – ok ***H*** is Happy's birth time so that is 12:03. That's what time – is that right? Are you happy with me? Learners generally: Yes. ***H*** is 12:03 is equal to (*writes on board 12.03 =*) Lucky's birth time so we don't know what Lucky's birth time is, they're asking us. I'm going to put a little square there (*draws square after = on board*). Ok plus 4 (*writes +4 on board*). Which is obviously 4 minutes isn't it? So that's going to give me 12:03 is equal to his birth time plus (*draws on board 12.03 = square + and then erases the square and +*) sorry which is going to be equal to 12:07 (*writes this on board*). Ok is 12:03 equal to 12:07?

Learners: No.

Teacher: Not it's not, so you can't just say I'm going to see this answer. This whole thing (*points to what she's written on board*) together must be equal to that together. But watch carefully now when we do it the other way. Ok if I had Lucky, Happy's birthday, let's take number *c*. Right let's go for number *c*. So Happy is born at 12:03 ok (*writes 12.03 = on board*). That is Happy's birth time – do you all agree?

Learners: Yes. (*Teacher erases = off board*)

Teacher: plus 4 (*writes = 4 on board*) is going to give us Lucky's birth time (*writes =* ***L*** *on board*). So 12.03 plus 4 is 12.07 (*Learners say the answer with T*) is equal to Lucky's birth time (*writes =* ***L*** *on board*). Is that right?

Learners: Yes.

Teacher: Ok

Learner: So a and c would have been right?

Teacher: No *a* is not right. I just showed you now that it's not right. So *c* is right. (*Sound of Learners saying "yes" in background – clearly pleased they had the correct answer!*) What about *b*? Somebody said it was *b* and *c*. Was it you Sue? Right let's test for number b now. Ok. Happy's birth time is 12.07 (*writes this on board*) is equal to (*writes =*) Lucky's birth time minus 4. I have now worked out (*points to previous line of sum on board*) that Lucky' birth time is. Sorry I'm making a mistake here (*erases 7*) Happy's birth time is 12.03 (*inserts 3 on board*). Right so Happy's birth time is 12.03 ok is equal to Lucky's birth time which we worked out was 12.07 minus 4 (*writes on board 12.03 = 12.07 – 4*). Ok that's what it says for number *b*. So 12.07 minus 4 gives me?

Learners: Happy's birth time.

Teacher: 12.03. So are those right?

Learners: No. (*Teacher looks puzzled and Learners start to say "yes".*)

Teacher: Is 12.03 the same as 12.07 minus 4?

Learners: Yes.

Teacher: If I take 12.07 minus 4 do I get 12.03?

Learners: Yes.

Teacher: Yes. So does it matter what side of the equals sign I put my things?

Activity

Activity 5

The problem deals with the calculation of time. The problem was posed in the form of an equation where the teacher used variables H and L for the birth times. Learners had to choose the correct equation.

a) Do you think the problem is relevant for grade 4 learners? Why or why not?
b) From the discussion in the transcript do you think that learners know what an equation is? Give a reason for your answer.
c) What is the difference between “the time” (as in the time of the day) and time elapsed?
d) What type of manipulatives could the teacher have used to explain the difference between these concepts?
e) There is a misconception in this lesson. What is it?
f) Do you think that this misconception interfered with the learning in this class? Explain your answer.
g) What would you have done differently in the lesson that you have seen a part of? Give at least three suggestions.
h) How would you have taught the concept of time and time elapsed to Grade 4 learners?
i) How would you describe the interaction between the learners and the teacher?

Commentary:

This was a difficult question for Grade 4s who are only just starting to learn about equations. They are expected to solve equations “by inspection”, which means that they need to be able to work out numeric values that make number sentences true (for example if 4 + __ = 9, the missing number is 5). Some learners had an explanation for how to find an equation to talk about the relationship between birth-times, but generally it seemed that learners had only a vague idea about the meaning of an equal sign and how to write equations.

The time of day is taken from the reading on a clock or watch, for example time of birth. Time elapsed is the time that has passed, for example a reading on a stop watch. We refer to time elapsed as time passed. The teacher could have used a clock on which she illustrated the time and time elapsed, by turning the long arm four minutes. This would have helped the learners to work with what they had been given (which was a question relating to both time and time elapsed). The confusion between these two time-related concepts is a misconception that may have been causing interference in this lesson.

The teacher wrote 12.03 + 4 = 12.07 on the board. Mathematically this is incorrect – she is not being consistent in her writing of the units and so the minutes and the hours can be confused. She should have written 12.03 + 0.04 = 12.07. The teacher should not have put four choices on the blackboard all at the same time. In itself the time and time elapsed concepts are difficult enough. She could have used a few examples where learners had to do the same type of calculations but using different ways of talking about the information and asking questions, such as:

- The time of Happy's birth is 12:03.
- *How many minutes after 12 is Happy born?*
- Four minutes later Lucky is born.
- *What will be the time on the watch when Lucky is born?*

In this lesson the teacher did most of the talking. Learners were involved by individually calculating the time on their white boards. It seems that most learners did not follow the explanations offered by the teacher.

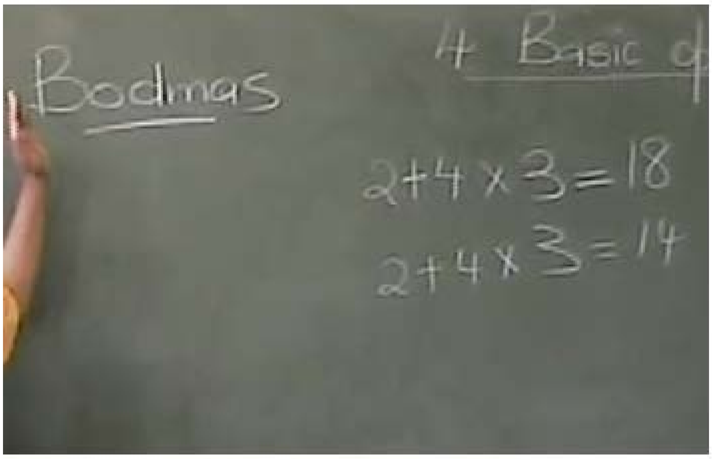
Reflection

- Have you ever taught the time and time elapsed?
- Will you teach these differently after looking at this video? If so, what will you do differently?

