Assessing Algebra in the Senior Phase: A Practical Guide

Saide

South African Institute
for Distance Education

Assessing Algebra
in the Senior Phase:
A Practical Guide

Saide

South African Institute
for Distance Education

This booklet demonstrates
how assessment can be used
in learning and teaching
Mathematics in
the Senior Phase.

Acknowledgments

Contents

About *Assessing Algebra in the Senior Phase: A Practical Guide*

This booklet was first developed in 2004, shortly after the introduction of the Revised National Curriculum Statement in South Africa. The intention of the booklet was to help teachers of senior phase (junior secondary) to integrate assessment into their teaching and learning.

Although the curriculum in South Africa has now changed, the purpose of assessment has not, and we think that teachers will find very useful the practical approach to planning, implementing and reflecting on assessment results.

As the booklet is an Open Educational Resource, we invite teachers or teacher educators to take, use and adapt it for whatever curriculum they are currently teaching. All we request is that you acknowledge the document as originally developed by Saide.

We would also like to hear what you think of this booklet, and especially how you use it. Please contact us at info@saide.org.za.

Saide

1. Introduction

In this booklet, we look at assessment in the Mathematics classroom. We focus particularly on algebra in the Senior Phase. Our main aim is to place assessment where it belongs – as an integral part of the whole teaching and learning cycle. We use some of the work of a teacher, Mrs Mothae, to illustrate some ideas about assessment and to help you to reflect on your own assessment practice.

We aim to provide you with resources to enable you to:

● Set tasks to assess whether a learner has achieved intended outcomes

● Assess learners' work, and record and interpret their results

● Use the results of assessment tasks to inform the ongoing teaching and learning process

---

Look out for these icons:

Task:
Complete an activity

Comment:
An idea is explained or described.

Reflect:
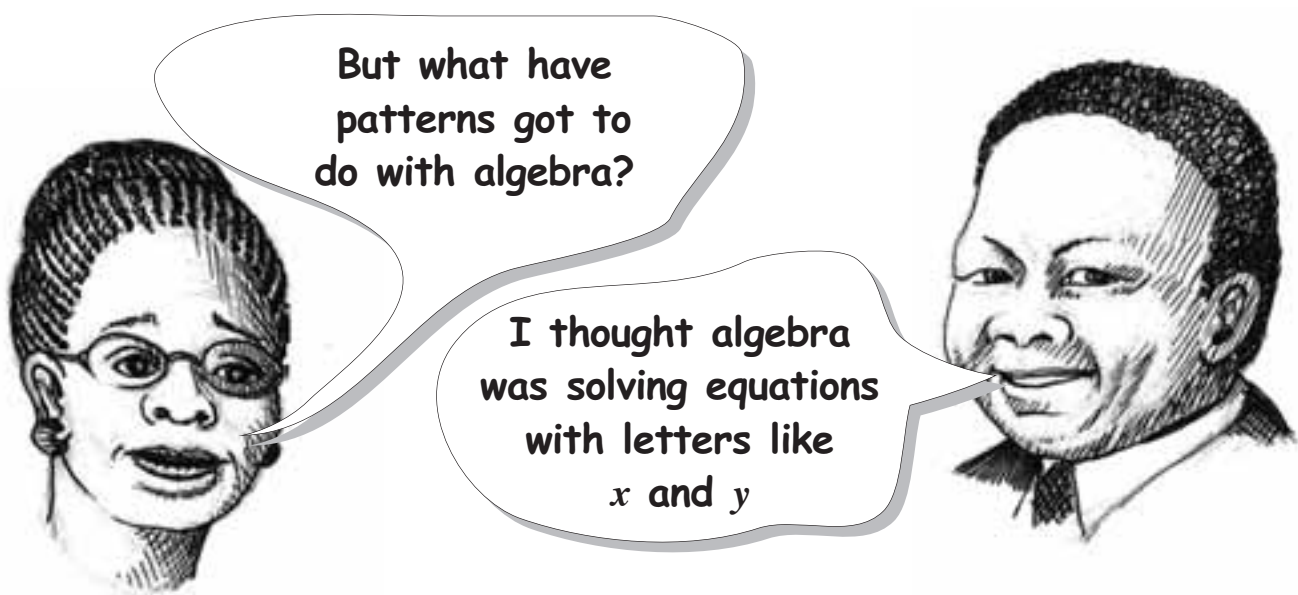Discuss and record your thoughts and ideas

---

Saide

2. The Mathematics Focus of this Booklet

In this booklet we focus on an outcome that is common to many Maths curricula:

> *The learner is able to recognise, describe and represent patterns and relationships, and solve problems using algebraic language and skills.*

This was Learning Outcome 2 in the South African National Curriculum Statement, and we think it is a meaningful way to introduce algebra in the Senior Phase.

But what have patterns got to do with algebra?

I thought algebra was solving equations with letters like x and y

Well, patterns provide a meaningful reason to use letter symbols. When learners **complete a pattern**, and then **describe** it, this enables them to find a **general rule** for the pattern. Then they can **use letter symbols** to put that general rule into mathematical language.

In the South African curriculum, learners are introduced to the teaching of algebra through patterns. Although there are other ways of introducing learners to algebra, this way has been found to be very successful.

Have a look at Table 1 on the next page. The outcome is unpacked into so-called 'assessment standards' for each of the grades in the Senior Phase.

● **Do you notice how similar the Assessment Standards are across the grades?**
● **Do you notice how the Assessment Standards in each grade build onto the previous grade's work?**
● **In general terms, what would you need to teach your learners in order for them to achieve these Assessment Standards? What do the Assessment Standards expect learners to be able to do as they progress toward the outcome?**

Saide

Table 1: Assessment standards for an introduction to algebra

Note: This table gives learning outcome 2 with the relevant assessment standards from the South African National Curriculum Statement. It can, however, be adapted for other national curricula.

Learning Outcome 2: The learner is able to recognise, describe and represent patterns and relationships, and solve problems using algebraic language and skills		
Grade 7 We know this when the learner:	**Grade 8** We know this when the learner:	**Grade 9** We know this when the learner:
• Investigates and extends numeric and geometric patterns looking for a relationship or rules, including patterns: 　• Represented in physical or diagrammatic form 　• Not limited to sequences involving constant difference or ratio 　• Found in natural and cultural contexts 　• Of the learner's own creation 　• Represented in tables • Describes, explains and justifies observed relationships or rules in own words • Represents and uses relationships between variables in order to determine input and/or output values in a variety of ways using: 　• verbal descriptions 　• flow diagrams 　• tables • Constructs mathematical models that represent, describe and provide solutions to problem situations, showing responsibility toward the environment and the health of others (including problems within human rights, social, economic, cultural and environmental contexts).	• Investigates and extends numeric and geometric patterns looking for a relationship or rules, including patterns: 　• Represented in physical or diagrammatic form 　• Not limited to sequences involving constant difference or ratio 　• Found in natural and cultural contexts 　• Of the learner's own creation 　• Represented in tables 　• Represented algebraically • Describes, explains and justifies observed relationships or rules in own words or algebraically • Represents and uses relationships between variables in order to determine input and/or output values in a variety of ways using: 　• verbal descriptions 　• flow diagrams 　• tables 　• formulae, equations and expressions • Constructs mathematical models that represent, describe and provide solutions to problem situations, showing responsibility toward the environment and the health of others (including problems within human rights, social, economic, cultural and environmental contexts).	• Investigates in different ways, a variety of geometric and numeric patterns and relationships by representing and generalising them and by explaining and justifying the rules that generate them (including patterns found in natural and cultural forms and patterns of the learner's own creation) • Represents and uses relationships between variables in order to determine input and/or output values in a variety of ways using: 　• verbal descriptions 　• flow diagrams 　• tables 　• formulae, equations and expressions • Constructs mathematical models that represent, describe and provide solutions to problem situations, showing responsibility toward the environment and the health of others (including problems within human rights, social, economic, cultural and environmental contexts).

When we looked at the Assessment Standards in Table 1, we identified several teaching and learning steps that are needed if learners are to achieve the standards:

1. Learners need to be able to **complete** patterns, by finding relationships between numbers. They also need to be able to complete patterns that are represented in tables.
2. Then they need to be able to **describe these patterns verbally** – this leads to 'generalising' or looking for the 'general rule'.
3. Then they need to be able to **use letter symbols** to write down their general rules in a mathematical way. They also sometimes need to use **flow diagrams** and graphs to do this.
4. Lastly, they need to use these steps to solve a problem.

The example below illustrates a problem that takes learners from patterns, to making rules, to using variables (expressed as letter symbols) to solve a problem:

Sharing out the bread

Mrs Moroko bakes a loaf of bread.

If she has to divide it between only herself and Mr Moroko, she would make **1** cut in the bread:
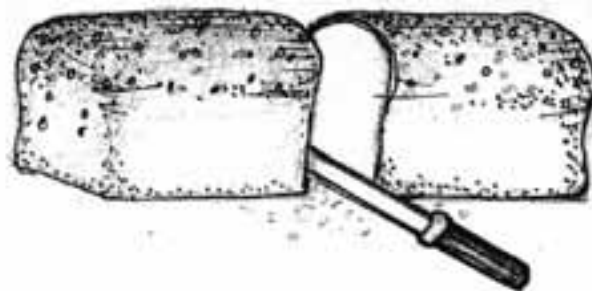
However, if her daughter comes to visit her that day, she would need to make **2** cuts in the bread in order to share it between **3** of them:

Her daughter brings her granddaughter with her! Now she needs to make **3** cuts in the bread to share it between **4** of them.

Saide

Complete this table:

Number of people	2	3	4	7	10	?
Number of cuts	1	2	3	?	?	14

- Write down a sentence to explain what the **pattern or general rule** is for working out how many cuts Mrs Moroko must make.
- If Mrs Moroko wants to share the bread between x people, how many cuts must she make?
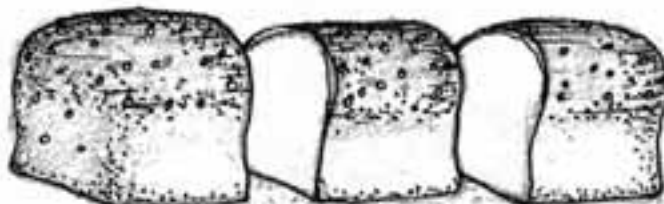- If Mrs Moroko makes y cuts in the bread, to how many people can she offer a piece of bread?

