

Solutions Unit Two: Developing Understanding in Mathematics

From the module:

Teaching and Learning Mathematics in Diverse Classrooms



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UNISA (2006). *Learning and teaching of Intermediate and Senior Mathematics (ACE ME1-C)*. Pretoria: UNISA

For permission to use in Unit Two

- UNISA (2006). Study Unit 3: *Learning and Teaching of Intermediate and Senior Phase Mathematics*.
- Penlington, T (2000). *The four basic operations*. ACE Lecture Notes. RUMEP, Rhodes University, Grahamstown.
- RADMASTE Centre, University of the Witwatersrand (2006). *Number Algebra and Pattern (EDUC 264)*.

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How the solutions for the module are presented

Overview of content

As an introduction to each activity notes about the teaching and learning of the mathematical content in the activity are given. These notes are intended to inform both lecturers using the materials in a teacher education context and teachers who may wish to use the materials in their own classes. The solutions to the activities are all given in full. Diagrams are given to provide visual explanations where necessary.

How the solutions unit is structured

The unit consists of the following:

- General points for discussion relating to the teaching of the mathematical content in the activities.
- Step-by-step mathematical solutions to the activities.
- Annotations to the solutions to assist teachers in their understanding the maths as well teaching issues relating to the mathematical content represented in the activities.
- Suggestions of links to alternative activities for the teaching of the mathematical content represented in the activities.

How to find alternative content material

The internet gives you access to a large body of mathematical activities, many of which are available for free downloading if you would like to use them in your classroom. There are different ways of searching the web for material, but a very easy way to do this is to use Google. Type in the address <http://www.google.co.za/> to get to the Google search page. You can then search for documents by typing in the topic you are thinking of in the space provided. You will be given many titles of articles (and so on) which may be appropriate for you to use. You need to open them in order to check which one actually suits your needs. To open an article you click on the title (on the screen) and you will be taken to

the correct web address for that article. You can then check the content and see whether or not it suits your needs. When you do this search you will see that there are many sites which offer worksheets open for use by anyone who would like to use them. You need to check carefully that the material is on the right level for your class and that there are no errors in the text. Anyone can miss a typographical error and web material may not be perfect, but you can easily correct small errors that you find on a worksheet that you download.

You can also use a Google image search, to find images relating to the topic you are thinking of. This usually saves you a lot of time, because you will quickly see which images actually relate to what you are thinking of and which do not. When you click on the image you like (on the screen), this will take you to the full page in which this image is actually found. In this way you can get to the worksheet of your choice. You can then copy and download the material.

Unit Two Solutions: Developing Understanding in Mathematics

Activity 2: Cognitive schema: a network of connections between ideas

According to the theory of constructivism, we construct our own knowledge. This happens essentially in two different ways. The first is that we add new knowledge to our existing knowledge. The second is that we fit the new knowledge into our existing networks of ideas, thereby changing the networks which we already have. All of this happens when we are involved in thinking actively about the new ideas, by wrestling with them, by challenging our ideas - new or old - and those of others. Through reflective thought, we can create an integrated network of connections between ideas. This represents a rich, connected understanding of ideas as opposed to isolated understandings of bits and pieces of theory.



Activity 2

Cognitive schema: a network of connections between ideas.

Select a particular skill (with operations, addition of fractions for example) that you would want your learners to acquire with understanding. Develop a cognitive schema (mental picture) for the newly emerging concept (or rule).