Transcript 5: Moving from misconceptions to correct conceptualizations is a process

Noma taught a lesson in which learners' incomplete knowledge of the order of operations (often summarised using the word "BODMAS") prevented them from solving equations. In this lesson, the teacher set up an example on the board to demonstrate that the order of operations matters. She then proceeded to work through the examples. The first example written (2 + 4 x 3 = 18) is the one which is wrong. The second one is correct (2 + 4 x 3 = 14). In the discussion that followed the learners answered the teacher's prompts in very short sentences. It is not clear that the meaning of what they were saying was clear to them. The episode took place in a grade 5 class, when Noma was teaching her learners how to operate on multiple number strings. Read the transcript and then complete Activity 6 below it.

Figure 5: Grade 5: "BODMAS"



Transcription of episode (minutes 11-15):

Teacher: Right and we have again let me write the very same number. It's 2 plus 4 times 3. What is the answer here? (*Teacher points to top sum.*) What is the answer here? What is the answer? We have 2 plus 4 times 3. What is the answer? What is the answer?
Learner: ...
Teacher: Yes you can try you can try – speak out.
Learner: 15.
Teacher: Is it 15? You say 2 plus 4?
Learners: 6.
Teacher: 2 plus 4?
Learners: 6.
Teacher: What is the answer, Pali?
Learner: The answer is 18.
Teacher: The answer is?
Learners: 18. (*Teacher writes 18 on board.*)
Teacher: The answer is?
Learners: 18.
Teacher: 18 ne? Well 2 plus 4 times 3 the answer is 18. Someone can get another answer. What is it? What is it? Let us add this one (*points to bottom sum which is exactly same as top one 2 + 4 × 3.*) Come … (*Teacher calls name of Learner*) you don't want to speak. Eh? You don't want to try? Speak out.
Learner: …?
Teacher: She thinks that it's 15. Ay your hands are up, that tells me that … they're telling me something else. What can you say (*points to Learner*)?
Learner: 14
Teacher: The answer is 14? How did you get 14? How did you get 14? How did you get 14? You don't know?
Learner: ?
Teacher: So you have not moved from left to right, you have started with multiplication. You have started with?
Learners generally: Multiplication.
Teacher: Right our question now – we have 2 answers. What is the correct answer there? What is the … what do you think is the correct answer? Ooh, let me hear Simon.
Learner: The correct answer is 14.
Teacher: The correct answer is 14 – is that true?
Learners: Yes.
Teacher: Is it true?
Learners: Yes.
Teacher: The correct answer is 14, yes. The correct answer is (*Learners say with her*) 14. Ne? So it is important for us to know that in mathematics we have rules. We have what class? We have
Learners: Rules.
Teacher: We have what class?
Learners: We have rules.
Teacher: We have rules for operation. Rules of operation. We've spoken about basic operation. We have rules of operation ne?
Learners: Yes.
Teacher: Where those rules for those operations that we have in mathematics. That we have in mathematics. That means that now I think (*points to Learner*) – eh what is the name? Aha Vusi. Vusi has applied the rule. We have that rule. We have this word, have you ever seen this word (*begins to write on board*)?
Learners: Yes.
Teacher: Have you ever seen this word? (*Continues writing "BODMAS" and underlines it.*)
Learners: Yes.
Teacher: Can you read the word for me? Can all of you.
Learners: BODMAS.
Teacher: Again.
Learners: BODMAS.
Teacher: I want you to put your hands up and tell me. We have this word. This word is an acronym. It is a?

Learners: Acronym.
Teacher: It is an acronym. We have the meaning of the word B, what is it? Huh? (*Speaks to Learner*.) Do you know anything? Come on let's try. You must try. No don't make a noise, you must try. Bona?
Learner: Brackets.
Teacher: Brackets, it's brackets. And O stands for ?
Learner: Of.
Teacher: Of. And D stands for? Bafana?
Learner: Division.
Teacher: Division. And M? Bongile?.
Learner: Multiplication.
Teacher: Multiplication. Dipua?
Learner: Addition.
Teacher: Addition and all of you?
Learners: Subtraction.
Teacher: And subtraction. This is what gives us that rule. That means we have when we are working up when we want to solve a problem we have to use the, we have to start with the brackets. Our brackets are always the first. Even brackets are always the ..?
Learners: First.
Teacher: Even if you don't see them. There is a way that you can see that I have to start with these numbers, ne?
Learners generally: Yes.
Teacher: Right we see brackets. Right brackets and multiplication and the of (*points to the O on board*) stands for multiplication, ne?
Learners: Yes.
Teacher: Multiplication and division are done together. They are done together. They are?
Learners: Done together.
Teacher: Yes and we always work from left to right ne? And then addition and subtraction again are done together. They are done together.

Activity

Activity 6

In this episode the teacher talks about the rules of operations.

a) What might have confused some learners when the teacher wrote the answer "18" on the board?
b) Did the teacher ask for explanations of the answer 18?
c) Another learner answered (twice) that the answer was 15. Did the teacher follow up on this incorrect response?
d) Do you think the teacher gave a good explanation of why we have rules of operations?
e) What would you have done differently to explain the rules of operations?
f) Are learners likely to understand mathematics if you tell them the rules?
g) Give two more examples in the mathematics curriculum where learners must remember rules.
h) What is the value of asking learners to repeat after you?
i) Can teaching by rules be avoided and if so, how?