In this problem, there are two things that change – the number of cuts and the number of people. These are called the variables. Learners should be able to discover the general rule that the number of cuts is always one less than the number of people. They should be able to write this rule using letter symbols for the variables. This would be $y = x - 1$
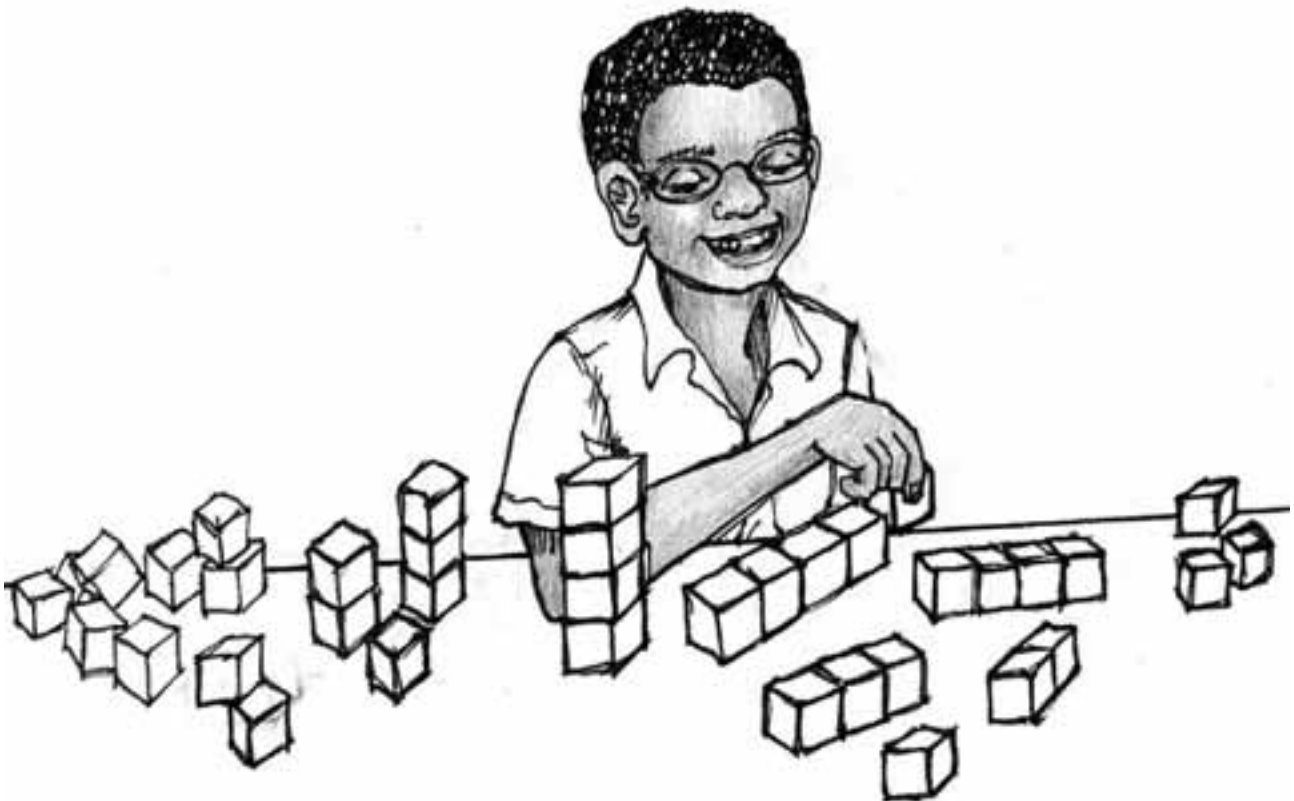
3. The Assessment Focus of this Booklet

3.1 The Purpose of Assessment

Even though the purposes for assessment of learners are many and varied, the main purpose is to give learners opportunities for growth and development. This includes assessing learners to facilitate their learning.

What this means is that assessment is only effective and purposeful if it is an integral part of the entire teaching and learning cycle. This is what we focus on in this booklet.

3.2 The teaching and learning cycle

How do we make assessment an integral part of the teaching and learning process?
The steps described below suggest how we can work with an ongoing cycle of planning, facilitating learning and assessing.

1. Plan

In planning our work, we start by asking: What do the learners need to know? In other words, what outcomes do we want them to achieve? We ask ourselves: What learning opportunities should we provide for learners so that they can achieve the learning, and produce evidence to show that they have achieved the outcome?

Planning involves three steps:
- Consult your term or year plan to see which learning outcomes/objectives to use for teaching and assessing. You need to plan your teaching to cover concrete, clearly defined assessment standards or sub-topics within the broad topic or outcome/objective.
- Plan a series of lessons, with teacher input and learner activities, that will give learners the opportunity to develop the knowledge and skills described in the assessment standards or sub-topics.
- Design an assessment task that gives learners a chance to prove (give evidence) that they have learnt and achieved the outcomes or mastered the topics you selected.

2. Facilitate Learning

In this part of the cycle, learners have an opportunity to work toward the outcomes. The learners **do** the learning activities that you have chosen. This is an ongoing process that includes teaching (giving them input), mediating their learning, observing their progress and assisting them to achieve the outcomes.

This step will equip the learners with the knowledge, skills and values needed in order to achieve the outcomes.

Saide

3. Assess

When the learners have had enough opportunity to acquire the skills and knowledge that you have planned, they are ready to complete an **assessment** task. The task should enable you to **assess** how far the learners are in achieving the outcomes.

In this part of the cycle you also:

- **Record** the results of the assessment.

- **Analyse** the group's results to see how they can be helped further towards achieving the outcomes and to inform your planning for the next step of learning needed. It is important to note trends in the class as a whole, their strengths and weaknesses.

- You provide **feedback to learners** to help them identify their own problem areas. If a learner can identify what her problem is, she can begin to work on improving her knowledge and skills. Feedback must be given in a spirit of encouragement and support, emphasising what the learner is able to do and building on this. Negative criticism can just entrench negative attitudes in learners.

- At times, **report** this assessment to parents, staff or others who need to know.

4. Reflect and Decide

In this part of the cycle you use learners' responses to **reflect** on your own teaching, and to **plan** the details of the next step of the Learning Programme. You may need to think of alternative ways to make the learning easier, provide more learning tasks or redo tasks to assist those learners who have not yet achieved the outcomes.

- *Do you think that the cycle described above reflects your understanding of assessment?*

- *Are here any other steps you think should be included?*

- *In what ways do you think this cycle reflects what you currently do in your classroom?*

4. Let's get Practical

The Teaching and Learning Cycle

1. Plan

Choose learning outcomes/topics and the more detailed assessment standards or sub-topics.
- What must learners know and be able to do?
- What are the outcomes?
- What learning opportunities can you provide?
- How can you assess what learners have achieved?

Plan

2. Facilitate Learning

Teaching and learning takes place.
- Learners engage in learning activities.
- You assess learners informally while you teach, observe and mediate.

Facilitate Learning

3. Assess

The learners' achievement is assessed and results recorded and interpreted.
- Record your assessment.
- Have the learners achieved the intended outcomes?
- Identify strengths and weaknesses.
- Provide constructive feedback.

Assess

4. Reflect and Decide

If the group of learners has satisfactory results, you can move to the next step in your plan. Individuals or the whole group may need concepts reinforced and another opportunity to be assessed in some way.

Reflect & decide

Saide

4.1 The teaching and learning cycle in Mrs Mothae's class

Meet Ms Sibongile Mothae. She teaches Maths at Rainbow Park Secondary School. This is how she has used the teaching and learning cycle.

"First, I used my year plan to plan in more detail for the term. We had planned to focus on Learning Outcome 2: 'The learner is able to recognise, describe and represent patterns and relationships, and solve problems using algebraic language and skills' for this term. I agreed with the four key steps described on page 10 and used them in my planning.

I looked at how this outcome was unpacked in my curriculum (see Table 1) across Grades 7, 8, and 9. I thought about what I wanted my Grade 8 learners to know and identified Assessment Standards that they needed to achieve. After designing learning activities, I set four tasks that would assess whether learners had achieved these Assessment Standards at certain key points in the programme."

COMPLETE PATTERNS	DESCRIBE AND GENERALISE	USE LETTER SYMBOLS	SOLVE PROBLEMS
Task 1:	Task 2:	Task 3:	Task 4:
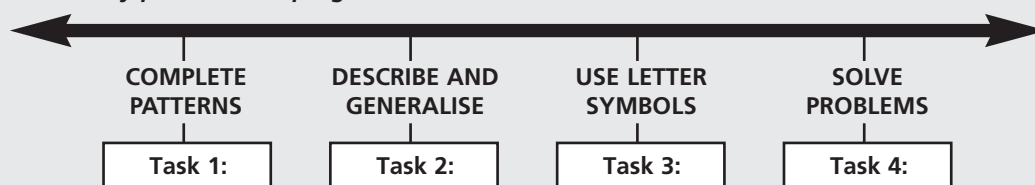
"For two weeks we worked with number and shape patterns in class. The learners completed patterns in a range of contexts and designed their own patterns, too. At first, I needed to help many of them. I also found that they needed to be reminded about how to use decimal fractions. I made notes about their progress as I went along."

"When I was confident that learners were ready to consolidate their work, I gave them Assessment Task 1 to complete. Then I marked their work, recorded their results and gave feedback. I considered the strengths and weaknesses of individuals, and suggested what they needed to do to improve their work."

"I then looked at the ways in which the class as a whole had managed the questions in the task. I thought about why many had problems with certain questions, and what I could do to help. I changed my original plan to accommodate these ideas."

4.2 Assessment in Mrs Mothae's Maths class

In the previous section we saw how Mrs Mothae used the teaching and learning cycle in her work. In the rest of this booklet we will consider in particular on Steps 3 and 4 of this cycle. In this section we pay greatest attention to step 3 – assessment. Our discussion focuses on Mrs Mothae's use of Assessment Task 1.

❖ ANALYSING ASSESSMENT TASK 1

All assessment tasks need to be carefully planned to:
- assess what was stated as the outcomes of the teaching and learning at the level that is expected from your grade.
- include enough questions that allow the "average" learner to succeed, with some challenging questions to extend the learners.
- provide opportunities for you to diagnose problem areas and consolidate previous Maths knowledge.

Mrs Mothae tried to set tasks that would put all of these principles into practice! We have included Task 1 and the other three Assessment Tasks in appendices in a size suitable for you to copy for your own class if you wish. In this booklet, however, we will only consider how Mrs Mothae uses Assessment Task 1 to assess her learners, to analyse their strengths and weaknesses and to adjust her Learning Programme based on this information. For easy reference, we have shown Assessment Task 1 on the next page.

Do Assessment Task 1

1. *A good way for you to get to grips with Assessment Task 1 is for you to do it yourself. So, as a first step, you should read through the task and answer the questions yourself.*

2. *As you do the task, think about some of the problems that might face your learners when they do the task.*

Saide

Assessment Task 1

Note: Marks will be awarded for showing that you can complete or describe the patterns correctly, and for the accuracy of your answers.

Total Marks: 50

(24 marks)

1. Fill in the missing numbers to complete these patterns

a) 15; 30; 45; ___; ___; ___; ___

b) 72; 65; 58; ___; ___; 64;

c) ___; ___; 1; 4; 16; 64;

d) ___; 81; 27; 9; 3; ___;

e) ___; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; ___

f) ___; 789; 678; 567; ___; ___; 23 379

g) ___; 23 079; 23 179; ___; ___

h) ___; ___; 1; 2; 4; 7; ___; ___;

i) ___; ___; 2.05; 2.1; 2.15; ___; ___

j) 10h50, 11h40, 12h30, ___;

2a) The numbers in this grid are arranged in patterns. Use this pattern to find the values of A, B, C, D and E

(1 × 5 = 5 marks)

```
        162   A           A .
   B   27  54  108  C     B .
        9   18             C .
        3       12         D .
     D      E   4          E .
```

(1 × 4 = 4 marks)

b) Describe the patterns in 2(a):

i) from left to right

ii) from right to left

iii) from top to bottom

iv) from bottom to top

3a) Gogo Zuma loves to cook her food in a three-legged pot over the fire. She discovers that many of her neighbours also use these pots. Work out how many legs there are on the number of pots listed in the table below

(7 marks)

Number of pots	1	2	3	4	8	13	50	200
Number of legs								

3b) Explain how you would work out what the number of legs is for any number of pots

(2 marks)

4. For each pattern below:

a) draw the part that comes immediately before and immediately after the three parts shown here

b) complete the matching number pattern for each shape

(3 × 2 = 6 marks)

(1 × 8 = 8 marks)

Pattern A

4   9   16

Pattern B

8   12   16

Pattern C

8   10   12

Think About Assessment Task 1

1. *Which questions In Assessment Task 1 did you find surprising, interesting, challenging or easy?*

2. *What can Assessment Task 1 tell you about the learner's ability to meet the Assessment Standards from Learning Outcome 2 (listed on page 5)?*

3. *What other Maths knowledge does the learner need in order to complete Assessment Task 1?*

4. *Suggest different ways in which you could use this assessment task in your classroom.*

5. *Match each of these statements about Task 1 correctly with the questions from Task 1.*
    Note: There may be more than one answer for each statement and the same question from the task may apply to more than one of the statements. We have matched the first statement with the questions as an example.