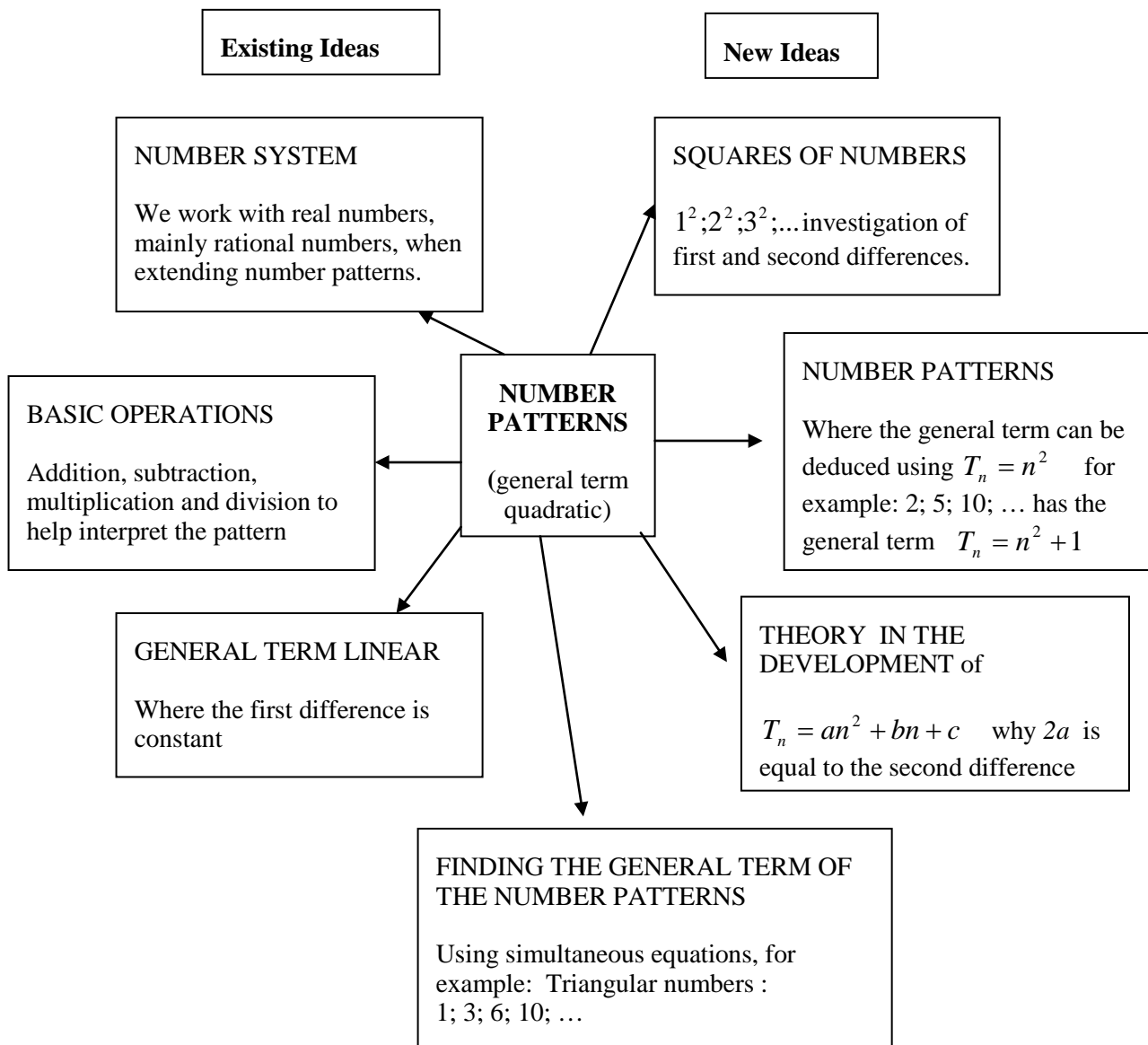
You should consider the following:

- Develop a network of connections between existing ideas (e.g. whole numbers, concept of a fraction, operations etc).
- Add the new idea (addition of fractions for example).

Draw in the connecting lines between the existing ideas and the new ideas used and formed during the acquisition of the skill.

Solutions to Activity 2: Cognitive schema

Below is a mindmap/metacog/web of association which could help to develop the concept of number patterns where the general term is a quadratic expression (Grade 11 level):



There are many number patterns with the second difference constant which can be presented to the learners. We can formulate a variety of questions for the learner once the pattern has been established.

Visual patterns are particularly effective because of their originality and the creative thinking which can be developed when learners spend time interpreting the patterns and converting them into mathematical formulae.