Commentary:

In the episode only a few learners gave the answers to the teacher's questions and after that the rest of the class repeated the answers in a chorus. If you count in the transcript, you can see that the incorrect answer (18) was repeated five times. By this time, some learners might have believed that the correct answer was 18, since they had to repeat it so many times and it was not challenged. Then the teacher moved to accepting the correct answer of 14, away from the incorrect answer of 18 (ignoring the answer of 15). The discussion about the correct answer proceeded in a similar way to the discussion about the incorrect answer. The teacher got the class to go through the same ritual of repeating the answer after her for several times. She did go on to explain the BODMAS rule (in general) but did not link it explicitly to the given correct and incorrect answers. This is an example of re-teaching the rule in the hope of replacing a wrong understanding with a right one – which may or may not be successful. The example BODMAS (as a rule) is one where evidence is very strong that "teaching a rule" does not mean that the rule will be understood or correctly applied. Most learners can say the word BODMAS but they do not know how to apply it correctly.

Misconceptions often occur because learners have to remember rules. They then get the rules mixed up, or forget them or how to apply them, which results in errors. When learners have taken up a 'rule' in an incomplete or incorrect way, it is not sufficient to re-teach the rule in order to 'replace' a wrong understanding with a right one. Instead teachers should guide learners through a process of understanding why a rule works and when and why it does not work. This involves assisting learners to understand concepts.

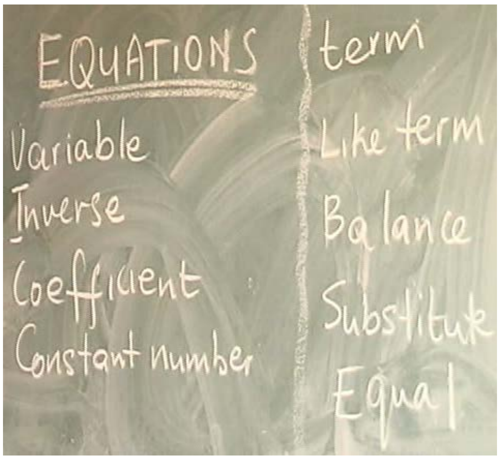
Reflection

- Reflect on where the rules of operations come from.
- Do you let learners repeat after you all the time, sometimes, never?
- Do you often think about "where the rules come from"?

Transcript 6: Some misconceptions are related to language

Sithembile wanted her learners to use a mathematics dictionary to find out the meaning of key terms related to the topic of equations. She wrote the words on the blackboard and handed out dictionaries to the learners. The episode took place in a grade 9 class, when she was teaching her learners the mathematical language of equations. Read the transcript and then complete Activity 7 below it.

Figure 6: Grade 9: Terminology relation to equations



Transcription of episode (minutes 10-15):

Teacher: Some look in the dictionary, some write the meanings. (*Repeats*) – a glossary is the meaning of words like… variable… etc. … *(she reads all the terms from her list in order… learners continue working more individually than together – but some sort of consultation going on. Learners Continue working in the same fashion… pretty silent. Very little communication. But they are working together, at least those who are close enough to each other and the dictionary. Books look new. They handle them with care. But they don't say much to each other. Some are writing, some are paging, work continues … One boy (row 2) sitting quite far away from the "action" is hardly involved at all.)*

Bell rings – class continues in silence …those far from the dictionaries look towards the action…

Teacher: The first group that has got the word variable to come and explain the meaning to the class … what does the word variable mean?" (*Teacher struggles with pronunciation of the word variable. Some hands go up… she calls one person to come up and explain… the first group that has the word variable*) (*A learner goes to the front – reads from her page what has been written there – sounds like it has been written directly from the dictionary.*)

Learner: A variable … is usually a letter such as x, y and z that may take any value from a given range … of the use ... unless the range of the value is stated then the variable can be any number" (*The learner reads in a sing-song voice.*)

Teacher: (*When the learner has finished the teacher repeats*) A variable can be any real number. (*The learner echoes this… goes back to her desk*)

Teacher: Inverse.

Learner: (*A learner stands up at his desk and reads.*) An inverse is the object in the set of binary operations with another object in the set which when combined with the original number produces the identity as a result. (*When he finishes the teacher asks him to repeat the definition.*)

Activity

Activity 7

In this lesson the teacher expects learners to use dictionaries to find the meanings of the list of words related to "equations".

a) Give your impression of what is happening in this classroom.
b) Why is it important for learners to understand the meanings of the words we use in mathematics?
c) Write down some mathematical terms for which the mathematical and 'everyday' meanings are different.
d) What would you have done differently to help learners to understand the meanings of mathematical terms?

Commentary:

It can be seen from the transcript that the atmosphere in the classroom was one of strict control with very little room for discussion. Even though the lesson was set up as a group discussion, it appears from the transcript that not much discussion took place. The learners did not all seem to be actively involved, as there were not enough dictionaries. The learners (some of them) used the dictionaries but this does not mean they really understood what they read (as is seen in the way they used and read from the dictionaries).

Mathematics can be considered to have its own language. It is very important that learners understand this language – a lot of which consists of mathematical terminology. To be able to do mathematics, the terminology must be understood and learners have to be taught to use mathematical terminology correctly. The mathematical terminology that teachers use is often very confusing for learners. Some mathematical terms have meanings in maths which are different in other contexts. Think of the term 'variable'. The general meaning of this word is 'change'. Something which is variable can change. In mathematics, the word variable does maintain the meaning of change but it is more specific. A variable in maths is what we use in algebra – the letters (for example a or b, x or y) which we write in the place of numbers are called variables. This is one of the terms that the teacher asked the learners to look up in their dictionaries. One learner read out a definition for the term "variable" from her dictionary. She read: "A variable … is usually a letter such as x, y and z that may take any value from a given range … of the use ... unless the range of the value is stated then the variable can be any number". It seems from the way in which the learner read the definition that she did not understand it herself. The teacher did not help or ask the learner to express the meaning using her own words, so we cannot see what level of understanding was reached here.