6. *Review your answers to questions 2 - 4 in the light of these statements.*

Learners need to understand simple fractions to be able to do these questions. They also need to multiply and divide accurately.	Questions 1(c), (d) and (e)
Learners need to understand decimals to be able to do these questions.	
Most learners in the Intermediate Phase should be able to complete these simple addition and subtraction patterns with little difficulty.	
To complete this pattern to the left, learners need to divide by 4.	
To complete the pattern to the left, learners need to multiply by 3; to complete the pattern to the right, they need to divide by 3 at each step.	
Learners must be able to add simple fractions. Even learners who are able to add quarters might not show the consistency in the pattern and fill in the zero (some may put one eighth as their answer)*.	
Learners must be able to add three digit numbers. There is a constant difference of 111 between consecutive numbers in the pattern. A good learner would show an understanding of place value and fill in 900 as the first number in the pattern.	
Some learners may have looked for the patterns **in the digits** instead of the whole numbers. The first digits are 7, 6, and 5; the second digits are 8,7 and 6 respectively. However, continuing the pattern in this way to the left would lead to the incorrect answer.	
This question needs an understanding of place value in large numbers.	
This question tests the learner's ability to be consistent in the pattern, but this time by subtracting zero from one to find the answer to the left of one. The result is that there are two consecutive '1s' in the pattern.	
In this question, learners have to show they can continue patterns in two dimensions or directions simultaneously.	
In these questions, the learner needs to explain the patterns, instead of just using the pattern. This is a necessary skill before being able to use variables in algebra.	
Learners have to extend shape patterns and match them to number patterns	

Saide

Here are some of our ideas about the questions asked on Assessment Task 1. Compare them with yours.

Comment

■ **Match each of these statements about Task 1 correctly with the questions from Task 1.**

We have given our ideas about question 5 first. We found that finding the links showed us some of the aspects of the Assessment Standards of Outcome 2 that Task 1 focuses on. They also give a sense of the other mathematical knowledge embedded in the questions. We found that by looking in this sort of detail at the questions in Task 1, we were better able to answer the more general questions that were asked about the task as a whole.

We thought that:

The statements and questions are linked like this:

Learners need to understand simple fractions to be able to do these questions They also need to multiply and divide accurately.	Questions 1(c), (d) and (e)
Learners need to understand decimals to be able to do these questions	Question 1(i)
Learners continue simple addition and subtraction patterns. Most learners in Intermediate Level should be able to complete these patterns with little difficulty.	Question 1(a) and (b)
To complete this pattern to the left, learners need to divide by 4	Question 1(c)
To complete the pattern to the left, learners need to multiply by 3; to complete the pattern to the right, they need to divide by 3 at each step	Question 1(d)
Learners must be able to add simple fractions. Even learners who are able to add quarters might not show the consistency in the pattern and fill in the zero (some may put one eighth as their answer).	Question 1(e)
Learners must be able to add three digit numbers. There is a constant difference of 111 between consecutive numbers in the pattern. A good learner would show an understanding of place value and fill in 900 as the first number in the pattern.	Question 1(f)
Some learners may have looked for the patterns **in the digits** instead of the whole numbers. The first digits are 7, 6, and 5; the second digits are 8,7 and 6 respectively. However, continuing the pattern in this way to the left would lead to the incorrect answer.	Question 1(f)
This question needs an understanding of place value in large numbers.	Question 1(g)
This question tests the learner's ability to be consistent in the pattern, but this time by subtracting zero from one to find the answer to the left of one. The result is that there are two consecutive '1s' in the pattern.	Question 1(h)
In this question, learners have to show they can continue patterns in two dimensions or directions simultaneously.	Question 2(a)
In these questions, the learner needs to explain the patterns, instead of just using the pattern. This is a necessary skill before being able to use variables in algebra.	Question 2(b) and 3
Learners have to extend shape patterns and match them to number patterns	Questions 3 and 4

■ **What can Assessment Task 1 tell you about the learner's ability to meet the Assessment Standards from Learning Outcome 2?**

We thought that:

- The task gives learners several opportunities to practise completing patterns, consolidate this ability and achieve what the first Assessment Standard for Grades 7 and 8 requires ('the learner investigates and extends numeric and geometric patterns looking for a relationship or rules').

- The patterns in the task include some that are in 'diagrammatic form', some that do not involve a constant difference or ratio (question 1(i), (j), (k)) and some that are 'represented in tables'. These are some, but not all of the kinds of patterns noted under this first Assessment Standard.

- Questions 2(b) and 3 require the learner to describe an observed relationship in words. These questions link to the second Assessment Standard. If a learner can explain patterns using words, it gives an insight into their reasoning. Reflecting on how the class as a whole manages Questions 2(b) and 3 will help you when you teach this aspect to them. (Assessment Task 2 deals more thoroughly with this)

> **Comment**
>
> **If a learner performs well on this task, this may show that he or she is able to extend simple patterns. However, in this task alone, we have not gathered enough reliable and valid evidence yet to say that the learner has achieved Learning Outcome 2 as a whole. Can you see that several of the Assessment Standards have not been included? In your planning for the year, you will need to be sure that for each grade you teach, all the relevant Assessment Standards have been covered – but they do not all have to be covered in one assessment task. Remember how Mrs Mothae thought about this in her planning.**

■ **What other Maths knowledge does the learner need in order to complete Assessment Task 1?**

We thought that:

- The number skills of earlier grades (covered in Learning Outcome 1) are needed to investigate and complete the patterns set in Assessment Task 1. We thought this was a good idea as it meant that the task can help identify learners with problems with these number skills.

- Some of the questions required learners to show that they understood that zero is a number in its own right. We think it is important that learners understand this, and that zero does not merely mean the absence of an amount. So, we thought it was a good idea to include several questions for which the answer is zero in patterns as there is a surprising reluctance to write zero as an answer.

Saide

- Some number concepts such as negative numbers and complex fractions, which are required in Grade 8, are not included in the task. The fraction knowledge expected here is simple. We thought this was a good idea as the main purpose of the exercise is to test pattern extension – and we would not have wanted to handicap learners who lacked more advanced fraction knowledge.

> **Comment**
>
> **When you are marking the learners' work, you need to keep the different skills and knowledge involved in mind. Although the primary focus in Task 1 is on learners' ability to recognise and complete patterns, at the same time it assesses learners' understanding of other mathematical knowledge and skills. You need to interpret learners' answers – to see exactly what it is they can and cannot do. Are they understanding patterning, but struggling with say, fractions? Are they able to add time accurately, but unable to see the patterns in the time question? Your analysis of this will help you know how to help the learners progress.**

■ Suggest different ways in which you could use this assessment task in your classroom.

Mrs Mothae used this task to assess her Grade 8 learners after two weeks of learning and teaching. It provided her with a **formal assessment** of the learners' achievement. But it could be used in other ways, too.

We thought that:

- It could be used as a **worksheet** rather than as a formal assessment task. Using the task in this way would give more opportunities to assist the learners when they have difficulties. We thought this would be particularly appropriate in a Grade 7 class.

- The task could also be used as a **baseline assessment** for Grade 9 learners who should be familiar with all the work covered. The task could provide a useful starting point for beginning algebra in Grade 9. It would give valuable information about the Grade 9 learners and their areas of difficulty.

> **Comment**
>
> **It is important to adapt learning materials and the way you use them according to the needs of your classes - and this is true, too, of Task 1 and your use of it. If you haven't yet covered a certain concept that is in Task 1 at a time when you want to use it for formal assessment, then you need to change the task accordingly. Leave out those items – replace them with something you have covered. Or you may decide to spend class time on that concept before giving learners the task. While you do need to be sure that all of the Assessment Standards for a grade are covered in the year, it is pointless rushing on to cover an Assessment Standard if you have not given learners enough opportunity to understand and achieve those standards which learners need to build the new standard on. Thus, if learners struggle with Assessment Standards from Outcome 1, such as those dealing with fractions, for example, they will probably not be able to recognise patterns involving these and you will need to consolidate this learning first.**

❖ MARKING ASSESSMENT TASK 1

> I would like you to meet three of my learners, Ahmed, Thembeka and Pedro. They have completed Task 1

As we have shown, the way learners answer the questions in Task 1 will give valuable information about how well they have achieved the learning that preceded their doing the task. Before such an analysis is possible, though, the work has to be marked. In order to make sense of Mrs Mothae's work, we would like you to complete the task below

Mark three learners' work

Pedro, Thembeka and Ahmed's completed assessment tasks are shown on the next pages. Of course, there will be a greater variety of learner responses than those shown here in any class. However, these three represent many of the most likely learner responses.

1. *Mark these learners' work as if they were in your class. Refer to the marking memorandum on page 36. To help each learner, write comments onto their scripts that will help them to see what they need to work on. Calculate their total mark out of 50.*

Saide

Themiseku

Assessment Task 1

Note: Marks will be awarded for showing that you can complete or describe the pattern correctly and for the accuracy of your answer.

Total Marks: 50

(24 marks)

1. Fill in the missing numbers to complete these patterns:

a) 15; 30; 45; 60 ; 75 . 90

b) 72; 68; 58; _51_ ; _34_ . _37_

c) _18_; 1; 4; 16; _1_ ; _4_

d) ÷3 ; 81; 27; 9; 3; _1_ ; _1/3_

e) ...

f) 870; 789; 678; 567. 456

g) 21179; 21 079; 21 179; _21 214_ ; 21 179

h) ; 1; 2; 4; 7; _10_ ; _13_

i) 2,15 ; 2 05; 2,1; 2,15; _2,20_ . _2,25_

j) 10450; 11840; 12450; _13L20_; _14h10_

2a) The numbers in this grid are arranged in patterns. Use the patterns to find the values of A, B, C, D and E.

(1 × 5 5 marks)

```
          162   A       A = 324
      8  27  54  108  C   B = 13 1/2
          9   18        C = 216
          3      12      D = 0
      D      E   4       E = 7
```

(1 × 4 = 4 marks)

a) Describe the patterns in 2(a)

i) from left to right   × 2   ii) from right to left   ÷ 2

iii) from top to bottom   × 3   iv) from bottom to top   ÷ 3

3a. Gogo Zuma loves to cook her food in a three-legged pot over the fire. She discovers that many of her neighbours also use these pots. Work out how many legs there are on the number of pots listed in the table below.

(3 marks)

Number of pots	1	2	3	4	5			
Number of legs	3	6	9	12	24	39	150	600

3b) Explain how you would work out what the number of legs is for any number of pots

× 3

(2 marks)

4. For each pattern below
a) draw the pots that comes immediately before and immediately after the three pots shown here (3 × 2 = 6 marks)
b) complete the matching number pattern for each shape (3 × 2 = 6 marks)

Pattern A

1 . 4 . 9 . 16 . 25

Pattern B

8 . 12 . 16

Pattern C

6 . 8 . 10 . 12 . 14

Assessment Task 1

Note: Marks will be awarded for showing that you can complete or describe the patterns correctly, and for the accuracy of your answers.

Total Marks: 60

(21 marks)

1. Fill in the missing numbers to complete these patterns.

a) 15; 30; 45; ___; ___; ___
b) 72; 68; 58; ___; ___; 64; ___
c) ___; ___; ___; 9; 5; ___
d) ___; 81; 27; ___; ___; ___
e) ___; ___; ___; ___; ___
f) ___; 789; 678; 567; ___; ___; 23 374
g) ___; 23 079; 23 179; ___; ___; ___
h) ___; 1; 2; 4; 7; ___; ___; ___
i) ___; 2 05; 2,1; 2,15; ___; ___; ___
j) 10h50; 11h50; 12h50; ___; ___

(1 × 5 = 5 marks)

2a) The numbers on this grid are arranged in patterns. Use the patterns to find the values of A, B, C, D and E

```
      162   A              A = 
   B   27  54  108   C     B = 
        9   18             C = 
          3    12          D = 
       D    E   4          E = 
```

(1 × 4 = 4 marks)

a) Describe the patterns in 2(a).

i) from left to right

ii) from top to bottom

a) from right to left

b) from bottom to top

3a. Gogo Zuma loves to cook her food in a three-legged pot over the fire. She discovers that many of her neighbours also use these pots. Work out how many legs there are on the number of pots listed in the table below.