Suggested links for other alternative activities:

- <http://kids.aol.com/homework-help/math> (on line activities, lessons and other material)
- <http://www.davidparker.com/janine/mathpage/patterns.html>
- http://www.aaastudy.com/pat_by5.htm (online interactive lessons and games)

Activity 3: Misconceptions in the addition of fractions

As teachers, we need to help learners throughout the grades to develop the many mathematical concepts we teach correctly. Whether the learner uses assimilation (taking in a new idea) or accommodation (adapting a new idea and/or an existing idea to form a broader concept), depends on the concept being integrated. It is possible that we have all encountered learners who make or who have made the same mistake as Daniel made in the activity below. There are learners at the FET stage (grades 10, 11 and 12) who still make this error because they have not truly understood the concept of adding two fractions.



Activity 3

Daniel, a learner in grade 4, gives the following incorrect response

$$\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$$

- 1 Explain the conceptual error made by the learner
- 2 What mental construct (or idea) needs to be modified by the learner to overcome this misconception? (Think of the addition of whole numbers and so on.)
- 3 Describe a useful constructive activity that Daniel could engage in to remedy the misconception. (He could use drawings, counters etc.)

What kind of process takes place as a result of the modification of Daniel's mental construct: accommodation or assimilation? Explain your answer.

Solutions to Activity 3: Misconceptions in the addition of two fractions

1. Daniel made a conceptual error. He thought he could add the denominators of the two fractions even though they are not the same.
2. The mental construct which needs to be modified by the learner is the concept of the fraction as a part of a whole. The learner needs to fully understand the meaning of a fraction before he attempts to add two fractions. If you compare this to the idea of adding whole numbers, learners are not ready to add whole numbers until their whole number concept is fully developed. The same applies to fraction number concept.
3. To remedy the misconception, the teacher could present Daniel with a 2-dimensional circle cut into two equal sized halves, so that he can work out the sum of two halves by actually doing the process using concrete manipulatives. This can then be extended to circles cut into various fraction parts of equal sizes. From there, Daniel can be encouraged to develop the mental construct required to add two

fractions which have the same denominator, and eventually those which do not have the same denominators.

4. **Accommodation** takes place as a result of the modification of Daniel's mental construct. Daniel knows how to add two whole numbers. We can see this because he correctly adds the whole numbers in the numerators and the denominators. Now that he is learning about adding two fractions, he has to alter his existing way of adding numbers to accommodate the fractions. He needs to learn that you cannot just add the numbers in the numerators and the denominators together because they are now in fractional notation, and have meanings according to that notation. The numerator tells us how many parts (which we do add) but the denominator tells us what kind of part (which will remain unchanged when we add).

Suggested links for other alternative activities:

- <http://www.visualfractions.com/add.htm> (on line and off line activities)
- <http://www.themathpage.com/Arith/add-fractions-subtract-fractions-1.htm> (lessons about adding fractions)
- <http://www.staff.vu.edu.au/mcaonline/units/fractions/fractadd.html> (on line lesson)
- <http://www.aaamath.com/fra57a-addfractld.html> (on line game)

Activity 4: Subtraction using the vertical algorithm

We need to be open to ideas and solutions presented by our learners. There is value in building confidence and self-esteem in learners when they are attempting to solve problems, even if the methods that they use for solving the problem are not necessarily the quickest or the 'best'.

However, if an algorithm is used incorrectly, it needs to be corrected. The calculation and the conceptual errors need to be corrected in the learner's existing knowledge, otherwise the learner will encounter major problems in the future. You as the teacher need to be on the lookout for errors and misconceptions which need to be addressed. These will become clear to you as you look closely at the learners work when you mark it. But even more importantly, you will find out about learners misconceptions in discussion with the learners, when you ask them to explain things that they have written or said.



Activity 4

Subtraction using the vertical algorithm

$$\begin{array}{r}
 5 \quad 13 \\
 603 \\
 - 257 \\
 \hline
 6
 \end{array}$$

There is nothing left in this next column, so I'll borrow from the 6.

- 1 What calculation error did the learner make in subtraction?
- 2 What conceptual error did the learner make? (Think of place-value concepts).
- 3 Was the rule 'borrow from the next column' clearly understood by the learner? Explain your answer.
- 4 In many instances, the learner's existing knowledge is incomplete or inaccurate – so he/she invents an incorrect meaning. Explain the subtraction error in the light of the above statement.