In planning this lesson the teacher should have prepared herself so that she could assist the learners to find the dictionary definitions and grapple with their meaning, not just read them out in a meaningless manner. Teachers need to be aware of the language they use in their classrooms. Meticulous planning is very important for the teaching of mathematics (so that teachers are sure that they themselves know and use correct terminology at all times). This will facilitate better learning and less confusion in mathematics lessons. Learners need to realise that there is sometimes a variety of expressions that a teacher uses to explain a concept.

Reflection

- Do you ever listen to yourself when you pronounce certain words?
- Do you take care to pronounce mathematical terms so that learners can also say the words correctly?
- Do you explain to your learners the difference between the mathematical and everyday meanings of words?

Keeping learners' misconceptions in mind when planning good quality lessons

The previous section of this unit highlighted different kinds of learner misconceptions that a mathematics teacher might encounter in the context of some actual lessons that were videoed during the DIPIP project. These were given as examples to highlight different ways in which misconceptions can be identified by teachers during teaching and different ways in which one can act when this happens. Lesson planning is essential for good teaching. When planning lessons, mathematics teachers need to be aware of the errors that learners commonly make when they are learning certain concepts. These errors are often based on misconceptions. In this part of the unit the focus is on planning lessons so that teachers are prepared to address and deal with errors when they arise in their classroom teaching.

Planning and preparation

Planning for a lesson means drawing up a plan of action. This plan could be based on one or more of the following:

- previous experiences of a teacher;
- resources such as the curriculum, textbooks or worksheets;
- lesson plans prepared by colleagues;
- lesson plans prepared by subject experts.

A good quality lesson plan should include the following:

- introduction (focus of the lesson);
- sequenced learner activities for whole class discussion;
- classwork activities for learners to work on individually or in groups;
- conclusion;
- homework activities to consolidate the learning of the day.

The plan is the starting point. Wherever the plan comes from, a teacher still has further work to do to prepare a good quality lesson.

Lesson preparation involves going through your lesson plan and making sure that you are ready to teach according to the plan. Preparation for a mathematics lesson involves:

- making sure that you know and understand the mathematical content being addressed in the lesson;
- working through each of the learner activities;
- making notes on likely learner misconceptions in relation to the activities;
- collecting any resources you need to use in the lesson (e.g. counters, a map, containers with different capacities.);
- knowing what board work will be done in the course of the lesson;
- drawing any essential diagrams on the board before the class begins (e.g. a number line labelled in 5s from 0 to 50).

Proactive / reactive responses to learner errors

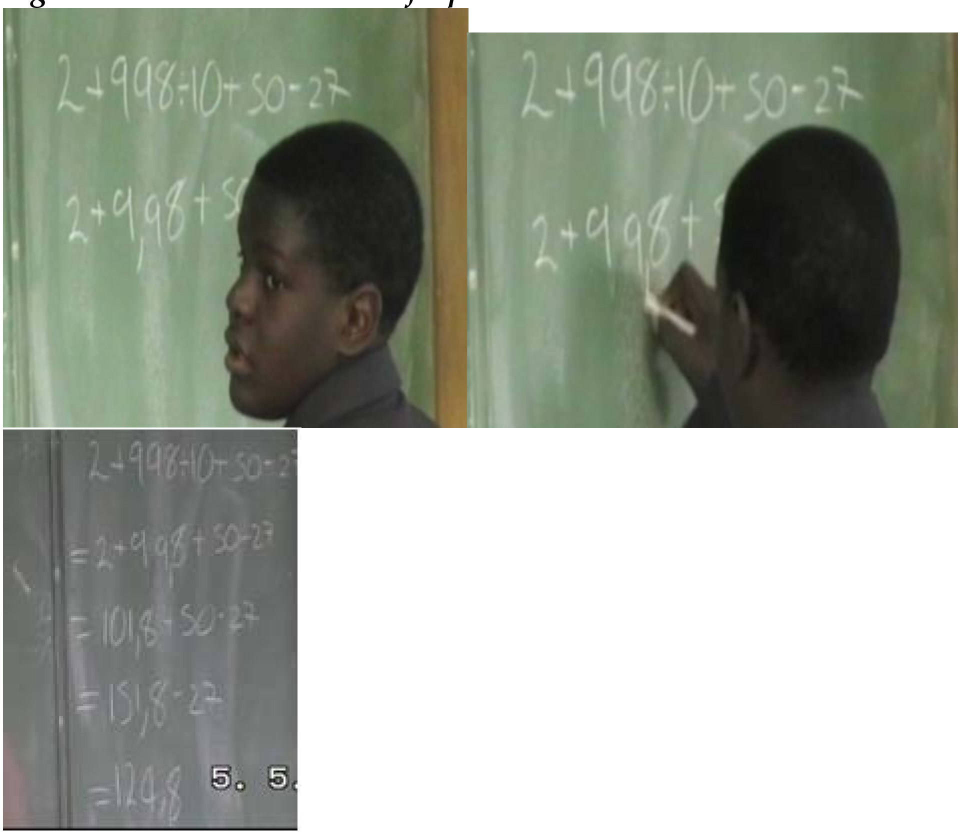
If teachers plan to teach a topic with learners' possible misconceptions in mind it is more likely that they will notice errors related to these misconceptions and be able to respond **proactively** (because they are already prepared for these errors**)** to them in the classroom. However, even if a teacher has prepared to respond to particular errors, during the lesson learners may make other errors to which the teacher will need to react without preparation.

The next episode gives you an opportunity to think about how to respond to errors 'in the moment' – that is, **reactively** (because you had not thought about this error when planning the lesson).

Transcript 7: Responding to errors in the moment

Theo's lesson (see transcript 2 in this unit) was one in which he was teaching his learners how to balance equations (LHS and RHS of the equal sign should balance) in a grade 9 class. During this lesson he responded proactively to certain errors that he anticipated. Other errors arose for which he had not specifically planned and to which he responded in the moment.