(3 marks)

Number of pots	1	2		4		8	13	50	200
Number of legs	3	6	9	12	15		21	24	

3b) Explain how you would work out what the number of legs is for any number of pots

(2 marks)

4. For each pattern below:
a) draw the part that comes immediately before and immediately after the three parts shown here
b) complete the matching number pattern for each shape
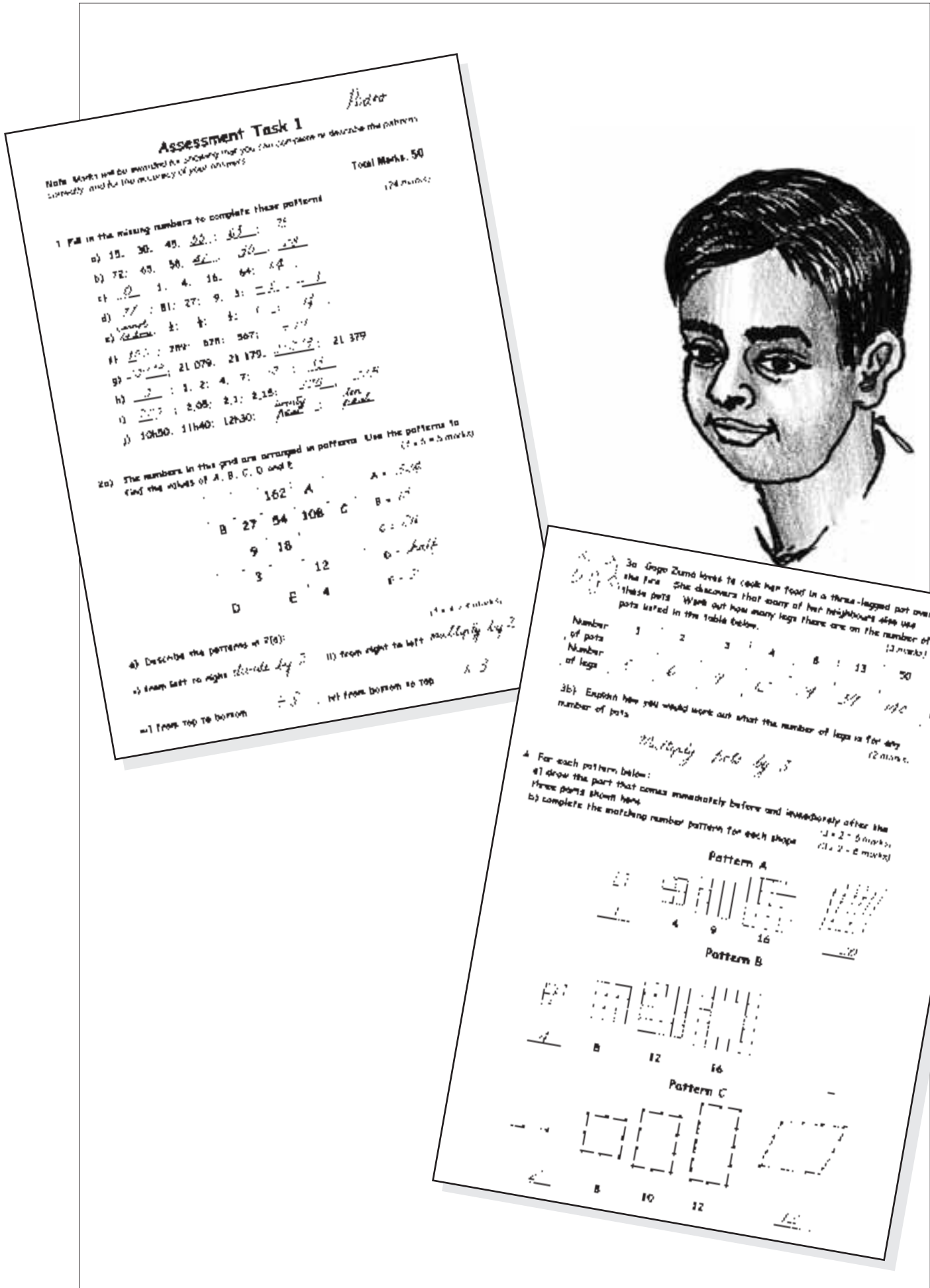
(3 × 2 = 6 marks)
(3 × 2 = 5 marks)

Pattern A

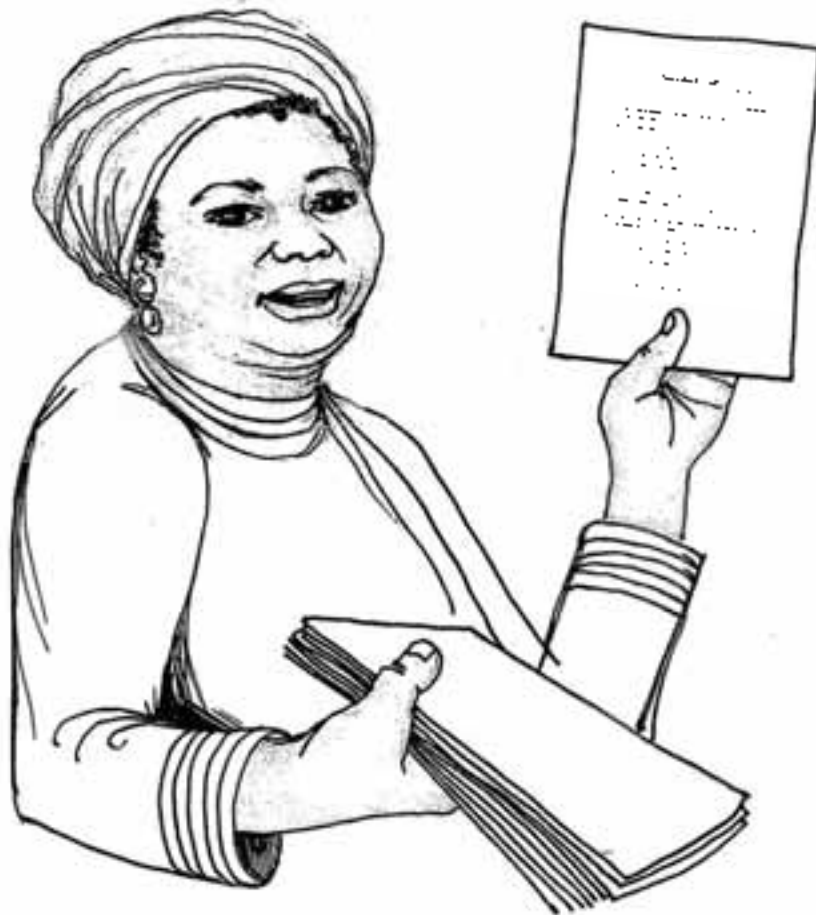1   4   9   16   ___

Pattern B

8   12   16   ___

Pattern C

6   8   10   12   ___

Saide

Assessment Task 1

Note: Marks will be awarded for showing that you can compare or describe the patterns correctly, and for the accuracy of your answers

Total Marks: 50

(24 marks)

1. Fill in the missing numbers to complete these patterns

a) 15, 30, 45, __, __, __

b) 72, 65, 58, __, __, __

c) __, 1, 4, 16, 64, __

d) __, 81, 27, 9, 3, __

e) __, __, __, __, __

f) __, __, 679, 567, __, __, 21 379

g) __, 21 079, 21 179, __, __

h) __, 1, 2, 4, 7, __, __

i) __, 2,05, 2,1, 2,15, __, __

j) 10h50, 11h40, 12h30, __, __

2a) The numbers in this grid are arranged in patterns. Use the patterns to find the values of A, B, C, D and E.

```
          162   A          A =
      B   27  54  108  C    B =
          9    18          C =
        3         12       D =
          D   E   4        E =
```

b) Describe the patterns in 2(a):
   i) from left to right: divide by 3   ii) from right to left: multiply by 2
   iii) from top to bottom: ×3   iv) from bottom to top: ÷3

3a) Gogo Zuma loves to cook her food in a three-legged pot over the fire. She discovers that many of her neighbours also use these pots. Work out how many legs there are on the number of pots listed in the table below. *(3 marks)*

Number of pots	1	2	3	4	8	13	50	200
Number of legs								

3b) Explain how you would work out what the number of legs is for any number of pots. *(2 marks)*

multiply pots by 3

4. For each pattern below:
   a) draw the part that comes immediately before and immediately after the three parts shown here
   b) complete the matching number pattern for each shape

Pattern A

1   4   9   16   __

Pattern B

4   8   12   16

Pattern C

8   10   12   __

Compare Mrs Mothae's marking with yours

Thembeka – 33

Ahmed – 31

Pedro – 26

- *Compare Mrs Mothae's marking of the three learners' papers with yours. Although your marks may differ slightly from hers, they should be similar.*

- *We have shown the comments she made on Pedro's script. In what ways do you think they are helpful? How could they be made more helpful?*

Saide

Themba

Assessment Task 1

Name _____

Total Marks: **50**   83

1. Fill in the missing numbers to complete these patterns.

a) 15; 30; 45; _6C_; _75_; _9C_.

b) 72; 68; 58; 51; _44_; _37_.

c) 1; 4; 16; 64; 256

d) 81; 27; 9; 3; _1_; _⅓_.

e) _4_; _-¼_; _4_

f) 789; 678; 567; _212_ ...; 21 379

g) 21 079; 21 179; _21 279_; _21 379_

h) 1; 2; 4; 7; _11_; _16_

i) _1.50_; 2.05; 2.1; 2.15; _2.20_; _2.25_

j) 10h50; 11h40; 12h30; 13h20; 14h13.

2a) The numbers in this grid are arranged in patterns. Use the patterns to find the values of A, B, C, D and E.

```
              162   A        A : 324
       B  27  54  108  C     B : 13½
          9   18             C : 216
          3       12         D : 0
       D       E   4         E : 2
```

b) Describe the patterns in 2(a)

i) from left to right   X 2

ii) from right to left   ÷ 2

iii) from top to bottom   _9_

3a. Gogo Zuma loves to cook her food in a three-legged pot over the fire. She discovers that many of her neighbours also use these pots. Work out how many legs there are on the number of pots listed in the table below.

Number of pots	1	2	3	4	8	13	50	200
Number of legs	3	6	9	12	24	39	150	600

3b) Explain how you would work out what the number of legs is for any number of pots.

_X 3_

4. For each pattern below:
 a) draw the part that comes immediately before and immediately after the three parts shown here.
 b) complete the matching number pattern for each shape.

Pattern A

_1_   4   9   16   _25_   25

Pattern B

8   12   16

Pattern C

_6_   8   10   12   _14_

Saide

Assessment Task 1

Note: Marks will be awarded for working that is complete or accurate to describe the patterns correctly and for the accuracy of your answers.

Total Marks: 50

(24 mins)

1 Fill in the missing numbers to complete these patterns

a) 15, 30, 45, ___, ___, ___
b) 72, 65, 58, ___, ___, ___
c) ___, 1, 4, 16, 64, ___
d) ___, 81, 27, ___, 3, ___
e) ___, ___, ___, ___
f) ___, 789, 678, 567, ___, ___
g) ___, 21 079, 21 179, ___
h) ___, 1, 2, 4, 7, ___
i) ___, 2.05, 2.1, 2.15, ___
j) 10h50, 11h40, 12h30, ___

2a) The numbers in this grid are arranged in patterns. Use the patterns to find the values of A, B, C, D and E.

```
          162   A
     B  27  54  108  C
         9   18
         3       12
      D       E   4
```

A =
B =
C =
D = half
E =

a) Describe the patterns in 2(a)
i) from left to right (multiply by 2)
ii) from top to bottom
iii) from right to left

3a. Gogo Zuma loves to cook her food in a three-legged pot over the fire. She discovers that many of her neighbours also use these pots. Work out how many legs there are on the number of pots listed in the table below.

(3 marks)

Number of pots	1	2	3	4	8	13	50	200
Number of legs	3	6	9	12	24	39	150	600

3b) Explain how you would work out what the number of legs is for any number of pots.

Multiply pots by 3

(2 marks)

4. For each pattern below:
   a) draw the part that comes immediately before and immediately after the three parts shown here.
   b) complete the matching number pattern for each shape.

(3 × 2 = 6 marks)
(3 × 2 = 6 marks)

Pattern A

___ , 4, 9, 16, 20

Pattern B

4, 8, 12, 16, ?