Solutions to Activity 4: Subtraction using the vertical algorithm

1. The calculation error made is that the learner borrowed from the hundred's column and not the ten's column.
2. The conceptual error made is that the learner thinks that zero in the ten's column means there is nothing there. The learner does not realize that the zero has representation in this column as a tens place holder. He/she does not realize that when taking one of the hundreds from the hundreds column, he/she has now got ten tens in the tens column.
3. The rule to “borrow from the next column” was not fully understood by the learner. If he/she understood he/she would not have made the error of borrowing from the hundred's column and not from the ten's when he/she is short of units in the units column. He/she has not realized that if there are no tens in the tens column (such as in this case) he/she does have to go to the hundreds column (which he/she did), but then that gives him/her ten tens in the tens column, from which he/she can borrow one ten to give him/her 13 units in the units column.
4. The learner's existing knowledge is incomplete or inaccurate in this case. He/she is trying to follow the subtraction algorithm without fully understanding it.


Suggested links for other alternative activities:

- <http://www.aaamath.com/fra57a-addfractld.html> (lesson ideas)

- <http://www.tullyschools.org/tfiles/folder204/Everyday%20Math%20Algorithms.pdf> (alternative algorithms)
- http://math.about.com/od/addingsubtracting/ss/3digsubre_7.htm (worksheets)
- http://www.softschools.com/math/games/subtraction_practice.jsp (online games)
- http://www.softschools.com/math/worksheets/subtraction_worksheets.jsp (worksheets that you can generate)

Activity 10: Procedures

Procedures and symbols should have a connection to concepts. However, the knowledge of a procedure can be learned without the understanding of a related concept. Procedures can be learnt through drill and practice without deep understanding of the basic concepts. What we as teachers need to remember is that these procedures which are developed without meaningful understanding can be easily forgotten or abused.

 <p>Activity 10</p>	<p>Procedures</p> <p>Reflect on the following example of a procedure:</p> <p>To add two three-digit numbers, first add the numbers in the right-hand column. If the answer is 10 or more, put the 1 above the second column, and write the other digit under the first column. Proceed in a similar manner for the second two columns in order.</p> <ul style="list-style-type: none"> ▪ Use an appropriate example to test the above procedure (or recipe) for the addition of two three-digit numbers. ▪ Give another example for a procedure for the purpose of computation. Describe the step-by-step procedure operative in the calculation.
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Solutions to Activity 10: Procedures

Let us test the procedure in this activity of adding two three-digit numbers:

$$\begin{array}{r}
 1 \\
 659 \\
 + 327 \\
 \hline
 986
 \end{array}$$

This procedure works. The learner may have no clue about the concept of place value (units, tens, hundreds, and so on) but she may manage simply by knowing the procedure and therefore correctly implementing it.

Another example of a procedure may be taken from the Grade 10 Assessment Standard for the adding of algebraic fractions.

The procedure is:

1. Find the Lowest Common Denominator.
2. Write it down with a long line above it where the numerator will go.
3. Multiply each number by the same factor as you did the denominator.
4. Simplify.

So, let's test the above procedure:

$$\begin{aligned} & \frac{3}{m-1} + \frac{5}{m+2} \\ &= \frac{3(m+2) + 5(m-1)}{(m-1)(m+2)} \\ &= \frac{3m + 6 + 5m - 5}{(m-1)(m+2)} \\ &= \frac{8m + 1}{(m-1)(m+2)} \end{aligned}$$

This procedure can be done through drill and practice. But it is far more likely that the learner will remember how to add algebraic fractions if he/she has grasped the basic concept of adding numeric fractions and then that of algebraic fractions. It is also much more meaningful and enjoyable to understand why a certain procedure is done! The development of conceptual understanding must be key in our teaching of methods and procedures throughout the grades.

Suggested links for other alternative activities:

- http://www.softschools.com/math/games/subtraction_practice.jsp (on line games)
- http://www.softschools.com/math/worksheets/subtraction_worksheets.jsp (worksheets that you can generate)
- http://www.teachervision.fen.com/tv/printables/botr/botr_111_26-27.pdf (worksheet with solutions)
- http://www.allacademic.com/meta/p_mla_apa_research_citation/1/1/7/5/5/p117552_index.html (academic article on regrouping and the difficulties learners experience using the addition algorithm)
- <http://www.mrnussbaum.com/dragadd5.htm> (online activity)

Activity 12: Thinking about mathematical models

Models can be used to help formulate mathematical ideas and they can be very useful in trying to help our learners to construct ideas and to understand concepts. But we should not use models uncritically in the teaching of mathematics because we need to develop our learners' abilities to form their own ideas as concepts, and these concepts will ultimately not be attached to any real object.