Figure 7: Grade 9: Solution of equations



Transcription of episode (minutes 1-4):

Teacher: Quiet please
Learner : (*Writes on board*)
2 + 998 ÷ 10 + 50 – 27
2 + 9,98 + 50 - 27
Teacher: 998 divide 10, is what? 99,8
Learner: (*Corrects 9,98 to 99,8. Proceeds to write on board*)
101,8 + 50 - 27
151,8 - 27
(*The learner pauses for a while*)
Teacher: Some of you could help with 151,8 minus 27?
Learners: 124,8 (*T now puts the equal signs on the board so that it reads*)
= 2 + 998 ÷ 10 + 50 – 27
= 2 + 99,8 + 50 - 27
= 101,8 + 50 - 27
= 151,8 - 27
= 124,8

Teacher: The solution here, which is something … does not give 123, right? So D (*one of the multiple choice options*) is not the correct solution. OK, so we are still using the order of operation to check equal to, what, 123. So order of operation, 998 divide 10. Ninety nine point eight. He start adding from left to right two plus ninety nine point eight is hundred and one point eight plus fifty is hundred fifty one point eight minus twenty seven.

Teacher: Did he cut up anything? Did he start branching up between the equal signs? No, he did not do that, right? So all these things… this step is equal to that, this is equal to that, this is equal to that and this is equal to that (*pointing to each of the steps on the board*). This will mean that this (*pointing to the 124,8*) is equal to that (*pointing to 2 + 998 ÷ 10 + 50 – 27*) So that's the end here.

Activity

Activity 8

After reading the transcript answer the following questions:

a) What is the error that arose in this episode?
b) How did the teacher respond to the learner's error?
c) When the teacher concluded the discussion of the question what was the focus of his discussion?
d) Would you have done anything differently? If so what?

Commentary:

The error that arose was one related to division by 10. The teacher just prompted the learner to think about what he had written by asking "998 divide 10, is what? 99,8" and the learner corrected himself (*Corrects 9,98 to 99,8. Proceeds to write on board*). The lesson continued smoothly. There was no conceptual discussion involved but it did not seem necessary at the time. It is very good that the teacher did not allow incorrect working to remain unchallenged on the board. His probing question focussed specifically on the error, which in this case did appear to be a mistake, since the learner was able to correct himself quickly.

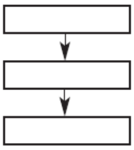
At the end the teacher gave another explanation of BODMAS (affirming how the learner had correctly applied the order of operations) to show how the correct solution needed to be calculated. He said: "So order of operation, 998 divide 10. Ninety nine point eight". He started adding from left to right: two plus ninety nine point eight is hundred and one point eight plus fifty is hundred fifty one point eight minus twenty seven. He showed the correct method but did not engage with the incorrect one in any detail since it did not seem necessary. The teacher also re-affirmed the correct use of the equal sign, saying "this step is equal to that, this is equal to that, this is equal to that and this is equal to that (*pointing to each of the steps on the board*)." Teaching in the moment can take many directions. Another teacher may have spent more time on the error – discussing it in relation to place value because the mistake that the learner made was that (initially) he divided by 100 instead of 10. A discussion that brought in place value concepts could have clarified this division in greater depth for the class than the simple correction (by the teacher or learner) may have done, but in the moment, the correction served its purpose.

Thinking about a lesson – using concept maps in lesson planning and preparation

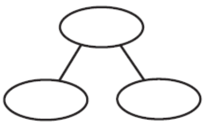
When planning for lessons with learners' errors in mind, it may be useful to use a concept map. A concept map is a way of representing relationships between ideas, images, or words. In a concept map, each word or phrase connects to another, and links back to the original idea, word, or phrase.

Concept maps are flexible. They can be simple or detailed, linear, branched, radiating, or cross-linked. (See http://www.energyeducation.tx.gov/pdf/223_inv.pdf)

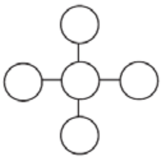
Linear concept maps are like flow charts that show how one concept or event leads to another.



Hierarchical concept maps represent information in a descending order of importance. The key concept is on top, and related concepts fall below.



Spider concept maps have a central or unifying theme in the centre of the map with subthemes surrounding it. Spider concept maps are useful for brainstorming or at other times when relationships between the themes need to be left open-ended.



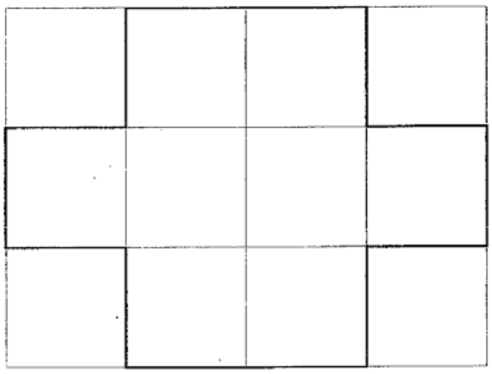
A concept map is useful for lesson planning because it helps teachers to think about:

- the lesson topic and all the related concepts learners will encounter in the lesson;
- learners' possible misconceptions in relation to the topic;
- logical sequencing of content and activities for the development of the lesson.

Planning a lesson with learners' misconceptions in mind

The planning and preparation of a lesson given here are based on Grade 5, Item 9 of the ICAS 2006[11] test. This is the same item that was used in the exemplar discussion in units 2, 3 and 4 of this module.

9. Holly drew this shape on 2 cm grid paper.



What is the area of Holly's shape?