Pattern C

2, 8, 10, 12, 12

❖ RECORDING AND INTERPRETING THE RESULTS

Focusing on individual learners

A formal assessment needs to be recorded as part of your measure of the learner's progress through the year. However, recording is not enough on its own. You need to use the results to understand each learner's areas of difficulty and to plan for the next cycle of teaching and learning. In marking the three learners' work, you have allocated them an overall mark for the task as a whole. How helpful is this? The questions below will help you think about this further.

Thinking about using marks to record assessment

1. *Does the learner's total mark indicate to what extent she is progressing towards achieving the outcome? In other words, does the mark show to what extent she is successfully completing patterns?*

2. *Does the learner's total mark indicate where she has managed well and where she has struggled?*

3. *Can the marks help you to identify misunderstandings or gaps in the learner's knowledge that might have led to incorrect answers?*

4. *Is there a way of marking this test that would show how the learner has done in each separate type of question? What could you do to record information about learner achievement that would be more helpful than a mark alone?*

Saide

Mrs Mothae found that the marks gave her a general idea of whether each learner could complete patterns, but they did not show her a learner's areas of strength and weakness, or where the gaps in their knowledge might be.

Mrs Mothae decided to identify and record how learners had done in the different skills that were being tested in Task 1. She used these as criteria to help her assess to what extent her learners had achieved the outcomes she had set for the task. We will focus on choosing criteria and using them for recording assessment in more detail in another issue of this series of booklets, but you may find it useful to think about her system here.

She noted 6 criteria that focus on patterning skills, and the questions that gave her evidence of learners' achievement of these. These are:

1.	Extend simple patterns to the right	1(a) – (j)
2.	Extend simple patterns to the left	1(a) – (j)
3.	Extend patterns horizontally and vertically	2(a)
4.	Describe simple patterns verbally	2(b) and 3(d)
5.	Complete a pattern in table form	3(a)
6.	Extend shape patterns	4

Mrs Mothae also chose 4 criteria to help her assess whether any learners need help with an understanding of fractions, decimals, large numbers or time.

7.	Use fractions to complete patterns	1(c), (d), (e)
8.	Use larger numbers to complete patterns	1(f) and (g)
9.	Use place value and decimal numbers correctly	1(i)
10.	Extend patterns of time	1(j)

Although this way of recording learner achievement looks complicated at first, it can be efficient and is not as complicated as it seems! Mrs Mothae kept her class list next to her and ticked off what her learners could do as she marked. Using this method, she decided not to use the marks at all! On the next page, you will see how Mrs Mothae completed her records for Pedro and some other learners.

Using a more qualitative system of recording

1. *Use Mrs Mothae's system and your marking to complete the records for Ahmed and Thembeka in Table 2 on the next page.*

2. *Then use the records and the learners' scripts to help you to identify each learner's strengths and weaknesses, and any gaps in knowledge or misunderstandings.*

3. *Suggest what information this system gives about the learners that only recording marks does not.*

Table 2: Mrs Mothae's record sheet

The learner can	Ahmed	Thembeka	Pedro	Portia	Natalie	Kobus
1. Extend simple patterns to the right 1(a) – (j)			✗	✓	✓	✓
2. Extend simple patterns to the left 1(a) – (j)			✗	✗	✓	✗
3. Extend patterns horizontally and vertically 2(a)			✓	✓	✓	✓
4. Describe simple patterns verbally 2(b) and 3(d)			✓	✗	✓	✓
5. Complete a pattern in table form 3(a)			✓	✗	✓	✓
6. Extend shape patterns Question 4			✗	✗	✓	✗
7. Use fractions to complete patterns 1(c), (d), (e)			✗	✗	✓	✗
8. Use large numbers to complete patterns 1(f) and (g)			✓	✓	✗	✓
9. Use place value and decimal numbers correctly 1(i)			✗	✗	✓	✓
10. Extend patterns of time 1(j)			✗	✗	✓	✓

Comment

- Each **column** gives you immediate information about how a **particular learner** has done on the set of criteria. This shows each learner's strengths and weaknesses across the range of skills and knowledge being assessed. It also gives you a picture of whether the learner has achieved what the whole task was assessing – can the learner extend and describe patterns?
- Each **row** indicates to you whether your **class** has grasped a particular skill or if they still need more work on this.

Analysing assessment information

Read what Mrs Mothae noticed about three of her learners.

1. Think about how her analysis compares with yours.

2. What would you say about Portia, Natalie and Kobus?

Saide

Ahmed

Understands patterning; struggles with fractions; needs to learn time notation; didn't notice that the pattern to the left is different from the pattern to the right in 2(b); struggled with patterning in table form and shape patterns.

Thembeka

Misunderstood that the pattern must be completed to the left as well in question 1 (c), (d), (e); struggled with large numbers; needs to be shown the pattern in (h); good work on shape patterns and tables (although needs help on 4(b)).

Pedro

Missed the pattern because he didn't look at all the numbers in 1(a) and (b); recognised patterns in (c) to (f) but wrote down rules instead of completing the pattern; needs to be reminded about conventions for decimal numbers and for time; stronger on finding the rule, shape patterns and completing the table.

Mrs Mothae used the information about each learner she has gathered thus far to decide on a general assessment of how well each learner is capable of completing and describing patterns (Learning Outcome 2) at this stage in her learning programme. She used ratings of 1 to 4 to record this general assessment of how well each learner has achieved this outcome. She also used the information she gained from the assessment of their work to write a comment about each learner's particular strengths and weaknesses. The table below shows how she did this.

Table 3: Mrs Mothae's rating sheet

	Ahmed	Thembeka	Pedro	Portia	Natalie	Kobus
Completing and describing number patterns	3	3	2	2	4	3
Completing and describing shape patterns	2	3	2	2	3	2
Comments	Needs practice using tables and shape patterns	Struggles with large numbers & completing patterns to left	Can recognise number patterns, but can't complete them			

The South African National Curriculum Statement provides the following codes for recording learners' levels of achievement of outcomes.

4 = Learner's performance has exceeded the requirements.

3 = Learner's performance has satisfied the requirements.

2 = Learner's performance has partially satisfied the requirements.

1 = Learner's performance has not satisfied the requirements.

Focusing on the group as a whole

Mrs Mothae looked at her learners' results to see if she could identify trends within the class as a whole. These would help her to decide how to move forward.

Finding trends

1. *Which questions did the three learners, Pedro, Ahmed and Thembeka, all do correctly? What does this indicate about their knowledge and skills?*

2. *Which questions did these three learners do poorly? What does this indicate about knowledge and skills that they are lacking?*

It is difficult to analyse general trends from only three learners' work. When you do this exercise with a larger group, the trends will be clearer. However, even with only these learners' work to consider, some general trends are clear.

- All three learners identified most of the number patterns in question 1, although only Ahmed saw the pattern in 1(h).

- They all saw the patterns in the grid in question 2, although they confused multiplying and dividing when asked to describe the patterns.

- Two of the learners completed the table correctly.

- All three learners were unable to do 1(c) and (d) where they needed to move from whole numbers to fractions;

- Two learners struggled with large numbers in 1(g);

- Two learners struggled with writing time correctly in 1(j);

- They all struggled to complete the gaps on the left of the patterns correctly, although they were able to continue them to the right.

- All the learners struggled with the shape patterns.

Thinking about what help learners need

1. *In what areas of the work do you think the learners in Mrs Mothae's class need help? Have a look again at the grid showing which standards each of the 6 learners has achieved to see the trends more clearly.*

There are three areas in which learners need help
- continuing patterns to the left;
- continuing shape patterns
- using fractions.

The learner can	Ahmed	Thembeka	Pedro	Portia	Natalie	Kobus
1. Extend simple patterns to the right 1(a) – (j)						
2. Extend simple patterns to the left 1(a) – (j)			✗	✓	✓	✓
3. Extend patterns horizontally and vertically 2(a)			✗	✗	✓	✗
4. Describe simple patterns verbally 2(b) and 3(d)			✓	✓	✓	
5. Complete a pattern in table form 3(a)			✓	✗	✓	✓
6. Extend shape patterns Question 4			✓	✗	✓	✓
7. Use fractions to complete patterns 1(c), (d), (e)			✗	✗	✓	✗
8. Use large numbers to complete patterns 1(f) and (g)			✗	✗	✓	✗
9. Use place value and decimal numbers correctly 1(i)			✓	✓	✗	✓
10. Extend patterns of time 1(j)			✗	✗	✓	✓
			✗	✗	✓	✓

Saide

❖ USING THE ASSESSMENT RESULTS TO PLAN

An essential part of the teaching and learning cycle is deciding how to use the knowledge you have gained from assessing the learners (Step 4 on pages 9 and 10) You need to do two things.

- Firstly, think about what the assessment suggests about how you might better teach this section of work another time. What strengths and weakness can you now anticipate, and how can you use this knowledge to plan your teaching better?
- Secondly, think about what you can do to help the learners whose work you have just assessed. How should you adjust your teaching plan to support their learning better? You cannot spend too long on going over work that has not been understood, but it can be damaging to the learners' progress to just ignore their difficulties. They are likely to struggle with the next section of the work. How can you help both individuals and the group as a whole? How can you extend those learners who are clearly able to do the work well?

Think about what to do to support learning

1. *Make suggestions about how you would address the problems that have been identified in the learners' work.*

2. *Compare your ideas with what Mrs Mothae did.*

This is what Mrs Mothae did to address these areas of concern:

- She showed the class how the patterns that they completed in the task continue in both directions. She helped the class see that the rule they find if they work to the right is the inverse of the rule they will use if they work to the left. She gave them some additional examples to work on in pairs to check that they had understood.

- She realised that she could not just give learners different versions of the same shape patterns, so she spent a whole lesson letting learners explore shape patterns by building the patterns with cardboard shapes, investigating their answers and explaining them in groups. This gave them a concrete experience of shape patterns.

- She had to remind them of previous work done with fractions and gave them a homework exercise on adding and subtracting simple fractions.

- She offered to make an extra time outside of classroom time to work with two learners who still struggled. She built up their confidence by working with simpler patterns first.

- She set some tasks to stretch the capabilities of the strongest learners, and arranged for them to work together in a group on some occasions when other groups of learners worked on simpler tasks.

❖ THE CYCLE IS ON-GOING

After some work such as that described above, Mrs Mothae was aware that she needed to move on to the next step in her plan – verbal and algebraic expression of patterns (ie generalisations). She continued with the planned classwork, but included more examples of shape patterns and patterns involving fractions. The learners worked in pairs and had to keep explaining what they were doing while they worked. They also began using letter symbols for variables to generalise the patterns. Mrs Mothae assessed them informally, mediating, assisting and observing their progress.

By the time she used Assessment Task 2, most of her learners were able to explain and use number and shape patterns, although some still struggled with using variables. Those who still struggled with fractions attended extra lessons after school.

In Assessment Task 2, Mrs Mothae set questions that asked learners to explain the relationship between different numbers in patterns. If you look at this task in Appendix 2, you will see that some of the same patterns of Assessment Task 1 are asked in a similar way in order to consolidate their knowledge. From question 2 on, the learners may start by using words to explain, but by the end should be able to show that they can use mathematical operations and variables (this addresses steps 2 and 3 identified on page 6).

As before, if the learners' performance on Assessment Task 2 showed that they could do this work, then Mrs Mothae felt that she could move on to the work planned next, leading up to Assessment Task 3 and finally Assessment Task 4. Learners need to have worked with flow diagrams and graphs before they can complete Assessment Task 3, and Mrs Mothae made sure she gave them plenty of opportunity to do this, mediating their learning.

Assessment Task 4 assesses learners' ability to apply their understanding to a problem, and again, learning opportunities were provided for this. While learners worked, Mrs Mothae checked on how they were doing, made informal notes about their progress and intervened to help and extend them where appropriate. As we described for Task 1, after each of the following assessment tasks Mrs Mothae checked more formally on what learners' work told about their achievement of the Assessment Standards their work was focusing on, and their strengths and weaknesses, and spent some time dealing with aspects of concern before moving on. She also built additional support of work already covered into her planned new work to help consolidate learning where she now realised this was needed.

You may want to use these assessment tasks in your teaching and learning cycle, adapting them to suit your needs

Conclusion

We hope that you found this booklet useful. It showed you how one teacher tried to use assessment as part of the teaching and learning cycle. Mrs Mothae did not find it easy, but she enjoyed trying new ideas - and then changing them if she finds something that seems to work better. We hope that you will be encouraged to do the same.