Activity 12

Models

- 1 You may talk of 100 people, 100 rand or 100 acts of kindness. Reflect on the above statement and then explain what is meant by the concept of 100. Discuss this concept of 100 with fellow colleagues. If you do not agree, establish why there is a difference of opinion in your understanding.
- 2 Explain what a 'model' for a mathematical concept refers to. Provide an example.
- 3 List some models (apparatus/manipulatives) that you have used in your mathematics teaching. Indicate in each case how you have used the particular model mentioned.
- 4 Why is it not correct to say that a model illustrates a concept? Explain your answer.

Solutions to Activity 12: Models

1. We may talk of 100 people, 100 rand or 100 acts of kindness. These are ideas for different "models" of the number 100. The number is constant (unchanging) - the models vary - they provide different ways of visualizing 100 things. If there are differences in opinion about the concept 100, we need to identify whether these differences indicate a lack of understanding of the number concept (which would be serious!) or simply a lack of understanding that there are different ways of representing the number 100.
2. Models are generally physical. They can be pictures, written symbols, oral language or real-world situations. A 'model' for a mathematical concept refers to the concrete or physical object which is used to help in the forming of ideas and in the understanding of the concept. An example of a model may be a cake cut into slices of equal size in the introduction of the fraction concept.
3. Some examples of models:
 - the abacus for counting
 - models of 3-D drawings to solve problems in trigonometry in Grade 12.
 - 3-D cardboard models of cylinders and other shapes so that the learners can deduce the surface area and volume of these shapes.

4. The idea that a model does not illustrate a concept is subtle but important. This is because concepts are ideas that are formed in our heads - models can help us to form these concepts. We can say that models embody a concept - they have the characteristics of the concept, and lead us towards the understanding or formulation of the concept.

Suggested links for other alternative activities:

- http://www.curriculum.org/csc/library/strategies/downloads/Managing_DI.doc (manipulatives and how to use them)
- http://images.google.co.za/imgres?imgurl=https://fcweb.limestone.on.ca/~kirkc/S03C9E5B3.7/pizza%2520fractions.jpg&imgrefurl=https://fcweb.limestone.on.ca/~kirkc/%3FOpenItemURL%3DS03C9E5B3&usq=__VzIMVVF5cno4BbthbDfAp0B385M=&h=438&w=400&sz=32&hl=en&start=3&tbnid=zRyBCLPoJFLarM:&tbnh=127&tbnw=116&prev=/images%3Fq%3Dmodels%2Bof%2Bfractions%26gbv%3D2%26hl%3Den (fractions models)
- http://images.google.co.za/imgres?imgurl=http://i.infoplease.com/images/tv/printables/scottforesman/Math_3_TTT_17.gif&imgrefurl=http://www.teachervision.fen.com/graphs-and-charts/graphic-organizers/44648.html&usq=__JQjwYUVLwKtd5V54xjedd_4UKYI=&h=209&w=160&sz=9&hl=en&start=16&tbnid=rkeSC4LB2w9kgM:&tbnh=106&tbnw=81&prev=/images%3Fq%3Dmodels%2Bof%2Bplace%2Bvalue%26gbv%3D2%26hl%3Den%26sa%3DG (place value models)
- <http://www.ics.uci.edu/~eppstein/junkyard/model.html> (incredible models of geometric (and other) shapes)
- <http://smasd.k12.pa.us/pssa/html/Math/manip.htm> (using models)
- <https://www.summitlearning.com/product/DG206079TS> (place value dice - an advert but look at the interesting product)

Activity 14: Using Diennes' blocks to explain grouping in tens up to 1 000

A knowledge and understanding of our numeration system is part of a learner's fundamental mathematical knowledge. Place value must be fully understood by learners. The Diennes' blocks are an excellent tool in the development of the concept of place value.