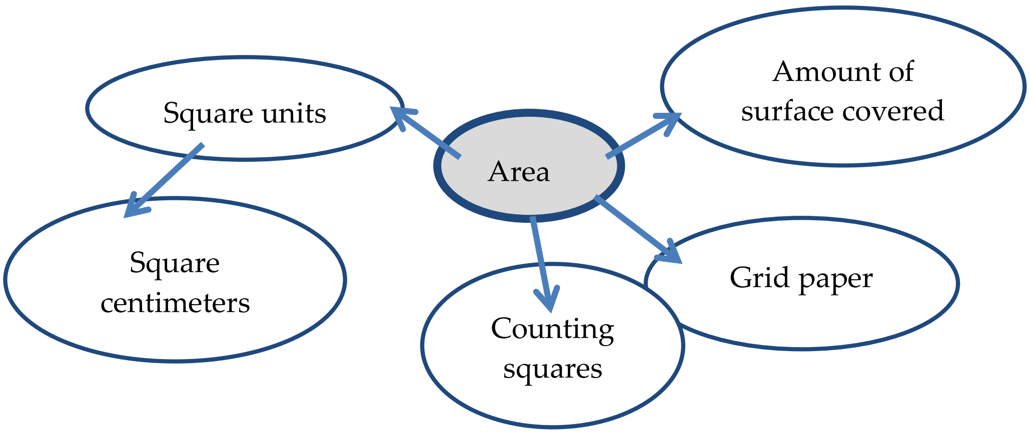
(A) 32 cm^2
(B) 28 cm^2
(C) 16 cm^2
(D) 8 cm^2

The planning process involves thinking about the content that will be taught. The example of planning below includes a number of actions or steps that are useful in planning a lesson:

1. Draw up a concept map related to the lesson concept to be taught.
2. Note the curriculum requirements for the grade being taught, in the curriculum context for the phase.
3. Note misconceptions that could arise in relation to the content to be covered in the lesson.
4. Plan the lesson, with learners in mind, thinking of tasks, questions to ask and activities for the learners to work on.

[11] International Competitions and Assessments for Schools (ICAS) is conducted by Educational Assessment Australia (EAA), University of New South Wales (UNSW) Global Pty Limited. Students from over 20 countries in Asia, Africa, Europe, the Pacific and the USA participate in ICAS each year. EAA produces ICAS papers that test students in a range of subjects including mathematics.

1. Concept map – Area (Grade 5)



2. Curriculum content (This is the South African CAPS document curriculum specification). The grade 5 content is highlighted since this is the level at which we are planning. Grade 4 content gives an indication of prior knowledge learners should have on the topic.

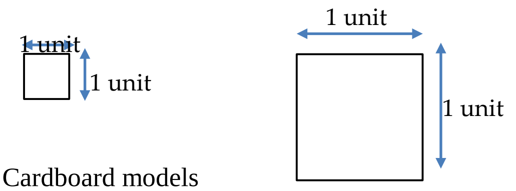
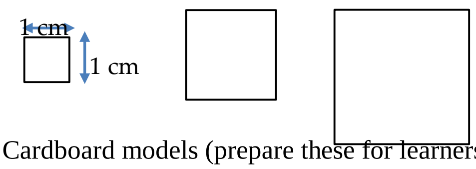
Measurement of area

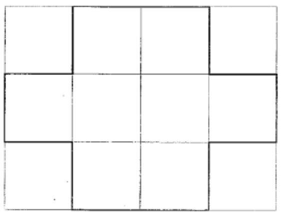
Grade 4	**Grade 5**	Grade 6
Find areas of regular and irregular shapes by counting squares on grids in order to develop an understanding of square units	**Find areas of regular and irregular shapes by counting squares on grids in order to develop an understanding of square units**	Continue to find areas of regular and irregular shapes by counting squares on grids Develop rules for calculating the areas of squares and rectangles

3. Misconceptions that may arise:
    a. Learners may confuse area (2-D surface covered) and perimeter (the distance around a 2-D shape).
    b. Learners may not know how to count units of area in a given shape – may not yet have a fully formed concept of area (amount of surface covered).
    c. Confusion about the size of the squares in the grid – what to count?

Lesson Plan: Grade 5 Area of shapes **Duration 40 minutes**

Teacher Action	**Learners' action**	**Teacher further action**	**Time**
Ask questions about area. Can you remember what the word "area" means? Write answers from learners on the board (Could be in the form of a concept map) Ask for illustrations, such as floor tiles, window panes, bricks, etc.	Learners recall what they have learned about area.	Learners might remember perimeter and confuse with area. Refer to examples to demonstrate the difference between the two concepts (area is amount of surface covered, perimeter	10 minutes.

Teacher Action	Learners' action	Teacher further action	Time
		is the length around the shape).	
Show some squares of different sizes on the board and discuss the concept: a square unit.  Cardboard models Talk about the surface covered by the square as a square unit.	Class discussion	Learners might not understand the word "unit" in this context. Do lots of counting of the square units that cover the shape – to show how these are being counted as "units of area". Use cardboard tiles and stick them onto the shape to make this even clearer.	
Activity 1 Hand out a worksheet which includes activities to calculate the areas of rectangular or square shapes drawn on grid paper. Question 1: 3 activities Discuss answers to activities	Learners will write down answers on the worksheet.	Circulate and assist. Allow learners to voice their own questions and try to get them to express solutions in their own words.	5 minutes.
Activity 2 Question 2 of worksheet Grid paper on which squares are shaded to create irregular shapes with different areas. Learners count the shaded squares to find the area of each shape.	As above	As above	5 minutes.
Activity 3 Question 3 of worksheet Show a square unit and a bigger block where learners have to superimpose the square unit on the bigger block to calculate the area.  Cardboard models (prepare these for learners) These units are square centimetres – since the dimensions are given.	As above	Assist learners to discover the relationship between the sizes of the blocks as they get bigger. Make sure they work with the cardboard models so that they can see this relationship concretely. Discuss the size of the units (square centimetres).	5 minutes.
Activity 4 Set the "Holly" task for the class. Put it up on the board (prepare on cardboard).	Leaners work in groups to solve the problem. Learners record their solutions on the chalk board.	Allow learners time to work on the activity and then discuss as a class.	10 minutes.