Saide

Appendices 1 - 4:
The Assessment Tasks and Answers

Saide

Assessment Task 1

Note: Marks will be awarded for showing that you can complete or describe the patterns correctly, and for the accuracy of your answers.

Total Marks: 50

1. Fill in the missing numbers to complete these patterns   (24 marks)

   a) 15; 30; 45; _____; _____; _____.

   b) 72; 65; 58; _____; _____; _____.

   c) _____; 1; 4; 16; 64; _____.

   d) _____; 81; 27; 9; 3; _____; _____.

   e) _____; $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{4}$; _____; _____.

   f) _____; 789; 678; 567; _____.

   g) _____ 21 079; 21 179; _____; 21 379

   h) _____; 1; 2; 4; 7; _____; _____.

   i) _____; 2,05; 2,1; 2,15; _____; _____.

   j) 10h50; 11h40; 12h30; _____; _____.

2a) The numbers in this grid are arranged in patterns. Use the patterns to find the values of
    A, B, C, D and E   (1 x 5 = 5 marks)

		162	A	
B	27	54	108	C
	9	18		
	3		12	
D		E	4	

A =

B =

C =

D =

E =

   b) Describe the patterns in 2(a):   (1 x 4 = 4 marks)

   i) from left to right: ………………………………   ii) from right to left: …………………………………

   iii) from top to bottom: …………………………   iv) from bottom to top: ………………………………..

Saide

3a. Gogo Zuma loves to cook her food in a three-legged pot over the fire. She discovers that many of her neighbours also use these pots. Work out how many legs there are on the number of pots listed in the table below.

(3 marks)

Number of Pots	1	2	3	4	8	13	50	200
Number of Legs								

3b) Explain how you would work out what the number of legs is for any number of pots.

(2 marks)

....
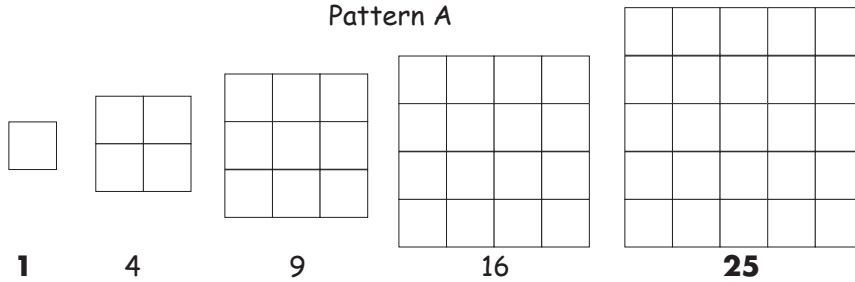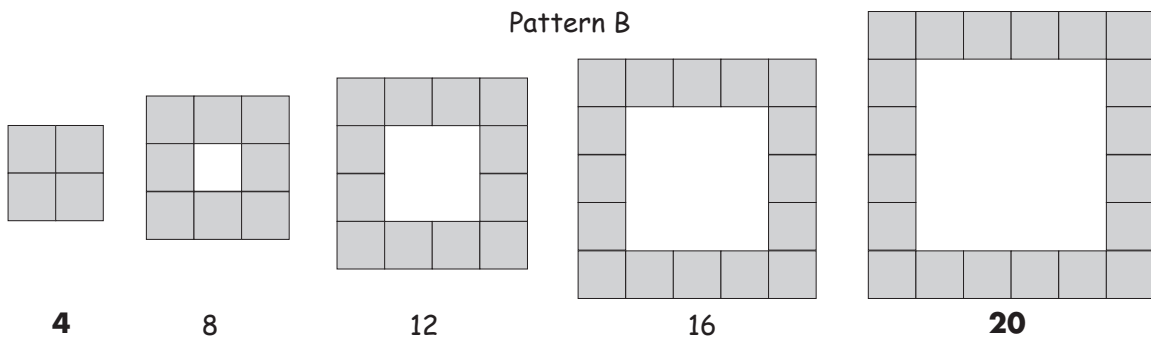
4. For each pattern below:

a) draw the part that comes immediately before and immediately after the three parts shown here.

(3 x 2 = 6 marks)

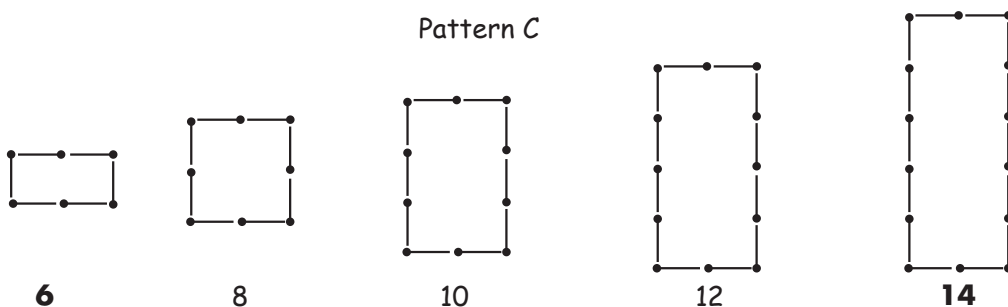b) complete the matching number pattern for each shape.

(3 x 2 = 6 marks)

Pattern A

—— 4   9   16   ——

Pattern B

—— 8   12   16   ——

Pattern C

—— 8   10   12   ——

Saide

Assessment Task 1
with answers and some of the working in bold

***Note:** Marks will be awarded for showing that you can complete or describe the patterns correctly, and for the accuracy of your answers.*

Total Marks: 50

1. Fill in the missing numbers to complete these patterns **(24 marks)**

1 mark for pattern, 1 for correct answer throughout. Extra mark for such things as pattern to left and for pattern with 0 where indicated.

a) 15; 30; 45; **60; 75; 90** (pattern: +15) 2 marks

b) 72; 65; 58; **51; 44; 37** (pattern: -7) 2 marks

c) $\frac{1}{4}$; 1; 4; 16; 64; **256** (pattern: x 4) 3 marks ($\frac{1}{4}$ in first space)

d) **243**; 81; 27; 9; 3; **1; $\frac{1}{3}$** (pattern: ÷ 3) 3 marks ($\frac{1}{3}$ in last space)

e) **0**; $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{4}$; 1; **1$\frac{1}{4}$** (pattern: +$\frac{1}{4}$) 3 marks (0 in first space)

f) **900**; 789; 678; 567; **456** (pattern: -111) 2 marks

g) **20 979**; 21 079; 21 179; **21 279**; 21 379 (pattern: +100) 2 marks

h) **1**; 1; 2; 4; 7; **11**; **16** (pattern: +0, +1, +2...) 3 marks (1 in first space)

i) **2**; 2,05; 2,1; 2,15; **2,2**; **2,25** (pattern: +0.05) 2 marks

j) 10h50; 11h40; 12h30; **13h20**; **14h10** (pattern: +50mins) 2 marks (check format)

2a) The numbers in this grid are arranged in patterns. Use the patterns to find the values of A, B, C, D and E **(1 x 5 = 5 marks)**

40,5	81	162	A	648
B	27	54	108	C
4,5	9	18	**36**	72
1,5	3	**6**	12	24
D	1	E	4	8

A = **324**

B = **13$\frac{1}{2}$ or 13,5**

C = **216**

D = **$\frac{1}{2}$ or 0,5**

E = **2**

b) Describe the patterns in 2(a): **(1 x 4 = 4 marks)**

i) from left to right: **Multiply by 2**   ii) from right to left: **Divide by 2**

iii) from top to bottom: **Divide by 3**   iv) from bottom to top: **Multiply by 3**

Saide

3a. Gogo Zuma loves to cook her food in a three-legged pot over the fire. She discovers that many of her neighbours also use these pots.
Work out how many legs there are on the number of pots listed in the table below.

(3 marks)

Number of Pots	1	2	3	4	8	13	50	200
Number of Legs	**3**	**6**	**9**	**12**	**24**	**39**	**150**	**600**

3b) Explain how you would work out what the number of legs is for any number of pots.

Multiply the number of pots by 3

(2 marks)

4. For each pattern below:

a) draw the part that comes immediately before and immediately after the three parts shown here. (3 x 2 = 6 marks)

b) complete the matching number pattern for each shape. (3 x 2 = 6 marks)

Pattern A

1   4   9   16   **25**

Pattern B

4   8   12   16   **20**

Pattern C

6   8   10   12   **14**

Assessment Task 2

1. Complete the patterns below

    a) _____; _____; 1; 4; 16; _____.

    b) _____; _____; _____; 3; 9; 27.

    c) 1; _____; 2; 4; 7; _____; _____; 22.

    d) 26; 18; 12; _____; _____; 6.

    e) _____; $1\frac{1}{5}$; $2\frac{2}{5}$; $3\frac{3}{5}$; _____; _____.

    f) _____; _____; 10h45; 11h30 12h15; _____; _____.

    g) _____; 4,055; 4,155; 4,255; _____.

    h) 2 250 m; 2 km; _____; _____; 1250 m; 1 km.

    i) _____; _____; 1; 3; 5; 7; _____.

2a) Explain how you would find:

    a) the number of **sides** on a given number of squares...

    b) the **area** of a square of any given side length...

3) Three glasses can be poured from one litre of milk. How many glasses of milk can be poured from a given number of litres?

                   ....

4) One car needs five tyres including the spare. How many cars can be fitted with a given number of tyres?

                   ....

5) Four people can sit around a square table like this one.:

    a) How many people can sit around two tables **set apart**

    b) How many people can sit around 5 tables **set apart**

    c) Describe in words how you would find the number of people that could sit at **any** number of tables **set apart**.

6) If two of the same tables are pushed together, then 6 people can sit around them

Saide

a) Complete the mathematical table below to show how many people can sit around the given number of tables pushed together.
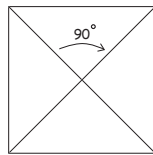
Number of Tables	1	2	3	4	8	15
Number of People						

b) Describe how you would find the number of people that could sit at **any** number of tables pushed together.

....

7. Look at the pattern in the shapes and in the numbers of degrees on each shape.

Shape 1	Shape 2	Shape 3	Shape 4	Shape 5
120°	90°		60°	≈51°
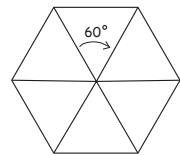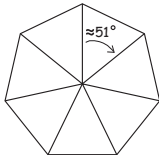
a) Draw Shape 3 in the space above

b) Find the number of degrees shown by the arrow in Shape 3

c) How many degrees would the arrow show if there were a Shape 6

d) How could you find the angle shown by the arrow in any shape in the same pattern?

....

8. In this pattern, three matches form 1 triangle, five matches form 2 triangles.

a) Draw and complete a table to show the number of matches needed to form 1, 2, 3, 4, 20 and 50 triangles

b) Describe how you would find the number of matches needed to form any number of triangles in this way.

c) Draw a line graph to show the relationship between the number of triangles (horizontal axis) and the number of matches (vertical axis). You can use the grid given here.
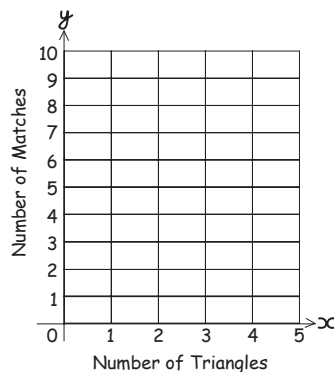
Assessment Task 2
with answers and some of the working in bold

1. Complete the patterns below

    a) $\frac{1}{16}$; $\frac{1}{4}$;   1;   4;   16;   **64**.

    b) $\frac{1}{9}$; $\frac{1}{3}$;   **1**;   3;   9;   27.

    c) 1;   **1**;   2;   4;   7;   **11**;   **16**;   22.

    d) 26;   18;   12;   **8**;   **6**;   6.

    e) **0**; $1\frac{1}{5}$; $2\frac{2}{5}$; $3\frac{3}{5}$;   **$4\frac{4}{5}$** .

    f) **09h15**;   **10h00**;   10h45;   11h30;   12h15;   **13h00**;   **13h45**.

    g) **3,955**;   4,055;   4,155;   4,255;   **4,355**.

    h) 2 250 m;   2 km;   **1750m**;   **1500m (or 1,5km)**;   1250 m;   1 km.

    i) **-3**;   **-1**;   1;   3;   5;   7;   **9**.