Activity 14

Using Diennes' blocks to explain grouping in tens up to 1 000

Establishing a very firm understanding of the place values up to 1 000 lays an excellent foundation for further understanding of place value. Activities with Diennes' blocks can be useful in this regard.

- 1 You could work with Diennes' blocks in the following type of exercise, to demonstrate the relationship between units in different places. Complete the following:
 - 60 tinies can be exchanged for _____ longs, so 60 units = _____ tens.
 - 480 tinies can be exchanged for _____ longs, so 480 units = _____ tens.
 - 40 longs can be exchanged for _____ flats, so 40 tens = _____ hundreds.
 - 500 longs can be exchanged for _____ flats, so 500 tens = _____ hundreds.
 - 33 longs can be exchanged for _____ tinies, so 33 tens = _____ units.
 - 83 flats can be exchanged for _____ tinies, so 83 hundreds = _____ units.
 - 765 tinies can be exchanged for _____ tinies, _____ longs, and _____ flats, and so 765 units = _____ units, _____ tens, and _____ hundreds.
 - 299 tinies can be exchanged for _____ tinies, _____ longs, and _____ flats, and so 299 units = _____ units, _____ tens, and _____ hundreds.
- 2 In what way do the Diennes' blocks clarify the ideas of face value, place value and total value? Explain your answer using an example.
- 3 Reflect on each of the concepts and the corresponding model.
- 4 Separate the physical model from the relationship embedded in the model and then explain, in each case, the relationship that you need to impose on the model in order to 'see' the concept.

Solutions to Activity 14: Using Diennes' blocks to explain grouping in tens up to 1000

1. You could work with Diennes' blocks in the following type of exercise, to demonstrate the relationship between units in different places. Complete the following:
 - 60 tinies can be exchanged for 6 longs, so 60 units = 6 tens.
 - 480 tinies can be exchanged for 48 longs, so 480 units = 48 tens.

- 40 longs can be exchanged for 4 flats, so 40 tens = 4 hundreds.
 - 500 longs can be exchanged for 50 flats, so 500 tens = 50 hundreds.
 - 33 longs can be exchanged for 330 tinies, so 33 tens = 330 units.
 - 83 flats can be exchanged for 8300 tinies, so 83 hundreds = 8300 units.
 - 765 tinies can be exchanged for 5 flats, 6 longs, and 7 hundreds, and so 765 units = 5 hundreds, 6 tens, and 7 units.
 - 299 tinies can be exchanged for 9 flats, 9 longs, and 2 hundreds, and so 299 units = 9 tens, 9 units, and 2 hundreds.
2. The Diennes' blocks clarify the ideas of face value, place value and total value because each different size block represents each place value. So the tinies represent the units, the longs the tens and the flats, the hundreds. For example, the number 567 will be represented by 5 flats, 6 longs and 7 tinies.
 3. A clever relationship is created with the Diennes' blocks. Units are perceived as small so they are represented by the tinies. Longs are bigger - so they represent the tens. And then the flats represent the hundreds. This is a concrete model of helping to formulate the concept place value (related to the "size" of the blocks), face value (related to "how many" of each of the blocks you see) and total value (worked out using the "size" (place value) of the block and "how many" (face value) of them there are).
 4. If we separate the physical model from the relationship embedded in the model, then all we have are three groups of blocks of different sizes. The relationship which we need to impose on the model in this case is that of matching units to tinies, tens to longs and hundreds to flats. This imposed relationship can easily be changed by calling the flat a unit. Then we have a model for units (flats), tenths (longs) and hundredths (tinies). The fact that we can change the model to suit our needs shows that the model is not (and never will be) the concept itself. Understanding of the concept becomes part of understanding why the model is effective in demonstrating the concept. It is in coming to this understanding of how the model embodies the concept that the learner forms his/her understanding of the concept involved.