Teacher Action	Learners' action	Teacher further action	Time
Holly drew this shape on 2 cm grid paper.  What is the area of Holly's shape? A) 32 cm^2 B) 28 cm^2 C) 16 cm^2 D) 8 cm^2 Discuss each group's solutions			
Wrapping up the lesson: Discuss the concept of area – • What is area? • How do we find it? • What is a square unit? • How do we measure area?	Class discussion	Allow learners to answer your questions. Look out for any learners who struggle to answer the questions or who cannot answer them correctly. Make a note of learners who struggle and work with them later to ensure that they grasp the concept fully.	5

Activity

Activity 9

Read carefully through the lesson plan given in the table above and then think about the following questions:

c) Why is it important to prepare concrete models before a lesson such as this one on area?
d) In what ways does the teacher plan to engage with learners in this lesson?
e) Does the lesson plan address the topics in the concept maps? Identify the topics that are addressed and the ways in which they are addressed.

Commentary:

Concrete aids are especially useful in mathematics lessons where the concepts can be modelled. The aids allow the teacher to demonstrate what she is teaching rather than just present ideas in an abstract manner. The teacher has prepared activities but has also readied herself to engage with the learners in class discussions as well as individually when they are working on the set activities. This lesson has covered all of the items in the concept map – all of which are related to the teaching and learning of the concept of area (area, amount of surface covered, grid, counting squares, squares units, square centimetres).

Activity

Activity 10

Plan your own lesson using the four steps given above to focus your thoughts. Select a topic that you are currently teaching for this activity and trial the lesson.

a) Draw up a concept map related to the lesson concept to be taught.
b) Note the curriculum requirements for the grade being taught, in the curriculum context for the phase.
c) Note misconceptions that could arise in relation to the content to be covered in the lesson.
d) Plan the lesson, with learners in mind, thinking of tasks, questions to ask and activities for the learners to work on.

Commentary:
This is an open ended task that will enable you to see if you have started to work with learners' misconceptions in your own lesson planning, preparation and teaching. After you have taught the lesson use the questions in the reflection box below to reflect on (think carefully about) the lesson planning, preparation and teaching.

Reflection

- Was it helpful to draw up a concept map related to the topic? If so, why?
- What insight did the curriculum document give you about what you had to teach?
- Which misconceptions did you anticipate would arise in the lesson?
- Did any anticipated (or other) misconceptions arise and if so, how did you deal with them?

Conclusion

This unit has:

- provided you with six examples of teachers dealing with errors during teaching;
- suggested different bases and foci one could follow when planning a lesson;
- introduced the idea of concept map as a tool for lesson planning;
- demonstrated a way of planning a lesson with misconceptions in mind; and
- concluded with an example of a detailed lesson plan.

Evidence shows that dealing with errors is a difficult task but it is one which can become easier with practice (see references below). Planning for lessons with errors in mind is critical so that teachers can be ready to address learners' misconceptions.

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Using Assessment Data to Improve Practice

Findings from the DIPIP project (2011-2013)

Karin Brodie

Introduction

In a time of increased accountability and monitoring of teachers and schools in the form of learner testing, particularly the Annual National Assessments (ANA), it is encouraging that calls to use assessment data for professional development are being made and heeded by government and others working in Education. This movement acknowledges that assessment in and of itself cannot improve the system. Agents (teachers, teacher-educators and district officials) within the system need to know what to do with assessment data – we cannot improve a system with only measurement and accountability demands.

In this learning brief, I take the argument one step forward, by drawing on experiences and findings from the Data Informed Practice Improvement Project (DIPIP), which was set up to help high school mathematics teachers use assessment data for their own professional development. I argue that assessment data needs to be utilised in particular ways in order to be useful for teachers and that it needs to be complemented with other forms of classroom data in order to support teachers in significantly shifting their practices.

Using data for professional learning

The assessment for learning movement suggests that assessment data can be used for individual learners, teachers and schools to establish areas of strength and weakness in learning, teaching and organisational practices, as well as to think about improving practice (Katz, Earl, & Ben Jaafar, 2009). This is consistent with the notion of professional learning, where professionals use data available to them together with knowledge from research, in order to develop new ideas about teaching and learning.

Key to professional learning is that local data provides for identification of local needs, while research data situates the local in terms of broader research findings (Jackson & Temperley, 2008). The connection of local with global happens in conversation, among teachers within and across schools, and with expertise from outside of the schools, from district subject advisors, non-government projects or university-based professional developers. Outside expertise is important in that it prevents the mere recycling of current explanations and practices and provides a means to refresh and re-invigorate practice (McLaughlin & Talbert, 2008). In the South African context, the DIPIP project is particularly interested in removing blame for learner under-achievement from both teachers and learners.

The DIPIP project has developed a set of activities, which supports teachers in using assessment and other classroom data to understand learner errors in mathematics (Brodie & Shalem, 2011). The focus on learner errors serves two purposes. First, teachers are already attuned to learner errors from their summative assessment practices, but do not necessarily know how to engage with learner errors productively to improve their teaching and their learners' learning. Second, learner errors often reveal partially valid reasoning among learners, which teachers can work with to develop more sophisticated mathematical understandings. It is important for teachers to access the reasoning behind learner errors in order to understand their learners' strengths and weaknesses in mathematics and subsequently improve their practice to build on the strengths and transform the weaknesses.

DIPIP activities and conversations

Data on learner errors comes from a number of sources, including tests, classroom tasks and talking to learners. Six DIPIP activities use these sources to support teachers to understand the thinking behind their learners' errors. The activities are:

1. Analyse test data. Teachers analyse ANA or other test scripts together, including international tests administered to their learners, and establish which errors are prevalent within and across grades, within school and across schools. Establishing that there are systematic and pervasive errors suggests underlying conceptual problems for the learners that go beyond particular teachers and learners (Nesher, 1987). Teachers also hypothesise what the reasoning behind the errors might be, looking for valid as well as invalid reasoning among learners.

2. Interview learners. Teachers select and interview learners who made particular errors. The interviews serve two purposes. First, they allow the teachers to understand some learners' thinking in more depth, i.e. to understand the reasoning behind errors on a script. Second, they serve as a model for teachers to talk with learners in class, where they can identify learner reasoning behind errors made in class.

3. Identify concept. Teachers identify a "leverage" concept, which underlies a number of errors they have seen. Examples of leverage concepts include the notion of a variable, the equal sign, factorization and ratio and rate.

4. Read and discuss research. Teachers are provided with research papers on the concept and errors they have identified. The papers often show how the particular errors are found among learners in other contexts, including "developed" countries, therefore removing blame from teachers and learners about making errors. The papers suggest reasons for the errors based on research, helping the teachers to identify whether their learners' reasoning is similar or different, and sometimes suggesting teaching methods to engage with and transform the errors into stronger mathematical understandings.

5. Plan and teach lessons. Drawing on the research and other resources, teachers collaborate to plan a series of four to five

lessons in order to surface and engage with the common learner errors in the chosen concept. They teach the lessons and are videotaped teaching them.

6. Reflect on lessons. Teachers reflect on their videotaped lessons within and across schools. They look for episodes where learner errors come up in class and reflect on whether they engaged with those errors in productive or non-productive ways. They discuss ways in which they could engage with learners' thinking more productively in the future.

All of the above activities take place in professional learning communities, where teachers talk to each other with the guidance of a facilitator. For the first two years the facilitators came from Wits, and in the third year we have been handing over facilitation to school-based facilitators, who have participated in the communities for some time and who are being trained for their facilitation roles.

The key function of the facilitator is to help teachers to develop their pedagogical content knowledge; particularly by predicting and engaging with learner thinking through diagnosing learner errors. Facilitators also help teachers to develop their content knowledge, when weaknesses in content knowledge emerge during discussions of learner thinking (Brodie, 2012).

Findings

An independent evaluation of the DIPIP project, the project team's own research and the experience of the project team and participating teachers over the past three years suggest the following:

Working in structured ways on the DIPIP activities in professional learning communities does produce gains in teacher knowledge and practices. These gains can be seen in the first year of participation for many teachers; however it takes another two years in the project for significant changes to strongly take root and be sustained.

Developing understandings of the reasons for learner errors, in order to inform teaching, is not easy. It takes time and strong facilitation to support teachers in developing this skill. This is very different from the kind of tick-the-box analysis that the department currently requires that teachers do of the ANA tests. These analyses tend to promote, rather than prevent, blaming and condemning of learners and teachers. In order to understand why learners make the errors that they do make, and for teachers to take responsibility for these without blaming themselves or learners, they need to understand the reasoning behind the errors.

Working with international tests is a useful exercise. These support teachers to see different kinds of questions and imagine different possibilities for assessment. They also enable teachers to see that these tests can be used as learning experiences, rather than for condemning our education system, as they have been used in the past.

The full range of activities may be necessary for teachers to begin engaging with learners' errors in class. The error analysis and learner interviews enable teachers to begin to see learner thinking in more depth. However, they are not often able to act on these new insights in the first round of classroom teaching. Reflection on their videotaped lessons during the first round helps teachers to see how they might have responded differently in class. So error analysis without subsequent follow-up lesson planning and reflection is unlikely to be useful.

Reading research papers is particularly empowering for teachers. They constantly report that the papers help them to see their learners and their mathematical knowledge in new ways, thus connecting local data with global research. In particular seeing common challenges across contexts helps teachers to remove blame from their learners and themselves and see that their challenges can be overcome through collaboration.

The analysis of Grade 9 ANA results at the end of the year means that the learning may only be in time for the next year's cohort of learners, or that teachers in subsequent years need to revisit the previous year's errors. Ongoing analysis of teacher's own tests means that lesson planning on the basis of the analysis will not fit into the curriculum and will require teachers to re-visit previous work. For this reason teachers often prefer to learn how to do ongoing analysis of their classroom teaching in order to act on learner errors sooner rather than later.

The support of a professional learning community within and across schools is vital to the process of learning from learner thinking. Teachers appreciate the support that other teachers give them, both in understanding their learners' thinking and in thinking about new ways in which to engage with learner errors.

Over time, the communities express the need to take ownership of the activities and conduct them in a different order, or focus on the activities that best suit their needs. Also, there is evidence of teachers conducting the activities in informal small meetings as well as in the formal DIPIP meetings, which indicates that the work is becoming increasingly part of the regular activities in the school, rather than an add-on professional development exercise.

Teachers find that the skill and knowledge of the facilitator is crucial in sustaining the professional learning that takes place in DIPIP, whether the facilitator is university or school-based. Teachers find that the facilitators support them to develop both their content knowledge, their knowledge of learning and teaching (pedagogical content knowledge) and their practice.

The school-based facilitators found that training to perform their roles was crucial in building their confidence and competence to facilitate the DIPIP activities. This training, as well as continued support from the DIPIP project is seen by most teachers to be essential in sustaining the communities and the activities.

Conclusion

The evidence and experiences of the DIPIP project over the past three years suggest that while teachers can and should learn to use assessment data in ways that inform their practice, this is not an easy process and requires time and thought from both government and professional developers. Teachers trying to analyse data on their own are unlikely to experience much benefit and may resort to simple compliance rather than pushing deeply into the data. Especially when teachers work collaboratively, time and energy are required for this form of professional learning. In particular, expertise in the form of facilitation and external input, through facilitators, research papers and teachers from other schools are necessary to sustain engagement and to bring in new ideas.

Most importantly, working with assessment data is not enough. The distance between understanding assessments and making changes in practice is too large for teachers to travel without the guidance of structured activities and facilitators. In fact it may be both more practical and more beneficial for teachers to learn to analyse errors that occur in classrooms on an on going basis, in order to identify learners' needs and therefore their own learning needs.

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