2a) Explain how you would find:

    a) the number of **sides** on a given number of squares   **4 x number of squares**

    b) the **area** of a square of any given side length   **Length x length (or ℓ^2)**

3) Three glasses can be poured from one litre of milk. How many glasses of milk can be poured from a given number of litres?

                                  3 x numbers of litres

4) One car needs five tyres including the spare. How many cars can be fitted with a given number of tyres?

                                    Number of tyres divided by 5

5) Four people can sit around a square table like this one.:

    a) How many people can sit around two tables **set apart**?   **2 x 4 = 8 people**

    b) How many people can sit around 5 tables **set apart**?   **5 x 4 = 20 people**

    c) Describe in words how you would find the number of people that could sit at **any** number of tables **set apart**.

                                     Number of people is 4 times the number of tables
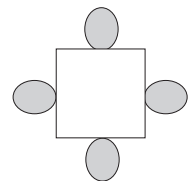
6) If two of the same tables are pushed together, then 6 people can sit around them
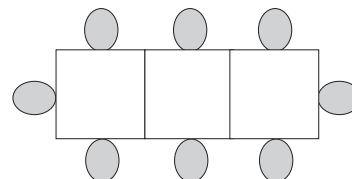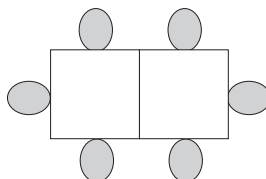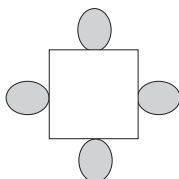
Saide

a) Complete the mathematical table below to show how many people can sit around the given number of tables pushed together.

Number of Tables	1	2	3	4	8	15
Number of People	**4**	**6**	**8**	**10**	**18**	**32**

b) Describe how you would find the number of people that could sit at **any** number of tables pushed together.
Learners answers will vary. One correct answer is:
'twice the number of tables and add 2'

7. Look at the pattern in the shapes and in the numbers of degrees on each shape.
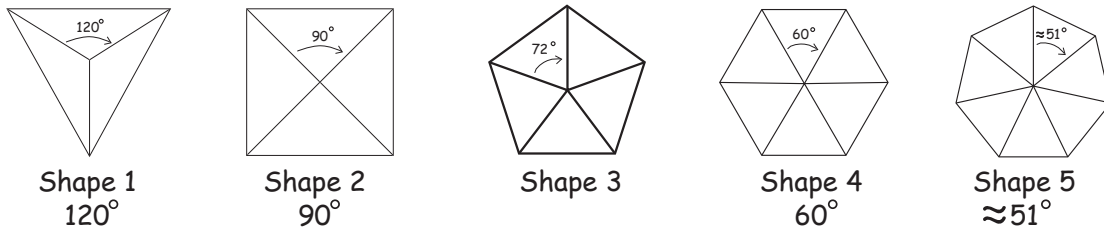
| Shape 1
120° | Shape 2
90° | Shape 3 | Shape 4
60° | Shape 5
≈51° |

a) Draw Shape 3 in the space above
b) Find the number of degrees shown by the arrow in Shape 3
c) How many degrees would the arrow show if there were a Shape 6? $\dfrac{360°}{8} = 45°$
d) How could you find the angle shown by the arrow in any shape in the same pattern?
Angle will be $\dfrac{360°}{\textbf{shape number}} + 2$ **or** $\dfrac{360°}{\textbf{number of sides}}$

8. In this pattern, three matches form 1 triangle, five matches form 2 triangles.

a) Draw and complete a table to show the number of matches needed to form 1, 2, 3, 4, 20 and 50 triangles

Number of triangles	1	2	3	4	20	50
Number of matches	3	5	7	9	41	101

b) Describe how you would find the number of matches needed to form any number of triangles in this way.
(Number of triangles x 2)+1 or 1+(2 x number of triangles)

c) Draw a line graph to show the relationship between the number of triangles (horizontal axis) and the number of matches (vertical axis). You can use the grid given here.
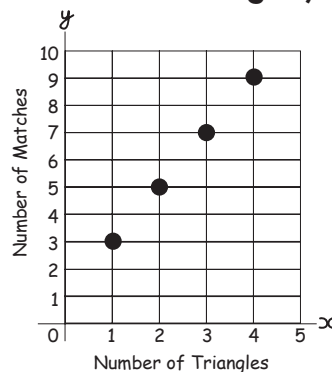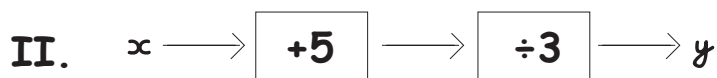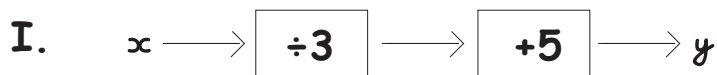
Assessment Task 3

1. Work out:

   a) How many days in x weeks?

   b) How many hours do you sleep in **m** days (If you sleep 8 hours each day)

   c) How many internal angles are there in **t** squares?

   d) What is the surface area of a cube with edge length x?

   e) How many legs on: i)   **p** chickens?

   ii)   **q** cows?

   iii)   **r** snakes?

   f) A taxi holds x people. How many people can fit into y taxis?

   g) How many cuts do you make to divide a sausage into x pieces?

   h) How many quarters in **k**?

2. Look at the two **flow diagrams** below.

   I. $x \longrightarrow \boxed{\div 3} \longrightarrow \boxed{+5} \longrightarrow y$

   II. $x \longrightarrow \boxed{+5} \longrightarrow \boxed{\div 3} \longrightarrow y$

   a) Put a **circle** around all the equations below that show **Flow Diagram I**

$y = \dfrac{x}{3} + 5$	$y = \dfrac{x+5}{3}$	$y = \dfrac{(x+5)}{3}$	$y = (x) + 5 \div 3$
$y = x + 5 \div 3$	$y = x \div 3 + 5$	$y = x \div (3+5)$	$y = \dfrac{x}{5 \div 3}$

   b) In **Flow Diagram I:**   i) if $x = 3$, what is y?   ii) If $x = -3$, what is y?

   c) In **Flow Diagram II:**   i) if $x = 3$, what is y?   ii) If $x = -3$, what is y?

3.   a) Draw a flow diagram to show $y = (2x + 3)^2$

   b) If $x = 9$, what is y?   c) If $y = 9$, what is x?

Saide

4

> To make a cube of edge length 1, you need one small cube
> To make a cube of edge length 2, you need 8 small cubes

a) How many small cubes are needed to make a cube of edge length 3?

b) Complete the table below to show the number of little cubes needed to build a large cube.

Edge length	1	2	3	4	5
Number of small cubes needed	1				

c) How would you find the number of small cubes needed to create a larger cube of any edge length?

d) On squared paper, draw a line graph to show the relationship between the edge length of a large cube (horizontal axis) and the number of little cubes you need to form a the large cube (vertical axis). You can use the grid given here.
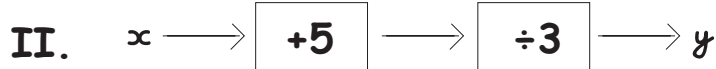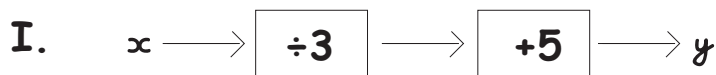
NUMBER OF SMALL CUBES (needed to make up a large cube)

LENGTH OF EDGE OF LARGE CUBE (counted in small cubes)
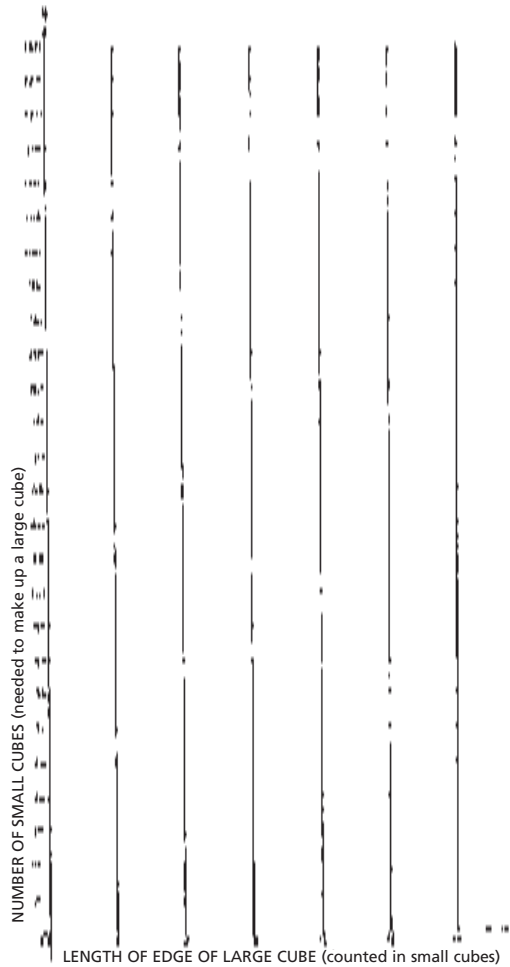
Assessment Task 3
with answers and some of the working in bold

1. Work out:

    a) How many days in x weeks?   **$7x$ days**

    b) How many hours do you sleep in **m** days (If you sleep 8 hours each day)   **8m hours**

    c) How many internal angles are there in **t** squares?   **4t internal angles**

    d) What is the surface area of a cube with edge lenghth x?   **Surface area is $6x^2$**

    e) How many legs on: i) **p** chickens?   **2p legs**

                            ii) **q** cows?   **4q legs**

                            iii) **r** snakes?   **0 (snakes have no legs!)**

    f) A taxi holds x people. How many people can fit into y taxis?   **xy people**

    g) How many cuts do you make to divide a sausage into x pieces?   **x -1 cuts**

    h) How many quarters in **k**? $\dfrac{k}{4}$

2. Look at the two **flow diagrams** below.

    I. $x \longrightarrow \boxed{÷3} \longrightarrow \boxed{+5} \longrightarrow y$

    II. $x \longrightarrow \boxed{+5} \longrightarrow \boxed{÷3} \longrightarrow y$

    a) Put a **circle** around all the equations below that show **Flow Diagram I**

$$y = \frac{x}{3} + 5 \qquad y = \frac{x+5}{3} \qquad y = \frac{(x+5)}{3} \qquad y = (x) + 5 ÷ 3$$

$$y = x + 5 ÷ 3 \qquad y = x ÷ 3 + 5 \qquad y = x ÷ (3+5) \qquad y = \frac{x}{5÷3}$$

    (circled: $y = \dfrac{x}{3} + 5$ and $y = x ÷ 3 + 5$)

    b) In **Flow Diagram I:**   i) if $x = 3$, what is y?   ii) If $x = -3$, what is y?

$$y = \frac{3}{3} + 5 = 1 + 5 = 6 \qquad y = \frac{-3}{3} + 5 = -1 + 5 = 4$$

    c) In **Flow Diagram II:**   i) if $x = 3$, what is y?   ii) If $x = -3$, what is y?

$$y = \frac{(3+5)}{3} = \frac{8}{3} = 2\frac{2}{3} \qquad y = \frac{(-3+5)}{3} = \frac{2}{3}$$

3.   a) Draw a flow diagram to show $y = (2x+3)^2$

    $x \longrightarrow \boxed{\textbf{x 2}} \longrightarrow \boxed{\textbf{+ 3}} \longrightarrow \boxed{\textbf{all squared}} \longrightarrow y$

    b) If $x = 9$, what is y?   c) If $y = 9$, what is x?