Suggested links for other alternative activities:

- http://images.google.co.za/imgres?imgurl=http://www.innovativeed.com/images/0924_0926.jpg&imgrefurl=http://www.innovativeed.com/baseten.htm&usq=_n-

[NQ_cwuqf02NuhvWbfed2GbA6o=&h=306&w=400&sz=36&hl=en&start=20&tbnid=WLRU33SYOO7BEM:&tbnh=95&tbnw=124&prev=/images%3Fq%3Dmodels%2Bof%2Bplace%2Bvalue%26gbv%3D2%26hl%3Den%26sa%3DG](http://www.abcteach.com/directory/clip_art/math/place_value_blocks/) (Diennes' s blocks – place value)

- http://www.abcteach.com/directory/clip_art/math/place_value_blocks/ (more on Diennes' blocks)
- <http://images.google.co.za/imgres?imgurl=http://mason.gmu.edu/~mmankus/Handson/b10blocks.gif&imgrefurl=http://mason.gmu.edu/~mmankus/Handson/b10blocks.htm&usq=XqVdAnj8OiCDYBS6MbTNTlor89I=&h=759&w=584&sz=9&hl=en&start=3&tbnid=6PSpz4TE9XExjM:&tbnh=142&tbnw=109&prev=/images%3Fq%3Dbase%2Bten%2Bblocks%26gbv%3D2%26hl%3Den> (base 10 blocks to cut out)

Activity 15: Using an abacus to think about place value

An abacus is another useful tool which can be used in the teaching of the number concept. It can be useful for very early counting activities as well as in the teaching of bigger numbers. It is also a very good model for helping to understand place value in our number system.

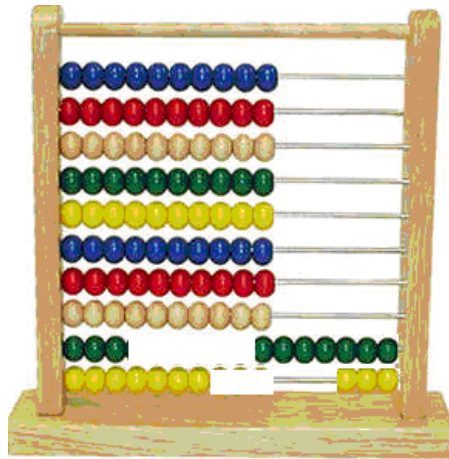


Activity 15

- 1 An abacus can be used to count and display numbers. If you use an abacus to count up to 37 (starting from one), which of the properties of our numeration system will this reveal?
- 2 If you display the number 752 on an abacus, which of the properties of our numeration system does this reveal?
- 3 Illustrate the following numbers on the abacus, and then write out the number in expanded notation.
 - 3
 - 68
 - 502
 - 594
- 4 In what way does an abacus clarify the ideas of face value, place value and total value?
- 5 Engage your learners in some of the examples given above. Reflect on whether they are able to separate the physical model from the concept.

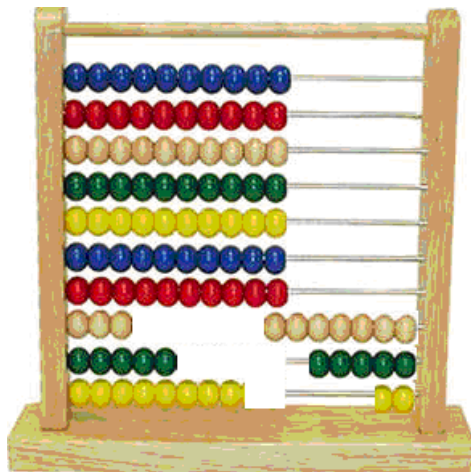
Solutions to Activity 15: Using an abacus to think about place value

1. If we use an abacus to count up to 37 ie:



The properties of our numeration system which are revealed is place value - those of tens (in this case 3) and units (in this case 7). This is because we see three beads pushed aside in the units place (first row of beads) and we see seven beads pushed aside in the tens place (second row of beads).

2. 752 displayed on an abacus reveals the properties of hundreds (7), tens (5) and units (2) - place value. This is because we see two beads pushed aside in the units place (first row of beads) and we see five beads pushed aside in the tens place (second row of beads) and we see five beads pushed aside in the hundreds place (third row of beads).



3. You should try to get hold of an abacus to use in the teaching of place value. Maybe there is one in a storeroom somewhere at school. It is also something that you can make using wood, wire and beads, quite easily, if you put your mind to it. The rows of beads represent the place value, the number of beads pushed aside represent the face value, and the person looking at the abacus can work out the total value represented by using the place value and face value they see on the abacus.

Here is each number in expanded notation:

$$3 = (3 \times 1)$$

$$68 = (6 \times 10) + (8 \times 1)$$

$$502 = (5 \times 100) + (2 \times 1)$$

$$594 = (5 \times 100) + (9 \times 10) + (4 \times 1)$$

4. An abacus does clarify the ideas of face value, place value and total value. Each row of beads represents an increasing position in our place value system: units, then tens, then hundreds, then thousands and so on. We start from the bottom with the units. The Diennes' blocks uses sizes to represent the place values, while the abacus uses the level of the rows for place value. As long as one how to use the abacus, the ideas will be clarified. The Diennes' blocks are limited to the sizes of blocks that we have, but an abacus such as the one above can go up to the thousand millions place.
5. The learners should be able to separate the physical model from the concept. Once they have learnt to use the abacus effectively, questions and problems should be posed to them to see whether or not they have developed the concepts of place value, face value and total value.

Suggested links for other alternative activities:

- <http://www.ictgames.com/abacusInteger.html> (abacus online)
- <http://www.galileo.org/math/puzzles/FactorialAbacus.htm> (multi base abacus problems)

Activity 16: Place value

The use of Flard cards is another effective way of helping the learners to formulate and understanding of the concepts of face value, place value and total value. Read again about Flard cards on page 49 and 50 of Unit Two of the SAIDE ACEMaths materials if you don't know or have forgotten what Flard cards are.



Activity 16

- 1 In the number 10 212 the 2 on the left is _____ times the 2 on the right.
- 2 In the number 10 212 the 1 on the left is _____ times the 1 on the right.
- 3 In the number 80 777 the 7 on the far left is _____ times the 7 immediately to the right of it.
- 4 In the number 80 777 the 7 on the far left is _____ times the 7 on the far right.
- 5 In the number 566 the 6 on the right is _____ times the 6 on the left.
- 6 In the number 202 the 2 on the right is _____ times the 2 on the left.
- 7 In the number 1 011 the 1 on the far right is _____ times the 1 on the far left.
- 8 In the number 387, the face values of the digits are _____, _____ and _____; the place value of the digits (from left to right) are _____, _____ and _____; and the total values represented by the digits (from left to right) are _____, _____ and _____.

Solutions to Activity 16: Place value

- 1 In the number 10 212 the 2 on the left is 100 times the 2 on the right.
- 2 In the number 10 212 the 1 on the left is 1000 times the 1 on the right.
- 3 In the number 80 777 the 7 on the far left is 10 times the 7 immediately to the right of it.
- 4 In the number 80 777 the 7 on the far left is 100 times the 7 on the far right.
- 5 In the number 566 the 6 on the right is $\frac{1}{10}$ times the 6 on the left.
- 6 In the number 202 the 2 on the right is $\frac{1}{100}$ times the 2 on the left.
- 7 In the number 1 011 the 1 on the far right is $\frac{1}{1000}$ times the 1 on the far left.
- 8 In the number 387, the face values of the digits are 3, 8 and 7; the place value of the digits (from left to right) are 100's, 10's and 1's; and the total values represented by the digits (from left to right) are 300, 80 and 7.

Suggested links for other alternative activities:

- <http://www.enasco.com/product/TB22978T> (an advert but an interesting number “flip stand” worth a look)

Unit mathematical content summary

Summary of content covered in this unit:

- Activity 2 involved mathematical concepts relating to the web of ideas connected to mathematical number patterns.
- Activity 3 involved mathematical concepts relating to addition of fractions.
- Activity 4 involved mathematical concepts relating to subtraction using the vertical algorithm.
- Activity 10 involved mathematical concepts relating to addition, particularly the procedures involved in addition of big numbers.
- Activity 12 involved thinking about using models as aids to the understanding of mathematical concepts.
- Activity 14 involved mathematical concepts relating to place value, particularly the use of Diennes' blocks in the teaching of place value.
- Activity 15 involved mathematical concepts relating to place value, particularly the use of an abacus in the teaching of place value.
- Activity 16 involved mathematical concepts relating to place value, particularly a comparison of the values of digits according to the place they are found in large numbers.