    $y = [(2 \times 9)] + 3]^2$ $9 = (2x+3)^2$

        $= (21)^2$ $\sqrt{9} = \sqrt{(2x+3)^2}$

        $= 441$ $\pm 3 = 2x+3$

                                      $2x+3 = 3$ or $2x+3 = -3$

                                      $2x = 0$ $2x = -6$
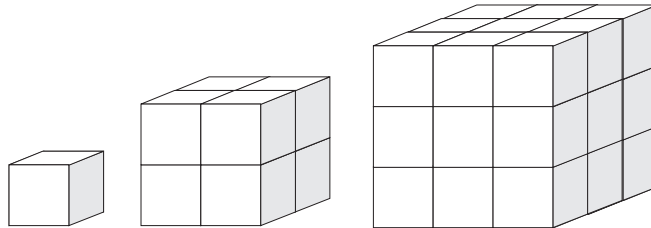
                                      $x = 0$ $x = -3$

Saide

4

To make a cube of edge length 1, you need one small cube
To make a cube of edge length 2, you need 8 small cubes

a) How many small cubes are needed to make a cube of edge length 3?   **27 small cubes**

b) Complete the table below to show the number of little cubes needed to build a large cube.

Edge length	1	2	3	4	5
Number of small cubes needed	**1**	**8**	**27**	**64**	**125**

c) How would you find the number of small cubes needed to create a large cube
of any edge length?   **(Edge length)^3 = x^3**

d) On squared paper, draw a line graph to
show the relationshop between the
edge length of a large cube (horizontal axis)
and the number of little cubes you need to
form a the large cube (vertical axis).

NUMBER OF SMALL CUBES (needed to make up a large cube)

LENGTH OF EDGE OF LARGE CUBE (counted in small cubes)

Saide
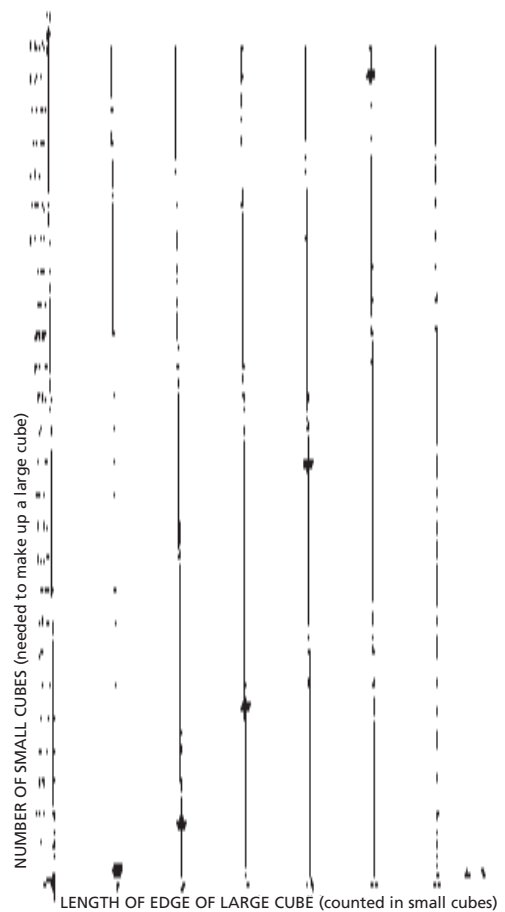
Assessment Task 4

1. Here is a plastic cup.
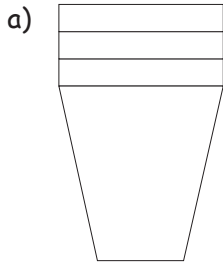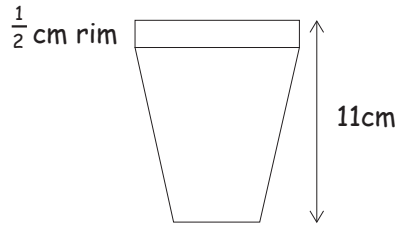
Its total height is 11cm
The rim of the cup is $\frac{1}{2}$ cm high.

$\frac{1}{2}$ cm rim

11cm

a)

When 3 cups are stacked inside each
other, they look like this.

    i)   What is the total height of 7 cups stacked in the same way?

    ii)   The total height of some cups stacked in this way is **17cm**.
        How many cups are there in the stack?

   iii) Write an algebraic formula to calculate the height of x cups stacked in this way.

  b)   The same cups are stacked in the pattern shown here:

    i)   Write an algebraic formula to find the
        height of x cups stacked in this way

    ii)   Find the difference in height
        between x cups stacked as shown in (a) and
        x cups stacked as shown in (b)
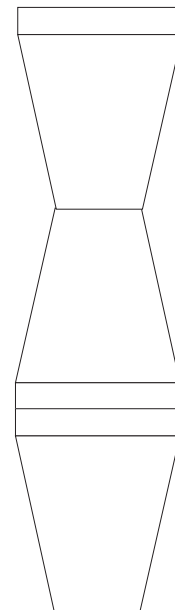
2. Here are two **rough** sketches of rectangles that have a perimeter of 36cm.

  a) Draw another 10 different rectangles (including a square), all of which have
      a perimeter of 36cm. Use 17cm as the longest possible length of a side and
   1cm as the shortest possible length of one side

Perimeter:
distance
around the
edge of
a shape

17cm

1cm

15cm

3cm

Saide
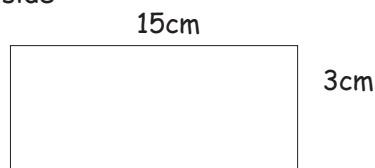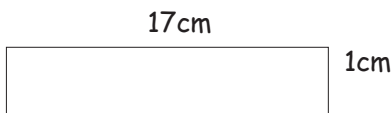
b) Find the area of each of your rectangles.

c) Copy and complete the table below to show the relationship between the length of one of the sides of each rectangle and its area.

x	17		15							
Area	17		45							

d) Plot the points from your table on a graph to show the relationship between the length of x in centimetres, and the area in cm^2. You can join these points because x can have any real value ($x \in R$).
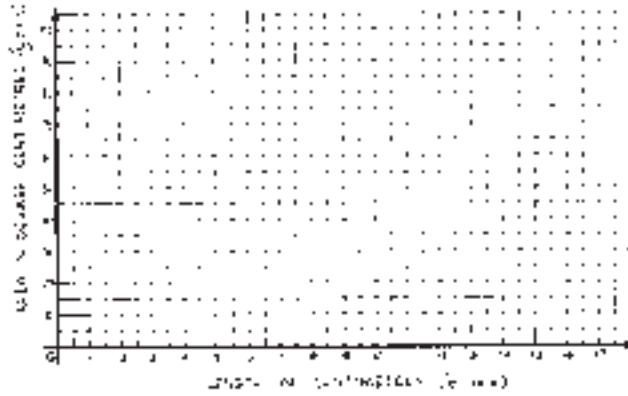
e) Write a sentence to describe how the area of the rectangle changes as x changes.

3. This famous number pattern below is called the Fibonacci sequence.
   To find the next number you add the previous two.

   1   1   2   3   5   8   13
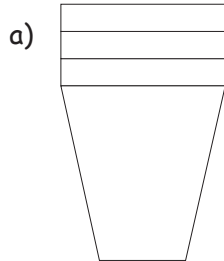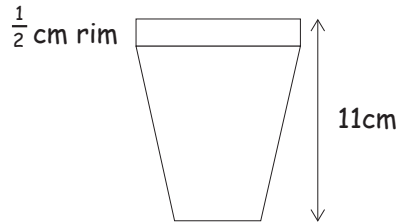
   a) Find the next two numbers in the series.

   b) Here are more Fibonacci numbers. Find the missing two.

   _____   89   144   233   377   _____

   c) Use the **same rule** as the one for the Fibonacci series to find the missing numbers in the following serries.

   i)   _____   2,17;   4,05;   6,22;   _____

   ii)   _____   155 000; $2\frac{1}{2}$ million;   _____

   iii)   _____ x; $x + y$; $2x + y$;   _____

   iv)   _____ $4x^2y$; $6x^2y$;   _____

   v)   _____   5;   -2;   3;   _____

Saide

Assessment Task 4
with answers and some of the working in bold

1. Here is a plastic cup.

   Its total height is 11cm
   The rim of the cup is $\frac{1}{2}$cm high.

   $\frac{1}{2}$ cm rim

   11cm

   a)

   When 3 cups are stacked inside each
   other, they look like this.

   i)   What is the total height of 7 cups stacked in the same way?
        $10\frac{1}{2}$ + (7 x $\frac{1}{2}$) $\Longrightarrow$ $10\frac{1}{2}$ + $3\frac{1}{2}$ = 14cm   or   11 + (6 x $\frac{1}{2}$) $\Longrightarrow$ 11 + 3 = 14cm

   ii) The total height of some cups stacked in this way is **17cm**.
        How many cups are there in the stack?
        13 cups

   iii) Write an algebraic formula to calculate the height of x cups stacked in this way.
        y = $10\frac{1}{2}$ + $\frac{1}{2}x$ or   11 + ($\frac{x-1}{2}$)cm
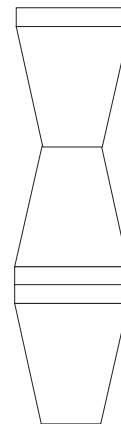
   b) The same cups are stacked in the pattern shown here:

   i)   Write an algebraic formula to find the
   height of x cups stacked in this way
        $11x$

   ii) Find the difference in height
        between x cups stacked as shown in (a) and
        x cups stacked as shown in (b)
        $11x$ - ($10\frac{1}{2}$ + $\frac{1}{2}x$)

2. Here are two **rough** sketches of rectangles that have a perimeter of 36cm.

   a) Draw another 10 different rectangles (including a square), all of which have
      a perimeter of 36cm. Use 17cm as the longest possible length of a side and
      1cm as the shortest possible length of one side.

   17cm

   1cm

   15cm

   3cm
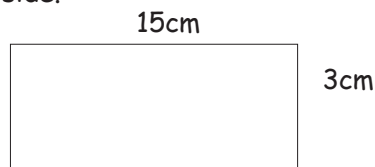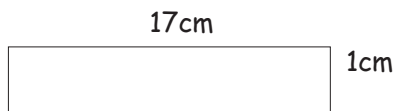
 | **Perimeter:** distance around the edge of a shape |
   |---|

   **There are 17 possible rectangles altogether. We have not drawn them here, but their
   dimensions are shown in the table. Learners will have chosen 10 from this set,
   but we have shown them all so as to include whichever any learner chooses. They
   do not have to show both sides – but we have done so to provide the range of answers
   possible in 2c. All learners should include the rectangle of sides 9 x 9 cm, as this will
   give the square asked for in the question.**

Saide

b) Find the area of each of your rectangles.

c) Copy and complete the table below to show the relationship between the length of one of the sides of each rectangle and its area.

y	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
x	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
Area	17	32	45	56	65	72	77	80	81	80	77	72	65	56	45	32	17

d) Plot the points from your table on a graph to show the relationship between the length of x in centimetres, and the area in cm^2. You can join these points because x can have any real value ($x \in R$).
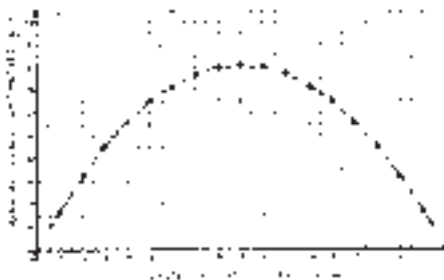
e) Write a sentence to describe how the area of the rectangle changes as x changes.
As x increases from 1 to 9, area increases. As x increases for $x > 9$, the area decreases.

3. This famous number pattern below is called the Fibonacci sequence.
To find the next number you add the previous two.

1   1   2   3   5   8   13

a) Find the next two numbers in the series.   **21   34**

b) Here are more Fibonacci numbers. Find the missing two.

55   89   144   233   377   **610**

c) Use the **same rule** as the one for the Fibonacci series to find the missing numbers in the following serries.

i)   **1,88;**   2,17;   4,05;   6,22;   **10,27**

ii)   **2 345 000;**   155 000; $2\frac{1}{2}$ million;   **2 655 000**

iii) y ; x ; $x+y$; $2x+y$; $3x+2y$

iv) $2x^2y$; $4x^2y$; $6x^2y$; $10x^2y$

v)   **-7;**   5;   -2;   3;   **1**

Notes: